Kinematics (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Differentiate / integrate → evaluate
78 problems

Lesson `apc.kinematics.session`

$v = dx/dt$ **at** $t = 2$

K1

A particle has $x(t) = 2t^2 + 3t$ (m). Find its velocity at $t = 2$ s.

$$v(t) = \frac{dx}{dt} = 4t + 3 \qquad \text{differentiate}$$

$$v(2) = 11 \text{ m/s} \qquad \text{evaluate}$$

Answer

11 m/s

$v = dx/dt$ **at** $t = 1$

K1

A particle has $x(t) = -5t^2 + 2$ (m). Find its velocity at $t = 1$ s.

$$v(t) = \frac{dx}{dt} = -10t \qquad \text{differentiate}$$

$$v(1) = -10 \text{ m/s} \qquad \text{evaluate}$$

Answer

-10 m/s

$v = dx/dt$ **at** $t = 2$

K1

A particle has $x(t) = t^3 + 4t$ (m). Find its velocity at $t = 2$ s.

$$v(t) = \frac{dx}{dt} = 3t^2 + 4 \qquad \text{differentiate}$$

$$v(2) = 16 \text{ m/s} \qquad \text{evaluate}$$

Answer

16 m/s

$v = dx/dt$ **at** $t = 3$

K1

A particle has $x(t) = 3t^2 - 2t + 1$ (m). Find its velocity at $t = 3$ s.

$$v(t) = \frac{dx}{dt} = 6t - 2 \qquad \text{differentiate}$$

$$v(3) = 16 \text{ m/s} \qquad \text{evaluate}$$

Answer

16 m/s

$v = dx/dt$ **at** $t = 1$

K1

A particle has $x(t) = 6t^2$ (m). Find its velocity at $t = 1$ s.

$$v(t) = \frac{dx}{dt} = 12t \qquad \text{differentiate}$$

$$v(1) = 12 \text{ m/s} \qquad \text{evaluate}$$

Answer

12 m/s

$v = dx/dt$ **at** $t = 2$

K1

A particle has $x(t) = -2t^2 + t + 5$ (m). Find its velocity at $t = 2$ s.

$$v(t) = \frac{dx}{dt} = -4t + 1 \qquad \text{differentiate}$$

$$v(2) = -7 \text{ m/s} \qquad \text{evaluate}$$

Answer

-7 m/s

$v = dx/dt$ **at** $t = 1$

K1

A particle has $x(t) = -4t^2 + 8t$ (m). Find its velocity at $t = 1$ s.

$$v(t) = \frac{dx}{dt} = -8t + 8 \qquad \text{differentiate}$$

$$v(1) = 0 \text{ m/s} \qquad \text{evaluate}$$

Answer

0 m/s

$v = dx/dt$ **at** $t = 1$

K1

A particle has $x(t) = 2t^3 + 3$ (m). Find its velocity at $t = 1$ s.

$$v(t) = \frac{dx}{dt} = 6t^2 \qquad \text{differentiate}$$

$$v(1) = 6 \text{ m/s} \qquad \text{evaluate}$$

Answer

6 m/s

Average velocity, $[1, 3]$

K2

For $x(t) = 2t^2 + 3t$ (m), find the average velocity over $t \in [1, 3]$ s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(3) - x(1)}{3 - 1}$$

displacement / time

$$\bar{v} = \frac{22}{2} = 11 \text{ m/s}$$

Answer

11 m/s

Average velocity, $[0, 2]$

K2

For $x(t) = -5t^2 + 2$ (m), find the average velocity over $t \in [0, 2]$ s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(2) - x(0)}{2 - 0}$$

displacement / time

$$\bar{v} = \frac{-20}{2} = -10 \text{ m/s}$$

Answer

-10 m/s

Average velocity, $[1, 2]$

K2

For $x(t) = t^3 + 4t$ (m), find the average velocity over $t \in [1, 2]$ s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(2) - x(1)}{2 - 1}$$

displacement / time

$$\bar{v} = \frac{11}{1} = 11 \text{ m/s}$$

Answer

11 m/s

Average velocity, $[0, 2]$

K2

For $x(t) = 3t^2 - 2t + 1$ (m), find the average velocity over $t \in [0, 2]$ s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(2) - x(0)}{2 - 0} \qquad \text{displacement / time}$$

$$\bar{v} = \frac{8}{2} = 4 \text{ m/s}$$

Answer

4 m/s

Average velocity, $[2, 4]$

K2

For $x(t) = 6t^2$ (m), find the average velocity over $t \in [2, 4]$ s.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(4) - x(2)}{4 - 2}$$

displacement / time

$$\bar{v} = \frac{72}{2} = 36 \text{ m/s}$$

Answer

36 m/s

$a = dv/dt$ **at** $t = 2$

K3

A particle has $x(t) = 2t^2 + 3t$ (m). Find its acceleration at $t = 2$ s.

$$
\begin{aligned}
v(t) &= 4t + 3 && \text{first derivative} \\
a(t) &= 4 && \text{second derivative} \\
a(2) &= 4\ \text{m/s}^2
\end{aligned}
$$

Answer

$4\ \text{m/s}^2$

$a = dv/dt$ **at** $t = 3$

K3

A particle has $x(t) = t^3$ (m). Find its acceleration at $t = 3$ s.

$$v(t) = 3t^2 \qquad \text{first derivative}$$

$$a(t) = 6t \qquad \text{second derivative}$$

$$a(3) = 18\ \text{m/s}^2$$

Answer

$18\ \text{m/s}^2$

$a = dv/dt$ **at** $t = 1$

K3

A particle has $x(t) = t^3 - 3t^2 + 2t + 1$ (m). Find its acceleration at $t = 1$ s.

$$
\begin{aligned}
v(t) &= 3t^2 - 6t + 2 && \text{first derivative} \\
a(t) &= 6t - 6 && \text{second derivative} \\
a(1) &= 0 \text{ m/s}^2
\end{aligned}
$$

Answer

0 m/s^2

$a = dv/dt$ **at** $t = 2$

K3

A particle has $x(t) = -t^3 + 5t$ (m). Find its acceleration at $t = 2$ s.

$$
\begin{aligned}
v(t) &= -3t^2 + 5 && \text{first derivative} \\
a(t) &= -6t && \text{second derivative} \\
a(2) &= -12 \text{ m/s}^2
\end{aligned}
$$

Answer

-12 m/s^2

$a = dv/dt$ **at** $t = 1$

K3

A particle has $x(t) = 2t^3 + 4t^2$ (m). Find its acceleration at $t = 1$ s.

$$
\begin{aligned}
v(t) &= 6t^2 + 8t && \text{first derivative} \\
a(t) &= 12t + 8 && \text{second derivative} \\
a(1) &= 20\ \text{m/s}^2
\end{aligned}
$$

Answer

$20\ \text{m/s}^2$

Tangent slope at $t = 2$

K4

On the graph of $x(t) = 3t^2$, what is the slope of the tangent line at $t = 2$ s?

$$\text{slope} = \frac{dx}{dt} = 6t \qquad \text{tangent slope} = \text{velocity}$$

$$\text{at } t = 2 = 12 \text{ m/s}$$

Answer

12 m/s

Tangent slope at $t = 3$

K4

On the graph of $x(t) = -t^2 + 4t + 1$, what is the slope of the tangent line at $t = 3$ s?

$$\text{slope} = \frac{dx}{dt} = -2t + 4 \qquad \text{tangent slope} = \text{velocity}$$

$$\text{at } t = 3 = -2 \text{ m/s}$$

Answer

-2 m/s

Tangent slope at $t = 1$

K4

On the graph of $x(t) = t^3 + 2t$, what is the slope of the tangent line at $t = 1$ s?

$$\text{slope} = \frac{dx}{dt} = 3t^2 + 2 \qquad \text{tangent slope} = \text{velocity}$$

$$\text{at } t = 1 = 5 \text{ m/s}$$

Answer

5 m/s

Tangent slope at $t = 2$

K4

On the graph of $x(t) = 2t^2 - 3t + 2$, what is the slope of the tangent line at $t = 2$ s?

$$\text{slope} = \frac{dx}{dt} = 4t - 3 \qquad \text{tangent slope} = \text{velocity}$$

$$\text{at } t = 2 = 5 \text{ m/s}$$

Answer

5 m/s

v from $a(t)$ at $t = 3$

K5

A particle has $a(t) = 6$ (m/s^2) and $v_0 = 0$ m/s. Find v at $t = 3$ s.

$$v(t) = v_0 + \int_0^t a\,dt' = 6t \qquad \text{integrate acceleration}$$
$$v(3) = 18 \text{ m/s}$$

Answer

18 m/s

v from $a(t)$ at $t = 2$

K5

A particle has $a(t) = 4t$ (m/s^2) and $v_0 = 2$ m/s. Find v at $t = 2$ s.

$$v(t) = v_0 + \int_0^t a\,dt' = 2t^2 + 2 \qquad \text{integrate acceleration}$$
$$v(2) = 10 \text{ m/s}$$

Answer

10 m/s

v from $a(t)$ at $t = 2$

K5

A particle has $a(t) = 6t + 2$ (m/s^2) and $v_0 = 1$ m/s. Find v at $t = 2$ s.

$$v(t) = v_0 + \int_0^t a\,dt' = 3t^2 + 2t + 1 \qquad \text{integrate acceleration}$$

$$v(2) = 17 \text{ m/s}$$

Answer

17 m/s

v from $a(t)$ at $t = 2$

K5

A particle has $a(t) = 3t^2$ (m/s^2) and $v_0 = 0$ m/s. Find v at $t = 2$ s.

$$v(t) = v_0 + \int_0^t a\,dt' = t^3 \qquad \text{integrate acceleration}$$
$$v(2) = 8 \text{ m/s}$$

Answer

8 m/s

v from $a(t)$ at $t = 3$

K5

A particle has $a(t) = -2t + 4$ (m/s^2) and $v_0 = 5$ m/s. Find v at $t = 3$ s.

$$v(t) = v_0 + \int_0^t a\,dt' = -t^2 + 4t + 5 \qquad \text{integrate acceleration}$$

$$v(3) = 8 \text{ m/s}$$

Answer

8 m/s

x from $v(t)$ at $t = 2$

K6

A particle has $v(t) = 6t$ (m/s) and $x_0 = 0$ m. Find its position at $t = 2$ s.

$$x(t) = x_0 + \int_0^t v\,dt' = 3t^2 \qquad \text{integrate velocity}$$
$$x(2) = 12 \text{ m}$$

Answer

12 m

x from $v(t)$ at $t = 2$

K6

A particle has $v(t) = 2t + 3$ (m/s) and $x_0 = 1$ m. Find its position at $t = 2$ s.

$$x(t) = x_0 + \int_0^t v\,dt' = t^2 + 3t + 1 \qquad \text{integrate velocity}$$

$$x(2) = 11 \text{ m}$$

Answer

11 m

x from $v(t)$ at $t = 2$

K6

A particle has $v(t) = 3t^2$ (m/s) and $x_0 = 0$ m. Find its position at $t = 2$ s.

$$x(t) = x_0 + \int_0^t v\,dt' = t^3 \qquad \text{integrate velocity}$$

$$x(2) = 8 \text{ m}$$

Answer

8 m

x from $v(t)$ at $t = 3$

K6

A particle has $v(t) = -2t + 4$ (m/s) and $x_0 = 0$ m. Find its position at $t = 3$ s.

$$x(t) = x_0 + \int_0^t v\,dt' = -t^2 + 4t \qquad \text{integrate velocity}$$
$$x(3) = 3 \text{ m}$$

Answer

3 m

x from $v(t)$ at $t = 1$

K6

A particle has $v(t) = 4t + 2$ (m/s) and $x_0 = 5$ m. Find its position at $t = 1$ s.

$$x(t) = x_0 + \int_0^t v\,dt' = 2t^2 + 2t + 5 \qquad \text{integrate velocity}$$

$$x(1) = 9 \text{ m}$$

Answer

9 m

Constant a: $v_0 = 0$, $a = 3.0$

With constant $a = 3.0$ m/s^2 and $v_0 = 0$ m/s, integrate to find v and x at $t = 5$ s.

$$v = v_0 + at = 0 + (3.0)(5) = 15.0 \text{ m/s} \quad \text{integrate } a$$

$$x = v_0 t + \tfrac{1}{2} a t^2 = 37.5 \text{ m} \quad \text{integrate } v$$

Answer

$v = 15.0$ m/s, $x = 37.5$ m

Constant a: $v_0 = 10$, $a = 2.0$

With constant $a = 2.0$ m/s^2 and $v_0 = 10$ m/s, integrate to find v and x at $t = 4$ s.

$$v = v_0 + at = 10 + (2.0)(4) = 18.0 \text{ m/s} \quad \text{integrate } a$$

$$x = v_0 t + \tfrac{1}{2} a t^2 = 56.0 \text{ m} \quad \text{integrate } v$$

Answer

$v = 18.0$ m/s, $x = 56.0$ m

Constant a: $v_0 = 20$, $a = -4.0$

With constant $a = -4.0$ m/s^2 and $v_0 = 20$ m/s, integrate to find v and x at $t = 3$ s.

$$v = v_0 + at = 20 + (-4.0)(3) = 8.0 \text{ m/s} \quad \text{integrate } a$$

$$x = v_0 t + \tfrac{1}{2} a t^2 = 42.0 \text{ m} \quad \text{integrate } v$$

Answer

$v = 8.0$ m/s, $x = 42.0$ m

Constant a: $v_0 = 5$, $a = 6.0$

With constant $a = 6.0$ m/s^2 and $v_0 = 5$ m/s, integrate to find v and x at $t = 2$ s.

$$v = v_0 + at = 5 + (6.0)(2) = 17.0 \text{ m/s} \quad \text{integrate } a$$

$$x = v_0 t + \tfrac{1}{2} a t^2 = 22.0 \text{ m} \quad \text{integrate } v$$

Answer

$v = 17.0$ m/s, $x = 22.0$ m

Constant a: $v_0 = 0$, $a = 9.8$

With constant $a = 9.8$ m/s^2 and $v_0 = 0$ m/s, integrate to find v and x at $t = 3$ s.

$$v = v_0 + at = 0 + (9.8)(3) = 29.4 \text{ m/s} \quad \text{integrate } a$$

$$x = v_0 t + \tfrac{1}{2} a t^2 = 44.1 \text{ m} \quad \text{integrate } v$$

Answer

$$v = 29.4 \text{ m/s}, \quad x = 44.1 \text{ m}$$

Jerk-limited $a = 6t$ at $t = 2$

K8

A particle starts with $v_0 = 0$ m/s and has $a(t) = 6t$ (m/s^2). Find v and x at $t = 2$ s ($x_0 = 0$).

$$\begin{aligned} v(t) &= v_0 + \int_0^t bt'\,dt' = 3t^2 && \tfrac{1}{2}bt^2 \text{ term} \\ x(t) &= \int_0^t v\,dt' = t^3 && \text{integrate again} \\ v, x\ (t{=}2) &= \text{12 m/s, 8 m} \end{aligned}$$

Answer

v = 12 m/s, x = 8 m

Jerk-limited $a = 4t$ at $t = 3$

K8

A particle starts with $v_0 = 2$ m/s and has $a(t) = 4t$ (m/s^2). Find v and x at $t = 3$ s ($x_0 = 0$).

$$\begin{aligned} v(t) &= v_0 + \int_0^t bt'\,dt' = 2t^2 + 2 && \tfrac{1}{2}bt^2 \text{ term} \\ x(t) &= \int_0^t v\,dt' = \tfrac{2}{3}t^3 + 2t && \text{integrate again} \\ v, x\ (t{=}3) &= \text{20 m/s, 24 m} \end{aligned}$$

Answer

v = 20 m/s, x = 24 m

Jerk-limited $a = 2t$ at $t = 2$

A particle starts with $v_0 = 5$ m/s and has $a(t) = 2t$ (m/s^2). Find v and x at $t = 2$ s ($x_0 = 0$).

$$
\begin{aligned}
v(t) &= v_0 + \int_0^t b t'\,dt' = t^2 + 5 && \tfrac{1}{2}bt^2 \text{ term} \\
x(t) &= \int_0^t v\,dt' = \tfrac{1}{3}t^3 + 5t && \text{integrate again} \\
v, x\ (t{=}2) &= 9 \text{ m/s},\ 12.67 \text{ m}
\end{aligned}
$$

Answer

$v = 9$ m/s, $x = 12.67$ m

Jerk-limited $a = 12t$ at $t = 2$

K8

A particle starts with $v_0 = 0$ m/s and has $a(t) = 12t$ (m/s^2). Find v and x at $t = 2$ s ($x_0 = 0$).

$$\begin{aligned} v(t) &= v_0 + \int_0^t bt'\,dt' = 6t^2 && \tfrac{1}{2}bt^2 \text{ term} \\ x(t) &= \int_0^t v\,dt' = 2t^3 && \text{integrate again} \\ v, x\ (t{=}2) &= 24\ \text{m/s},\ 16\ \text{m} \end{aligned}$$

Answer

$v = 24$ m/s, $x = 16$ m

Jerk-limited $a = 3t$ at $t = 4$

K8

A particle starts with $v_0 = 1$ m/s and has $a(t) = 3t$ (m/s^2). Find v and x at $t = 4$ s ($x_0 = 0$).

$$\begin{aligned} v(t) &= v_0 + \int_0^t bt'\,dt' = \tfrac{3}{2}t^2 + 1 && \tfrac{1}{2}bt^2 \text{ term} \\ x(t) &= \int_0^t v\,dt' = \tfrac{1}{2}t^3 + t && \text{integrate again} \\ v, x\ (t{=}4) &= \text{25 m/s, 36 m} \end{aligned}$$

Answer

v = 25 m/s, x = 36 m

Vector components, 10 at 30°

Write $\vec{A}$ of magnitude 10 at 30° above $+x$ in component form.

$$
\begin{aligned}
A_x &= A\cos\theta = 8.66 && \hat{i} \text{ part} \\
A_y &= A\sin\theta = 5.00 && \hat{j} \text{ part} \\
\vec{A} &= 8.66\,\hat{i} + 5.00\,\hat{j}
\end{aligned}
$$

Answer

$8.66\,\hat{i} \quad + \quad 5.00\,\hat{j}$

Vector components, 20 at 60°

Write $\vec{A}$ of magnitude 20 at 60° above $+x$ in component form.

$$
\begin{aligned}
A_x &= A\cos\theta = 10.00 && \hat{i} \text{ part} \\
A_y &= A\sin\theta = 17.32 && \hat{j} \text{ part} \\
\vec{A} &= 10.00\,\hat{i} + 17.32\,\hat{j}
\end{aligned}
$$

Answer

$10.00\,\hat{i} \quad + \quad 17.32\,\hat{j}$

Vector components, 15 at 45°

Write $\vec{A}$ of magnitude 15 at 45° above $+x$ in component form.

$$
\begin{aligned}
A_x &= A\cos\theta = 10.61 && \hat{i} \text{ part} \\
A_y &= A\sin\theta = 10.61 && \hat{j} \text{ part} \\
\vec{A} &= 10.61\,\hat{i} + 10.61\,\hat{j}
\end{aligned}
$$

Answer

$10.61\,\hat{i} \quad + \quad 10.61\,\hat{j}$

Vector components, 25 at 53°

Write $\vec{A}$ of magnitude 25 at 53° above $+x$ in component form.

$$
\begin{aligned}
A_x &= A\cos\theta = 15.05 && \hat{i} \text{ part} \\
A_y &= A\sin\theta = 19.97 && \hat{j} \text{ part} \\
\vec{A} &= 15.05\,\hat{i} + 19.97\,\hat{j}
\end{aligned}
$$

Answer

$15.05\,\hat{i} \;+\; 19.97\,\hat{j}$

Vector components, 8 at 90°

Write $\vec{A}$ of magnitude 8 at 90° above $+x$ in component form.

$$
\begin{aligned}
A_x &= A\cos\theta = 0.00 && \hat{i} \text{ part} \\
A_y &= A\sin\theta = 8.00 && \hat{j} \text{ part} \\
\vec{A} &= 0.00\,\hat{i} + 8.00\,\hat{j}
\end{aligned}
$$

Answer

$0.00\,\hat{i} \;+\; 8.00\,\hat{j}$

Vector components, 12 at 37°

V1

Write $\vec{A}$ of magnitude 12 at 37° above $+x$ in component form.

$$\begin{aligned} A_x &= A\cos\theta = 9.58 && \hat{i} \text{ part} \\ A_y &= A\sin\theta = 7.22 && \hat{j} \text{ part} \\ \vec{A} &= 9.58\,\hat{i} + 7.22\,\hat{j} \end{aligned}$$

Answer

$9.58\,\hat{i} \;+\; 7.22\,\hat{j}$

Vector components, 30 at 20°

Write $\vec{A}$ of magnitude 30 at 20° above $+x$ in component form.

$$\begin{aligned} A_x &= A\cos\theta = 28.19 && \hat{i} \text{ part} \\ A_y &= A\sin\theta = 10.26 && \hat{j} \text{ part} \\ \vec{A} &= 28.19\,\hat{i} + 10.26\,\hat{j} \end{aligned}$$

Answer

$28.19\,\hat{i} \quad + \quad 10.26\,\hat{j}$

$\vec{v} = d\vec{r}/dt$ **at** $t = 2$

A particle has $\vec{r}(t) = (3t)\hat{i} + (2t^2)\hat{j}$ (m). Find $\vec{v}$ and the speed at $t = 2$ s.

$$\begin{aligned} \vec{v} &= 3\,\hat{i} + 4t\,\hat{j} && \text{differentiate each component} \\ \vec{v}(2) &= 3\,\hat{i} + 8\,\hat{j} && \text{evaluate} \\ |\vec{v}| &= 8.54\ \text{m/s} && \text{speed} \end{aligned}$$

Answer

8.54 m/s

$\vec{v} = d\vec{r}/dt$ **at** $t = 1$

A particle has $\vec{r}(t) = (2t+1)\hat{i} + (-t^2+4t)\hat{j}$ (m). Find $\vec{v}$ and the speed at $t = 1$ s.

$$\begin{aligned} \vec{v} &= 2\hat{i} + -2t + 4\hat{j} && \text{differentiate each component} \\ \vec{v}(1) &= 2\hat{i} + 2\hat{j} && \text{evaluate} \\ |\vec{v}| &= 2.83 \text{ m/s} && \text{speed} \end{aligned}$$

Answer

2.83 m/s

$\vec{v} = d\vec{r}/dt$ **at** $t = 2$

A particle has $\vec{r}(t) = (t^2)\hat{i} + (5t)\hat{j}$ (m). Find $\vec{v}$ and the speed at $t = 2$ s.

$$\begin{aligned} \vec{v} &= 2t\,\hat{i} + 5\,\hat{j} && \text{differentiate each component} \\ \vec{v}(2) &= 4\,\hat{i} + 5\,\hat{j} && \text{evaluate} \\ |\vec{v}| &= 6.40 \text{ m/s} && \text{speed} \end{aligned}$$

Answer

6.40 m/s

$\vec{v} = d\vec{r}/dt$ **at** $t = 1$

A particle has $\vec{r}(t) = (3t^2 + 2)\hat{i} + (-2t^2)\hat{j}$ (m). Find $\vec{v}$ and the speed at $t = 1$ s.

$$\begin{aligned} \vec{v} &= 6t\,\hat{i} + -4t\,\hat{j} && \text{differentiate each component} \\ \vec{v}(1) &= 6\,\hat{i} + -4\,\hat{j} && \text{evaluate} \\ |\vec{v}| &= 7.21\ \text{m/s} && \text{speed} \end{aligned}$$

Answer

7.21 m/s

For $\vec{r}(t) = (3t)\hat{i} + (2t^2)\hat{j}$, find the acceleration vector.

$$\vec{v} = 3\hat{i} + 4t\hat{j} \quad \text{first derivative}$$
$$\vec{a} = 0\hat{i} + 4\hat{j} \quad \text{second derivative}$$

Answer

$$\vec{a} = 0\hat{i} + 4\hat{j}$$

For $\vec{r}(t) = (3t^2 + 1)\hat{i} + (-2t^2)\hat{j}$, find the acceleration vector.

$$\vec{v} = 6t\,\hat{i} + -4t\,\hat{j} \quad \text{first derivative}$$
$$\vec{a} = 6\,\hat{i} + -4\,\hat{j} \quad \text{second derivative}$$

Answer

$$\vec{a} = 6\,\hat{i} + -4\,\hat{j}$$

For $\vec{r}(t) = (4t^2)\hat{i} + (t^2 + 5t)\hat{j}$, find the acceleration vector.

$$\vec{v} = 8t\,\hat{i} + 2t + 5\,\hat{j} \qquad \text{first derivative}$$

$$\vec{a} = 8\,\hat{i} + 2\,\hat{j} \qquad \text{second derivative}$$

Answer

$$\vec{a} = 8\,\hat{i} + 2\,\hat{j}$$

Projectile, 20 m/s at 30°

V4

A projectile launches at 20 m/s, 30° above horizontal, with $\vec{a} = -g\hat{j}$. Find the hang time, max height, and range ($g = 9.8$).

$$v_{0x}, v_{0y} = 17.32,\ 10.00 \text{ m/s} \qquad \text{constant } v_x,\ a_y = -g$$

$$T = \frac{2v_{0y}}{g} = 2.04 \text{ s} \qquad v_y = 0 \text{ at top}$$

$$H, R = 5.1 \text{ m},\ 35.3 \text{ m}$$

Answer

$$T = 2.04 \text{ s},\ H = 5.1 \text{ m},\ R = 35.3 \text{ m}$$

Projectile, 25 m/s at 53°

V4

A projectile launches at 25 m/s, 53° above horizontal, with $\vec{a} = -g\hat{j}$. Find the hang time, max height, and range ($g = 9.8$).

$$v_{0x}, v_{0y} = 15.05,\ 19.97 \text{ m/s} \qquad \text{constant } v_x,\ a_y = -g$$

$$T = \frac{2v_{0y}}{g} = 4.07 \text{ s} \qquad v_y = 0 \text{ at top}$$

$$H, R = 20.3 \text{ m},\ 61.3 \text{ m}$$

Answer

$$T = 4.07 \text{ s},\ H = 20.3 \text{ m},\ R = 61.3 \text{ m}$$

Projectile, 30 m/s at 45°

V4

A projectile launches at 30 m/s, 45° above horizontal, with $\vec{a} = -g\hat{j}$. Find the hang time, max height, and range ($g = 9.8$).

$$v_{0x}, v_{0y} = 21.21,\ 21.21 \text{ m/s} \qquad \text{constant } v_x,\ a_y = -g$$

$$T = \frac{2v_{0y}}{g} = 4.33 \text{ s} \qquad v_y = 0 \text{ at top}$$

$$H, R = 23.0 \text{ m},\ 91.8 \text{ m}$$

Answer

$$T = 4.33 \text{ s},\ H = 23.0 \text{ m},\ R = 91.8 \text{ m}$$

Projectile, 15 m/s at 60°

V4

A projectile launches at 15 m/s, 60° above horizontal, with $\vec{a} = -g\hat{j}$. Find the hang time, max height, and range ($g = 9.8$).

$$v_{0x}, v_{0y} = 7.50,\ 12.99 \text{ m/s} \qquad \text{constant } v_x,\ a_y = -g$$

$$T = \frac{2v_{0y}}{g} = 2.65 \text{ s} \qquad v_y = 0 \text{ at top}$$

$$H, R = 8.6 \text{ m},\ 19.9 \text{ m}$$

Answer

$$T = 2.65 \text{ s},\ H = 8.6 \text{ m},\ R = 19.9 \text{ m}$$

Projectile, 40 m/s at 37°

V4

A projectile launches at 40 m/s, 37° above horizontal, with $\vec{a} = -g\hat{j}$. Find the hang time, max height, and range ($g = 9.8$).

$$v_{0x}, v_{0y} = 31.95,\ 24.07 \text{ m/s} \qquad \text{constant } v_x,\ a_y = -g$$

$$T = \frac{2v_{0y}}{g} = 4.91 \text{ s} \qquad v_y = 0 \text{ at top}$$

$$H, R = 29.6 \text{ m},\ 156.9 \text{ m}$$

Answer

$$T = 4.91 \text{ s},\ H = 29.6 \text{ m},\ R = 156.9 \text{ m}$$

Parametric range, $v_x = 3$

V5

A projectile has $x(t) = 3t$ and $y(t) = 20t - 4.9t^2$ (m). Eliminate t to find where it lands ($y = 0$).

$$y = 0 \;=\; t = \frac{20}{4.9} = 4.08 \text{ s} \qquad \text{landing time (nonzero root)}$$

$$x \;=\; 3(4.08) = 12.24 \text{ m} \qquad \text{range}$$

Answer

12.24 m

Parametric range, $v_x = 4$

V5

A projectile has $x(t) = 4t$ and $y(t) = 15t - 4.9t^2$ (m). Eliminate t to find where it lands ($y = 0$).

$$y = 0 \;=\; t = \frac{15}{4.9} = 3.06 \text{ s} \qquad \text{landing time (nonzero root)}$$

$$x \;=\; 4(3.06) = 12.24 \text{ m} \qquad \text{range}$$

Answer

12.24 m

Parametric range, $v_x = 5$

V5

A projectile has $x(t) = 5t$ and $y(t) = 25t - 4.9t^2$ (m). Eliminate t to find where it lands ($y = 0$).

$$y = 0 \;=\; t = \frac{25}{4.9} = 5.10 \text{ s} \qquad \text{landing time (nonzero root)}$$

$$x \;=\; 5(5.10) = 25.51 \text{ m} \qquad \text{range}$$

Answer

25.51 m

Parametric range, $v_x = 2$

V5

A projectile has $x(t) = 2t$ and $y(t) = 10t - 4.9t^2$ (m). Eliminate t to find where it lands ($y = 0$).

$$y = 0 \;=\; t = \frac{10}{4.9} = 2.04 \text{ s} \qquad \text{landing time (nonzero root)}$$

$$x \;=\; 2(2.04) = 4.08 \text{ m} \qquad \text{range}$$

Answer

4.08 m

a_t & a_c, v=10, r=5

A car moves at 10 m/s on a radius-5 m curve while speeding up at $a_t = 2\ m/s^2$. Find the centripetal and total acceleration.

$$a_c = \frac{v^2}{r} = 20.00\ \text{m/s}^2 \qquad \text{toward the center}$$

$$a = \sqrt{a_t^2 + a_c^2} = 20.10\ \text{m/s}^2 \qquad \text{perpendicular pieces}$$

Answer

$20.10\ \text{m/s}^2$

a_t & a_c, v=20, r=8

A car moves at 20 m/s on a radius-8 m curve while speeding up at $a_t = 3\ m/s^2$. Find the centripetal and total acceleration.

$$a_c = \frac{v^2}{r} = 50.00\ \text{m/s}^2 \qquad \text{toward the center}$$

$$a = \sqrt{a_t^2 + a_c^2} = 50.09\ \text{m/s}^2 \qquad \text{perpendicular pieces}$$

Answer

$50.09\ \text{m/s}^2$

a_t & a_c, v=6, r=3

A car moves at 6 m/s on a radius-3 m curve while speeding up at $a_t = 1$ m/s^2. Find the centripetal and total acceleration.

$$a_c = \frac{v^2}{r} = 12.00 \text{ m/s}^2 \qquad \text{toward the center}$$

$$a = \sqrt{a_t^2 + a_c^2} = 12.04 \text{ m/s}^2 \qquad \text{perpendicular pieces}$$

Answer

12.04 m/s^2

a_t & a_c, v=15, r=45

A car moves at 15 m/s on a radius-45 m curve while speeding up at $a_t = 0$ m/s^2. Find the centripetal and total acceleration.

$$a_c = \frac{v^2}{r} = 5.00 \text{ m/s}^2 \qquad \text{toward the center}$$

$$a = \sqrt{a_t^2 + a_c^2} = 5.00 \text{ m/s}^2 \qquad \text{perpendicular pieces}$$

Answer

5.00 m/s^2

Circular, R=2.0 m, $\omega = 3$

A particle moves as $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$ with $R = 2.0$ m, $\omega = 3$ rad/s. Find the speed and centripetal acceleration.

$$|\vec{v}| = R\omega = 2.0(3) = 6.00 \text{ m/s} \qquad \vec{v} = d\vec{r}/dt$$

$$|\vec{a}| = R\omega^2 = 18.00 \text{ m/s}^2 \qquad \vec{a} = -\omega^2\vec{r}, \text{ inward}$$

Answer

$$v = 6.00 \text{ m/s}, \; a = 18.00 \text{ m/s}^2$$

Circular, R=0.5 m, $\omega = 10$

A particle moves as $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$ with $R = 0.5$ m, $\omega = 10$ rad/s. Find the speed and centripetal acceleration.

$$|\vec{v}| = R\omega = 0.5(10) = 5.00 \text{ m/s} \qquad \vec{v} = d\vec{r}/dt$$

$$|\vec{a}| = R\omega^2 = 50.00 \text{ m/s}^2 \qquad \vec{a} = -\omega^2\vec{r}, \text{ inward}$$

Answer

$$v = 5.00 \text{ m/s}, \quad a = 50.00 \text{ m/s}^2$$

Circular, R=4.0 m, $\omega = 2$

A particle moves as $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$ with $R = 4.0$ m, $\omega = 2$ rad/s. Find the speed and centripetal acceleration.

$$|\vec{v}| = R\omega = 4.0(2) = 8.00 \text{ m/s} \qquad \vec{v} = d\vec{r}/dt$$

$$|\vec{a}| = R\omega^2 = 16.00 \text{ m/s}^2 \qquad \vec{a} = -\omega^2\vec{r}, \text{ inward}$$

Answer

$$v = 8.00 \text{ m/s}, \; a = 16.00 \text{ m/s}^2$$

Circular, R=1.5 m, $\omega = 4$

A particle moves as $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$ with $R = 1.5$ m, $\omega = 4$ rad/s. Find the speed and centripetal acceleration.

$$|\vec{v}| = R\omega = 1.5(4) = 6.00 \text{ m/s} \qquad \vec{v} = d\vec{r}/dt$$

$$|\vec{a}| = R\omega^2 = 24.00 \text{ m/s}^2 \qquad \vec{a} = -\omega^2\vec{r}, \text{ inward}$$

Answer

$$v = 6.00 \text{ m/s}, \; a = 24.00 \text{ m/s}^2$$

Circular, R=3.0 m, $\omega = 5$

A particle moves as $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$ with $R = 3.0$ m, $\omega = 5$ rad/s. Find the speed and centripetal acceleration.

$$|\vec{v}| = R\omega = 3.0(5) = 15.00 \text{ m/s} \qquad \vec{v} = d\vec{r}/dt$$

$$|\vec{a}| = R\omega^2 = 75.00 \text{ m/s}^2 \qquad \vec{a} = -\omega^2\vec{r}, \text{ inward}$$

Answer

$v = 15.00$ m/s, $a = 75.00$ m/s^2

Relative velocity

$\vec{v}_A = (3, 0)$ *and* $\vec{v}_B = (0, 4)$ *m/s. Find* $\vec{v}_{A/B}$ *and its magnitude.*

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = (3, -4)\ \text{m/s} \qquad \text{subtract componentwise}$$

$$|\vec{v}_{A/B}| = 5.00\ \text{m/s}$$

Answer

5.00 m/s

Relative velocity

$\vec{v}_A = (5, 2)$ *and* $\vec{v}_B = (1, -1)$ *m/s. Find* $\vec{v}_{A/B}$ *and its magnitude.*

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = (4, 3)\ \text{m/s} \qquad \text{subtract componentwise}$$

$$|\vec{v}_{A/B}| = 5.00\ \text{m/s}$$

Answer

5.00 m/s

Relative velocity

V8

$\vec{v}_A = (0, 6)$ *and* $\vec{v}_B = (3, 0)$ *m/s. Find* $\vec{v}_{A/B}$ *and its magnitude.*

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = (-3, 6)\ \text{m/s} \qquad \text{subtract componentwise}$$

$$|\vec{v}_{A/B}| = 6.71\ \text{m/s}$$

Answer

6.71 m/s

Relative velocity

V8

$\vec{v}_A = (4, 4)$ *and* $\vec{v}_B = (2, 1)$ *m/s. Find* $\vec{v}_{A/B}$ *and its magnitude.*

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = (2, 3)\ \text{m/s} \qquad \text{subtract componentwise}$$

$$|\vec{v}_{A/B}| = 3.61\ \text{m/s}$$

Answer

3.61 m/s

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Newton's Laws & Dynamics (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Set up → solve → answer
77 problems

Net force from components

F1

Forces have x-components $5, -2$ N and y-components $3, 3$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 3,\ 6 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 6.71 \text{ N at } 63^\circ \qquad \text{resultant}$$

Answer

6.71 N

Net force from components

F1

Forces have x-components $10, -4, -3$ N and y-components $0, 0, 0$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 3,\ 0 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 3.00 \text{ N at } 0^\circ \qquad \text{resultant}$$

Answer

3.00 N

Net force from components

F1

Forces have x-components $6,0$ N and y-components $0,8$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 6,\ 8 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 10.00 \text{ N at } 53^\circ \qquad \text{resultant}$$

Answer

10.00 N

Net force from components

F1

Forces have x-components $4, 4, -2$ N and y-components $1, -1, 0$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 6,\ 0 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 6.00 \text{ N at } 0^\circ \qquad \text{resultant}$$

Answer

6.00 N

Net force from components

F1

Forces have x-components $12, -5$ N and y-components $9, -9$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 7,\ 0 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 7.00 \text{ N at } 0^\circ \qquad \text{resultant}$$

Answer

7.00 N

Net force from components

F1

Forces have x-components $8, -3$ N and y-components $6, 0$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 5,\ 6 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 7.81 \text{ N at } 50^\circ \qquad \text{resultant}$$

Answer

7.81 N

Net force from components

F1

Forces have x-components $2, 2, 2$ N and y-components $5, -2, 0$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 6,\ 3 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 6.71 \text{ N at } 27^\circ \qquad \text{resultant}$$

Answer

6.71 N

Net force from components

F1

Forces have x-components $15, -10$ N and y-components $8, 4$ N. Find the net force.

$$\Sigma F_x, \Sigma F_y = 5,\ 12 \text{ N} \qquad \text{add each axis}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 13.00 \text{ N at } 67^\circ \qquad \text{resultant}$$

Answer

13.00 N

$\Sigma F = ma$, 5 kg

A net force accelerates a 5 kg body at 2.0 m/s^2. Find the force.

$$\Sigma F = ma = 5(2.0) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 10.0 \text{ N}$$

Answer
10.0 N

$\Sigma F = ma$, 10 kg

A net force accelerates a 10 *kg body at* 3.0 m/s^2. *Find the force.*

$$\Sigma F = ma = 10(3.0) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 30.0 \text{ N}$$

Answer

30.0 N

$\Sigma F = ma$, 2 kg

A net force accelerates a 2 kg body at 6.0 m/s^2. Find the force.

$$\Sigma F = ma = 2(6.0) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 12.0 \text{ N}$$

Answer
12.0 N

$\Sigma F = ma$, 20 kg

A net force accelerates a 20 kg body at 0.5 m/s^2. Find the force.

$$\Sigma F = ma = 20(0.5) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 10.0 \text{ N}$$

Answer
10.0 N

$\Sigma F = ma$, 0.5 kg

A net force accelerates a 0.5 *kg body at* 8.0 m/s^2. *Find the force.*

$$\Sigma F = ma = 0.5(8.0) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 4.0 \text{ N}$$

Answer

4.0 N

$\Sigma F = ma$, 8 kg

A net force accelerates a 8 kg body at 2.5 m/s^2. Find the force.

$$\Sigma F = ma = 8(2.5) \qquad \text{Newton's 2nd law}$$
$$\Sigma F = 20.0 \text{ N}$$

Answer
20.0 N

Accel from 50 N

A net 50 N acts on a 10 kg body. Find the acceleration.

$$a = \frac{\Sigma F}{m} = \frac{50}{10} \qquad \text{solve for } a$$

$$a = 5.00\ \text{m/s}^2$$

Answer

$5.00\ \text{m/s}^2$

Accel from 30 N

A net 30 N acts on a 6 kg body. Find the acceleration.

$$a = \frac{\Sigma F}{m} = \frac{30}{6} \qquad \text{solve for } a$$

$$a = 5.00\ \text{m/s}^2$$

Answer

$5.00\ \text{m/s}^2$

Accel from 100 N

A net 100 N acts on a 4 kg body. Find the acceleration.

$$a = \frac{\Sigma F}{m} = \frac{100}{4} \qquad \text{solve for } a$$

$$a = 25.00\ \text{m/s}^2$$

Answer

$25.00\ \text{m/s}^2$

Accel from 24 N

A net 24 N acts on a 3 kg body. Find the acceleration.

$$a = \frac{\Sigma F}{m} = \frac{24}{3} \qquad \text{solve for } a$$

$$a = 8.00\ \text{m/s}^2$$

Answer

$8.00\ \text{m/s}^2$

$F = dp/dt$, 2.000 kg

A 2.000 kg body's velocity changes from 0 to 10 m/s in 4.000 s. Find the average net force.

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

Newton: $F = dp/dt$

$$F = 5.0\ \text{N}$$

Answer

5.0 N

$F = dp/dt$, 0.500 kg

A 0.500 kg body's velocity changes from 20 to 0 m/s in 0.500 s. Find the average net force.

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

Newton: $F = dp/dt$

$$F = -20.0 \text{ N}$$

Answer

-20.0 N

$F = dp/dt$, 1000.000 kg

A 1000.000 kg body's velocity changes from 30 to 20 m/s in 5.000 s. Find the average net force.

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

Newton: $F = dp/dt$

$$F = -2000.0 \text{ N}$$

Answer

-2000.0 N

$F = dp/dt$, 0.145 kg

A 0.145 kg body's velocity changes from 40 to −40 m/s in 0.002 s. Find the average net force.

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

Newton: $F = dp/dt$

$$F = -5800.0 \text{ N}$$

Answer

−5800.0 N

Impulse of $F = 6t$

A time-varying force $F(t) = 6t$ (N) acts from $t = 0$ to 3 s. Find the impulse (change in momentum).

$$J = \int_0^3 6t\,dt = \frac{6}{2}t^2\Big|_0^3 \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 27.0\ \text{N}\cdot\text{s}$$

Answer

27.0 N · s

Impulse of $F = 10t$

A time-varying force $F(t) = 10t$ (N) acts from $t = 0$ to 2 s. Find the impulse (change in momentum).

$$J = \int_0^2 10t\,dt = \frac{10}{2}t^2\Big|_0^2 \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 20.0\ \text{N}\cdot\text{s}$$

Answer

20.0 N · s

Impulse of $F = 4t$

A time-varying force $F(t) = 4t$ (N) acts from $t = 0$ to 5 s. Find the impulse (change in momentum).

$$J = \int_0^5 4t\,dt = \tfrac{4}{2}t^2\Big|_0^5 \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 50.0\ \text{N}\cdot\text{s}$$

Answer

50.0 N · s

Impulse of $F = 8t$

A time-varying force $F(t) = 8t$ (N) acts from $t = 0$ to 4 s. Find the impulse (change in momentum).

$$J = \int_0^4 8t\,dt = \frac{8}{2}t^2\Big|_0^4 \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 64.0\ \text{N}\cdot\text{s}$$

Answer

64.0 N · s

Impulse of $F = 12t$

A time-varying force $F(t) = 12t$ (N) acts from $t = 0$ to 2 s. Find the impulse (change in momentum).

$$J = \int_0^2 12t\,dt = \frac{12}{2}t^2\Big|_0^2 \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 24.0\ \text{N}\cdot\text{s}$$

Answer

24.0 N · s

Kinetic friction, $\mu = 0.30$

A 10 kg block slides on level ground, $\mu_k = 0.30$. Find the friction force ($g = 9.8$).

$$N = mg = 98.0 \text{ N} \qquad \text{normal} = \text{weight}$$

$$f_k = \mu_k N = 0.30(98.0) = 29.4 \text{ N} \qquad \text{opposes sliding}$$

Answer

29.4 N

Kinetic friction, $\mu = 0.50$

A 4 kg block slides on level ground, $\mu_k = 0.50$. Find the friction force ($g = 9.8$).

$$N = mg = 39.2 \text{ N} \qquad \text{normal} = \text{weight}$$

$$f_k = \mu_k N = 0.50(39.2) = 19.6 \text{ N} \qquad \text{opposes sliding}$$

Answer

19.6 N

Kinetic friction, $\mu = 0.20$

A 20 kg block slides on level ground, $\mu_k = 0.20$. Find the friction force ($g = 9.8$).

$$N = mg = 196.0 \text{ N} \qquad \text{normal} = \text{weight}$$

$$f_k = \mu_k N = 0.20(196.0) = 39.2 \text{ N} \qquad \text{opposes sliding}$$

Answer

39.2 N

Kinetic friction, $\mu = 0.40$

A 5 kg block slides on level ground, $\mu_k = 0.40$. Find the friction force ($g = 9.8$).

$$N = mg = 49.0 \text{ N} \qquad \text{normal} = \text{weight}$$

$$f_k = \mu_k N = 0.40(49.0) = 19.6 \text{ N} \qquad \text{opposes sliding}$$

Answer

19.6 N

Kinetic friction, $\mu = 0.25$

A 8 kg block slides on level ground, $\mu_k = 0.25$. Find the friction force ($g = 9.8$).

$$N = mg = 78.4 \text{ N} \quad \text{normal} = \text{weight}$$

$$f_k = \mu_k N = 0.25(78.4) = 19.6 \text{ N} \quad \text{opposes sliding}$$

Answer

19.6 N

Max static friction, $\mu_s = 0.6$

A 5 kg block sits on level ground, $\mu_s = 0.6$. Find the maximum static friction before it slips.

$$f_{s,\max} = \mu_s N = \mu_s mg = 0.6(49.0) \qquad \text{the slip threshold}$$

$$f_{s,\max} = 29.4 \text{ N}$$

Answer

29.4 N

Max static friction, $\mu_s = 0.4$

A 10 kg block sits on level ground, $\mu_s = 0.4$. Find the maximum static friction before it slips.

$$f_{s,\max} = \mu_s N = \mu_s mg = 0.4(98.0) \qquad \text{the slip threshold}$$

$$f_{s,\max} = 39.2 \text{ N}$$

Answer

39.2 N

Max static friction, $\mu_s = 0.5$

A 2 kg block sits on level ground, $\mu_s = 0.5$. Find the maximum static friction before it slips.

$$f_{s,\max} = \mu_s N = \mu_s mg = 0.5(19.6) \qquad \text{the slip threshold}$$
$$f_{s,\max} = 9.8 \text{ N}$$

Answer

9.8 N

Slip on incline? $\mu_s = 0.5$, 30°

A block rests on a 30° incline, $\mu_s = 0.5$. Does it slide?

$$\tan\theta = \text{vs } \mu_s: \ 0.58 > 0.5 \qquad \text{slips if } \tan\theta > \mu_s$$

$$\text{result} = \text{slides}$$

Answer

slides

Slip on incline? $\mu_s = 0.3$, 20°

A block rests on a 20° incline, $\mu_s = 0.3$. Does it slide?

$$\tan\theta = \text{vs } \mu_s: \ 0.36 > 0.3 \qquad \text{slips if } \tan\theta > \mu_s$$

$$\text{result} = \text{slides}$$

Answer

slides

Slip on incline? $\mu_s = 0.7$, 45°

A block rests on a 45° incline, $\mu_s = 0.7$. Does it slide?

$$\tan\theta \;=\; \text{vs } \mu_s:\; 1.00 > 0.7 \qquad \text{slips if } \tan\theta > \mu_s$$

$$\text{result} \;=\; \text{slides}$$

Answer

slides

Atwood, 3 & 5 kg

Two masses 3 kg and 5 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 2.45 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 36.8 \text{ N}$$ same tension throughout

Answer

$a = 2.45 \text{ m/s}^2, \; T = 36.8 \text{ N}$

Atwood, 2 & 6 kg

Two masses 2 kg and 6 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 4.90 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 29.4 \text{ N}$$ same tension throughout

Answer

$a = 4.90 \text{ m/s}^2, \; T = 29.4 \text{ N}$

Atwood, 4 & 4 kg

Two masses 4 kg and 4 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 0.00 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 39.2 \text{ N}$$ same tension throughout

Answer

$a = 0.00 \text{ m/s}^2, \; T = 39.2 \text{ N}$

Atwood, 1 & 9 kg

Two masses 1 kg and 9 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 7.84 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 17.6 \text{ N}$$ same tension throughout

Answer

$a = 7.84 \text{ m/s}^2, \; T = 17.6 \text{ N}$

Atwood, 5 & 10 kg

Two masses 5 kg and 10 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 3.27 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 65.3 \text{ N}$$ same tension throughout

Answer

$a = 3.27 \text{ m/s}^2$, $T = 65.3 \text{ N}$

Atwood, 2 & 8 kg

Two masses 2 kg and 8 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 5.88 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 31.4 \text{ N}$$ same tension throughout

Answer

$a = 5.88 \text{ m/s}^2, \; T = 31.4 \text{ N}$

Atwood, 6 & 4 kg

Two masses 6 kg and 4 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 1.96 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 47.0 \text{ N}$$ same tension throughout

Answer

$a = 1.96 \text{ m/s}^2, \; T = 47.0 \text{ N}$

Atwood, 3 & 7 kg

F5

Two masses 3 kg and 7 kg hang over an ideal pulley (Atwood machine). Find the acceleration and rope tension ($g = 9.8$).

$$a = \frac{|m_1 - m_2|g}{m_1 + m_2} = 3.92 \text{ m/s}^2$$ net weight / total mass

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 41.2 \text{ N}$$ same tension throughout

Answer

$a = 3.92 \text{ m/s}^2, \; T = 41.2 \text{ N}$

Table+hanging, 4 & 2 kg

A 4 kg block on a frictionless table is connected over a pulley to a hanging 2 kg mass. Find the acceleration and tension ($g = 9.8$).

$$a = \frac{mg}{M+m} = \frac{19.6}{6} = 3.27\ \text{m/s}^2 \qquad \text{hanging weight / total mass}$$

$$T = Ma = 13.1\ \text{N} \qquad \text{pulls the table block}$$

Answer

$$a = 3.27\ \text{m/s}^2,\quad T = 13.1\ \text{N}$$

Table+hanging, 6 & 3 kg

A 6 kg block on a frictionless table is connected over a pulley to a hanging 3 kg mass. Find the acceleration and tension ($g = 9.8$).

$$a = \frac{mg}{M+m} = \frac{29.4}{9} = 3.27\ \text{m/s}^2 \qquad \text{hanging weight / total mass}$$

$$T = Ma = 19.6\ \text{N} \qquad \text{pulls the table block}$$

Answer

$$a = 3.27\ \text{m/s}^2,\quad T = 19.6\ \text{N}$$

Table+hanging, 5 & 1 kg

F5

A 5 kg block on a frictionless table is connected over a pulley to a hanging 1 kg mass. Find the acceleration and tension ($g = 9.8$).

$$a = \frac{mg}{M+m} = \frac{9.8}{6} = 1.63 \text{ m/s}^2 \quad \text{hanging weight / total mass}$$

$$T = Ma = 8.2 \text{ N} \quad \text{pulls the table block}$$

Answer

$$a = 1.63 \text{ m/s}^2, \; T = 8.2 \text{ N}$$

Terminal velocity, m=2.00, b=0.50

F6

An object ($m = 2.00$ kg) falls with linear drag $F_d = -bv$, $b = 0.50$ kg/s. Find its terminal velocity ($g = 9.8$).

$$0 = mg - bv_t \qquad \text{at terminal velocity } a = 0$$

$$v_t = \frac{mg}{b} = \frac{19.6}{0.50} = 39.20 \text{ m/s} \qquad \text{drag balances gravity}$$

Answer

39.20 m/s

Terminal velocity, m=5.00, b=1.00

An object ($m = 5.00$ kg) falls with linear drag $F_d = -bv$, $b = 1.00$ kg/s. Find its terminal velocity ($g = 9.8$).

$$0 = mg - bv_t \qquad \text{at terminal velocity } a = 0$$

$$v_t = \frac{mg}{b} = \frac{49.0}{1.00} = 49.00 \text{ m/s} \qquad \text{drag balances gravity}$$

Answer

49.00 m/s

Terminal velocity, m=0.10, b=0.20

An object ($m = 0.10$ kg) falls with linear drag $F_d = -bv$, $b = 0.20$ kg/s. Find its terminal velocity ($g = 9.8$).

$$0 = mg - bv_t \qquad \text{at terminal velocity } a = 0$$

$$v_t = \frac{mg}{b} = \frac{1.0}{0.20} = 4.90 \text{ m/s} \qquad \text{drag balances gravity}$$

Answer

4.90 m/s

Terminal velocity, m=80.00, b=12.00

An object ($m = 80.00$ kg) falls with linear drag $F_d = -bv$, $b = 12.00$ kg/s. Find its terminal velocity ($g = 9.8$).

$$0 = mg - bv_t \qquad \text{at terminal velocity } a = 0$$

$$v_t = \frac{mg}{b} = \frac{784.0}{12.00} = 65.33 \text{ m/s} \qquad \text{drag balances gravity}$$

Answer

65.33 m/s

Terminal velocity, m=1.00, b=0.25

F6

An object ($m = 1.00$ kg) falls with linear drag $F_d = -bv$, $b = 0.25$ kg/s. Find its terminal velocity ($g = 9.8$).

$$0 = mg - bv_t \qquad \text{at terminal velocity } a = 0$$

$$v_t = \frac{mg}{b} = \frac{9.8}{0.25} = 39.20 \text{ m/s} \qquad \text{drag balances gravity}$$

Answer

39.20 m/s

Accel at v=10 with drag

A 2.0 kg object falling with drag $F_d = -bv$ ($b = 0.50$) moves at $v = 10$ m/s. Find its acceleration right then ($g = 9.8$).

$$ma = mg - bv$$ Newton's 2nd law with drag

$$a = g - \frac{b}{m}v = 9.8 - \frac{0.50}{2.0}(10) = 7.30 \text{ m/s}^2$$ shrinks toward 0

Answer

7.30 m/s^2

Accel at v=20 with drag

A 5.0 kg object falling with drag $F_d = -bv$ ($b = 1.00$) moves at $v = 20$ m/s. Find its acceleration right then ($g = 9.8$).

$$ma = mg - bv$$ Newton's 2nd law with drag

$$a = g - \frac{b}{m}v = 9.8 - \frac{1.00}{5.0}(20) = 5.80 \text{ m/s}^2$$ shrinks toward 0

Answer

5.80 m/s^2

Accel at v=8 with drag

A 1.0 kg object falling with drag $F_d = -bv$ ($b = 0.25$) moves at $v = 8$ m/s. Find its acceleration right then ($g = 9.8$).

$$ma = mg - bv$$ Newton's 2nd law with drag

$$a = g - \frac{b}{m}v = 9.8 - \frac{0.25}{1.0}(8) = 7.80 \text{ m/s}^2$$ shrinks toward 0

Answer

7.80 m/s^2

Accel at v=30 with drag

A 2.0 kg object falling with drag $F_d = -bv$ ($b = 0.50$) moves at $v = 30$ m/s. Find its acceleration right then ($g = 9.8$).

$$ma = mg - bv$$ Newton's 2nd law with drag

$$a = g - \frac{b}{m}v = 9.8 - \frac{0.50}{2.0}(30) = 2.30 \text{ m/s}^2$$ shrinks toward 0

Answer

2.30 m/s^2

Accel at v=40 with drag

A 80.0 kg object falling with drag $F_d = -bv$ ($b = 12.00$) moves at $v = 40$ m/s. Find its acceleration right then ($g = 9.8$).

$$ma = mg - bv$$ Newton's 2nd law with drag

$$a = g - \frac{b}{m}v = 9.8 - \frac{12.00}{80.0}(40) = 3.80 \text{ m/s}^2$$ shrinks toward 0

Answer

3.80 m/s^2

$v(t)$ with drag at t=4 s

Released from rest with drag $F_d = -bv$ ($m = 2.0$ kg, $b = 0.50$), the solution is $v(t) = v_t(1 - e^{-bt/m})$. Find v at $t = 4$ s ($g = 9.8$).

$$v_t = \frac{mg}{b} = 39.20 \text{ m/s} \quad \text{terminal speed}$$

$$v(4) = v_t(1 - e^{-bt/m}) = 24.78 \text{ m/s} \quad \text{approaches } v_t$$

Answer

24.78 m/s

$v(t)$ with drag at t=5 s

Released from rest with drag $F_d = -bv$ ($m = 5.0$ kg, $b = 1.00$), the solution is $v(t) = v_t(1 - e^{-bt/m})$. Find v at $t = 5$ s ($g = 9.8$).

$$v_t = \frac{mg}{b} = 49.00 \text{ m/s} \qquad \text{terminal speed}$$

$$v(5) = v_t(1 - e^{-bt/m}) = 30.97 \text{ m/s} \qquad \text{approaches } v_t$$

Answer

30.97 m/s

$v(t)$ with drag at t=4 s

Released from rest with drag $F_d = -bv$ ($m = 1.0$ kg, $b = 0.25$), the solution is $v(t) = v_t(1 - e^{-bt/m})$. Find v at $t = 4$ s ($g = 9.8$).

$$v_t = \frac{mg}{b} = 39.20 \text{ m/s} \qquad \text{terminal speed}$$

$$v(4) = v_t(1 - e^{-bt/m}) = 24.78 \text{ m/s} \qquad \text{approaches } v_t$$

Answer

24.78 m/s

$v(t)$ with drag at t=2 s

Released from rest with drag $F_d = -bv$ ($m = 2.0$ kg, $b = 0.50$), the solution is $v(t) = v_t(1 - e^{-bt/m})$. Find v at $t = 2$ s ($g = 9.8$).

$$v_t = \frac{mg}{b} = 39.20 \text{ m/s} \qquad \text{terminal speed}$$

$$v(2) = v_t(1 - e^{-bt/m}) = 15.42 \text{ m/s} \qquad \text{approaches } v_t$$

Answer

15.42 m/s

$v(t)$ with drag at t=10 s

F6

Released from rest with drag $F_d = -bv$ ($m = 5.0$ kg, $b = 1.00$), the solution is $v(t) = v_t(1 - e^{-bt/m})$. Find v at $t = 10$ s ($g = 9.8$).

$$v_t = \frac{mg}{b} = 49.00 \text{ m/s} \qquad \text{terminal speed}$$

$$v(10) = v_t(1 - e^{-bt/m}) = 42.37 \text{ m/s} \qquad \text{approaches } v_t$$

Answer

42.37 m/s

Newton's third law

F7

A hammer hits a nail with 200 N. What force does the nail exert on the hammer?

rule = 200 N, opposite equal, opposite, different bodies

Answer

200 N, opposite

Newton's third law

F7

Earth pulls a 10 kg rock down with 98 N. What does the rock do to Earth?

rule = 98 N up on Earth equal, opposite, different bodies

Answer

98 N up on Earth

Newton's third law

Do the two forces of a third-law pair ever cancel?

rule = no — different bodies equal, opposite, different bodies

Answer

no — different bodies

Frictionless incline, 5 kg, 30°

A 5 kg block is on a frictionless 30° incline. Tilt the axes and find N and the acceleration ($g = 9.8$).

$$
\begin{aligned}
N &= mg\cos\theta = 42.4\ \text{N} && y\text{: perpendicular} \\
ma &= mg\sin\theta && x\text{: along the surface} \\
a &= g\sin\theta = 4.90\ \text{m/s}^2 && \text{mass cancels}
\end{aligned}
$$

Answer

$$N = 42.4\ \text{N},\quad a = 4.90\ \text{m/s}^2$$

Frictionless incline, 10 kg, 20°

A 10 kg block is on a frictionless 20° incline. Tilt the axes and find N and the acceleration ($g = 9.8$).

$$N = mg\cos\theta = 92.1\ \text{N} \qquad y\text{: perpendicular}$$

$$ma = mg\sin\theta \qquad x\text{: along the surface}$$

$$a = g\sin\theta = 3.35\ \text{m/s}^2 \qquad \text{mass cancels}$$

Answer

$$N = 92.1\ \text{N},\ a = 3.35\ \text{m/s}^2$$

Frictionless incline, 2 kg, 45°

A 2 kg block is on a frictionless 45° incline. Tilt the axes and find N and the acceleration ($g = 9.8$).

$$
\begin{aligned}
N &= mg\cos\theta = 13.9\ \text{N} && \text{y: perpendicular} \\
ma &= mg\sin\theta && \text{x: along the surface} \\
a &= g\sin\theta = 6.93\ \text{m/s}^2 && \text{mass cancels}
\end{aligned}
$$

Answer

$$N = 13.9\ \text{N},\quad a = 6.93\ \text{m/s}^2$$

Frictionless incline, 8 kg, 37°

A 8 kg block is on a frictionless 37° incline. Tilt the axes and find N and the acceleration ($g = 9.8$).

$$N = mg\cos\theta = 62.6\ \text{N} \qquad y\text{: perpendicular}$$

$$ma = mg\sin\theta \qquad x\text{: along the surface}$$

$$a = g\sin\theta = 5.90\ \text{m/s}^2 \qquad \text{mass cancels}$$

Answer

$$N = 62.6\ \text{N},\ a = 5.90\ \text{m/s}^2$$

Frictionless incline, 4 kg, 53°

A 4 kg block is on a frictionless 53° incline. Tilt the axes and find N and the acceleration ($g = 9.8$).

$$
\begin{aligned}
N &= mg\cos\theta = 23.6\ \text{N} && \text{y: perpendicular} \\
ma &= mg\sin\theta && \text{x: along the surface} \\
a &= g\sin\theta = 7.83\ \text{m/s}^2 && \text{mass cancels}
\end{aligned}
$$

Answer

$$N = 23.6\ \text{N},\quad a = 7.83\ \text{m/s}^2$$

Incline w/ friction, 30°, $\mu = 0.2$

A 5 kg block slides down a 30° incline with $\mu_k = 0.2$. Find its acceleration ($g = 9.8$).

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$ gravity along minus friction

$$a = g(\sin\theta - \mu_k\cos\theta) = 3.20\ \text{m/s}^2$$ mass cancels

Answer

$3.20\ \text{m/s}^2$

Incline w/ friction, 37°, $\mu = 0.3$

A 10 kg block slides down a 37° incline with $\mu_k = 0.3$. Find its acceleration ($g = 9.8$).

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$ gravity along minus friction

$$a = g(\sin\theta - \mu_k\cos\theta) = 3.55\ \text{m/s}^2$$ mass cancels

Answer

$3.55\ \text{m/s}^2$

Incline w/ friction, 45°, $\mu = 0.1$

A 4 kg block slides down a 45° incline with $\mu_k = 0.1$. Find its acceleration ($g = 9.8$).

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$ gravity along minus friction

$$a = g(\sin\theta - \mu_k\cos\theta) = 6.24\ \text{m/s}^2$$ mass cancels

Answer

$6.24\ \text{m/s}^2$

Incline w/ friction, 30°, $\mu = 0.1$

A 8 kg block slides down a 30° incline with $\mu_k = 0.1$. Find its acceleration ($g = 9.8$).

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$ gravity along minus friction

$$a = g(\sin\theta - \mu_k\cos\theta) = 3.63\ \text{m/s}^2$$ mass cancels

Answer

$3.63\ \text{m/s}^2$

Incline w/ friction, 53°, $\mu = 0.2$

A 6 kg block slides down a 53° incline with $\mu_k = 0.2$. Find its acceleration ($g = 9.8$).

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$ gravity along minus friction

$$a = g(\sin\theta - \mu_k\cos\theta) = 6.35\ \text{m/s}^2$$ mass cancels

Answer

$6.35\ \text{m/s}^2$

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Work, Energy & Power (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Integrate / differentiate → answer
73 problems

Lesson app energy session

Work $\int F\,dx$, $[0, 3]$

E1

A force $F(x) = 6x$ (N) acts as the object moves from $x = 0$ to $x = 3$ m. Find the work done.

$$W = \int_0^3 (6x)\,dx \qquad \text{work = line integral of force}$$
$$W = 27 \text{ J}$$

Answer
27 J

Work $\int F\,dx$, $[1, 4]$

A force $F(x) = 2x + 4$ (N) acts as the object moves from $x = 1$ to $x = 4$ m. Find the work done.

$$W = \int_1^4 (2x + 4)\,dx \qquad \text{work} = \text{line integral of force}$$

$$W = 27 \text{ J}$$

Answer

27 J

Work $\int F\,dx$, $[0, 2]$

E1

A force $F(x) = 3x^2$ (N) acts as the object moves from $x = 0$ to $x = 2$ m. Find the work done.

$$W = \int_0^2 (3x^2)\,dx \qquad \text{work = line integral of force}$$

$$W = 8 \text{ J}$$

Answer

8 J

Work $\int F\,dx$, $[0, 5]$

A force $F(x) = -x + 2$ (N) acts as the object moves from $x = 0$ to $x = 5$ m. Find the work done.

$$W = \int_0^5 (-x + 2)\,dx \qquad \text{work} = \text{line integral of force}$$

$$W = -2.50 \text{ J}$$

Answer

-2.50 J

Work $\int F\,dx$, $[0, 4]$

E1

A force $F(x) = 8x$ (N) acts as the object moves from $x = 0$ to $x = 4$ m. Find the work done.

$$W = \int_0^4 (8x)\,dx \qquad \text{work = line integral of force}$$
$$W = 64 \text{ J}$$

Answer

64 J

Work $\int F\,dx$, $[1, 3]$

E1

A force $F(x) = x^2 + 3$ (N) acts as the object moves from $x = 1$ to $x = 3$ m. Find the work done.

$$W = \int_1^3 (x^2 + 3)\,dx \qquad \text{work = line integral of force}$$

$$W = 14.67 \text{ J}$$

Answer

14.67 J

Work $\int F\,dx$, $[0, 2]$

A force $F(x) = 2x^2 + 4x$ (N) acts as the object moves from $x = 0$ to $x = 2$ m. Find the work done.

$$W = \int_0^2 (2x^2 + 4x)\,dx \qquad \text{work = line integral of force}$$

$$W = 13.33 \text{ J}$$

Answer

13.33 J

Work $\int F\,dx$, $[0, 2]$

E1

A force $F(x) = 6x^2$ (N) acts as the object moves from $x = 0$ to $x = 2$ m. Find the work done.

$$W = \int_0^2 (6x^2)\,dx \qquad \text{work = line integral of force}$$

$$W = 16 \text{ J}$$

Answer

16 J

Work $\int F\,dx$, $[1,3]$

E1

A force $F(x) = 2x + 5$ (N) acts as the object moves from $x = 1$ to $x = 3$ m. Find the work done.

$$W = \int_1^3 (2x + 5)\,dx \qquad \text{work = line integral of force}$$
$$W = 18 \text{ J}$$

Answer

18 J

Work $\int F\,dx$, $[0, 3]$

E1

A force $F(x) = 10x$ (N) acts as the object moves from $x = 0$ to $x = 3$ m. Find the work done.

$$W = \int_0^3 (10x)\,dx \qquad \text{work} = \text{line integral of force}$$

$$W = 45 \text{ J}$$

Answer

45 J

Constant force work, 0°

E1

A constant 50 *N force acts over* 4 *m at* 0° *to the motion. Find the work.*

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 50(4)\cos 0^\circ$$ constant-force special case

$$W = 200.0 \text{ J}$$

Answer

200.0 J

Constant force work, 60°

E1

A constant 100 N force acts over 3 m at 60° to the motion. Find the work.

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 100(3)\cos 60^\circ$$ constant-force special case

$$W = 150.0 \text{ J}$$

Answer

150.0 J

Constant force work, 30°

E1

A constant 40 N force acts over 5 m at 30° to the motion. Find the work.

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 40(5)\cos 30^\circ$$ constant-force special case

$$W = 173.2 \text{ J}$$

Answer

173.2 J

Constant force work, 90°

A constant 200 N force acts over 2 m at 90° to the motion. Find the work.

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 200(2)\cos 90^\circ$$ constant-force special case

$$W = 0.0\ \text{J}(\textit{perpendicular} : \textit{nowork})$$

Answer

0.0 J

Constant force work, 0°

E1

A constant 60 N force acts over 8 m at 0° to the motion. Find the work.

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 60(8)\cos 0^\circ$$ constant-force special case

$$W = 480.0 \text{ J}$$

Answer

480.0 J

Work-energy, 2 kg

E2

A 2 kg object at 0 m/s receives 100 J of net work. Find its final speed.

$$
\begin{aligned}
W_{\text{net}} &= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 && \text{work-energy theorem} \\
KE_f &= 0.0 + (100) = 100.0 \text{ J} && \text{add the work} \\
v_f &= \sqrt{2KE_f/m} = 10.00 \text{ m/s}
\end{aligned}
$$

Answer

10.00 m/s

Work-energy, 4 kg

E2

A 4 kg object at 3 m/s receives 50 J of net work. Find its final speed.

$$W_{\text{net}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \text{work-energy theorem}$$

$$KE_f = 18.0 + (50) = 68.0 \text{ J} \qquad \text{add the work}$$

$$v_f = \sqrt{2KE_f/m} = 5.83 \text{ m/s}$$

Answer

5.83 m/s

Work-energy, 1 kg

E2

A 1 kg object at 10 m/s receives −32 J of net work. Find its final speed.

$$W_{\text{net}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \text{work-energy theorem}$$

$$KE_f = 50.0 + (-32) = 18.0 \text{ J} \qquad \text{add the work}$$

$$v_f = \sqrt{2KE_f/m} = 6.00 \text{ m/s}$$

Answer

6.00 m/s

Work-energy, 5 kg

E2

A 5 kg object at 2 m/s receives 90 J of net work. Find its final speed.

$$W_{\text{net}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \text{work-energy theorem}$$

$$KE_f = 10.0 + (90) = 100.0 \text{ J} \qquad \text{add the work}$$

$$v_f = \sqrt{2KE_f/m} = 6.32 \text{ m/s}$$

Answer

6.32 m/s

Work-energy, 0.5 kg

E2

A 0.5 kg object at 8 m/s receives 20 J of net work. Find its final speed.

$$W_{\text{net}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \text{work-energy theorem}$$

$$KE_f = 16.0 + (20) = 36.0 \text{ J} \qquad \text{add the work}$$

$$v_f = \sqrt{2KE_f/m} = 12.00 \text{ m/s}$$

Answer

12.00 m/s

Kinetic energy, 2.000 kg

E3

Find the kinetic energy of a 2.000 *kg object moving* 10 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.000)(10)^2 \qquad \text{energy of motion}$$

$$KE = 100.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

100.0 J

Kinetic energy, 0.500 kg

E3

Find the kinetic energy of a 0.500 *kg object moving* 8 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.500)(8)^2 \qquad \text{energy of motion}$$

$$KE = 16.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

16.0 J

Kinetic energy, 1000.000 kg

E3

Find the kinetic energy of a 1000.000 *kg object moving* 20 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1000.000)(20)^2 \qquad \text{energy of motion}$$

$$KE = 200000.0 \text{ J} \qquad \text{scalar, } \geq 0$$

Answer

200000.0 J

Kinetic energy, 4.000 kg

E3

Find the kinetic energy of a 4.000 *kg object moving* 15 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(4.000)(15)^2 \qquad \text{energy of motion}$$

$$KE = 450.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

450.0 J

Kinetic energy, 0.145 kg

Find the kinetic energy of a 0.145 *kg object moving* 40 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.145)(40)^2 \qquad \text{energy of motion}$$

$$KE = 116.0 \text{ J} \qquad \text{scalar, } \geq 0$$

Answer

116.0 J

Kinetic energy, 10.000 kg

E3

Find the kinetic energy of a 10.000 *kg object moving* 5 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(10.000)(5)^2 \qquad \text{energy of motion}$$

$$KE = 125.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

125.0 J

Kinetic energy, 70.000 kg

E3

Find the kinetic energy of a 70.000 kg object moving 4 m/s.

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(70.000)(4)^2 \qquad \text{energy of motion}$$

$$KE = 560.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

560.0 J

Kinetic energy, 0.200 kg

Find the kinetic energy of a 0.200 *kg object moving* 30 *m/s.*

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.200)(30)^2 \qquad \text{energy of motion}$$

$$KE = 90.0\ \text{J} \qquad \text{scalar, } \geq 0$$

Answer

90.0 J

Conservative forces

A force does 5 J moving an object out and -5 J bringing it back by another path. Is it conservative?

test = yes (path-independent) closed-loop work = 0?

Answer

yes (path-independent)

Conservative forces

What is the work done by a conservative force around any closed path?

test = 0 closed-loop work = 0?

Answer

0

Conservative forces

E4

Is kinetic friction conservative or non-conservative?

test = non-conservative closed-loop work = 0?

Answer

non-conservative

$F = -dU/dx$ **at** $x = 2$

A particle has potential energy $U(x) = 3x^2$ (J). Find the force at $x = 2$ m.

$$F(x) = -\frac{dU}{dx} = -6x$$

force is minus the slope of U

$$F(2) = -12 \text{ N}$$

Answer

-12 N

$F = -dU/dx$ **at** $x = 2$

A particle has potential energy $U(x) = x^3$ (J). Find the force at $x = 2$ m.

$$F(x) = -\frac{dU}{dx} = -3x^2$$

force is minus the slope of U

$$F(2) = -12 \text{ N}$$

Answer

-12 N

$F = -dU/dx$ **at** $x = 3$

A particle has potential energy $U(x) = x^2 - 4x$ (J). Find the force at $x = 3$ m.

$$F(x) = -\frac{dU}{dx} = -2x + 4$$

force is minus the slope of U

$$F(3) = -2 \text{ N}$$

Answer

-2 N

$F = -dU/dx$ **at** $x = 1$

A particle has potential energy $U(x) = x^3 + 2x^2$ (J). Find the force at $x = 1$ m.

$$F(x) = -\frac{dU}{dx} = -3x^2 - 4x \qquad \text{force is minus the slope of } U$$

$$F(1) = -7 \text{ N}$$

Answer

-7 N

$F = -dU/dx$ **at** $x = 2$

A particle has potential energy $U(x) = -2x^2 + 5$ (J). Find the force at $x = 2$ m.

$$F(x) = -\frac{dU}{dx} = 4x \qquad \text{force is minus the slope of } U$$
$$F(2) = 8 \text{ N}$$

Answer

8 N

Equilibrium point

A particle has $U(x) = 2x^2$ (J). Find the equilibrium position (where $dU/dx = 0$).

$$\frac{dU}{dx} = 4x = 0 \qquad \text{force is zero at equilibrium}$$

$$x_{\text{eq}} = 0 \text{ m}$$

Answer

$$x = 0 \text{ m}$$

Equilibrium point

E5

A particle has $U(x) = x^2 - 8x$ (J). Find the equilibrium position (where $dU/dx = 0$).

$$\frac{dU}{dx} = 2x - 8 = 0 \qquad \text{force is zero at equilibrium}$$

$$x_{\text{eq}} = 4 \text{ m}$$

Answer

$$x = 4 \text{ m}$$

Equilibrium point

A particle has $U(x) = 3x^2$ (J). Find the equilibrium position (where $dU/dx = 0$).

$$\frac{dU}{dx} = 6x = 0 \qquad \text{force is zero at equilibrium}$$

$$x_{\text{eq}} = 0 \text{ m}$$

Answer

$$x = 0 \text{ m}$$

Equilibrium point

A particle has $U(x) = 3x^2 - 12x$ (J). Find the equilibrium position (where $dU/dx = 0$).

$$\frac{dU}{dx} = 6x - 12 = 0 \qquad \text{force is zero at equilibrium}$$

$$x_{\text{eq}} = 2 \text{ m}$$

Answer

$x = 2$ m

KE from energy diagram, $x = 2$

A particle with total energy $E = 16$ J moves in $U(x) = 2x^2$ (J). Find its kinetic energy at $x = 2$ m.

$$E = KE + U \qquad \text{total energy is constant}$$

$$KE = E - U(2) = 16 - 8 = 8.0 \text{ J} \qquad \text{the rest is motion}$$

Answer

8.0 J

KE from energy diagram, $x = 3$

A particle with total energy $E = 25$ J moves in $U(x) = x^2$ (J). Find its kinetic energy at $x = 3$ m.

$$E = KE + U \qquad \text{total energy is constant}$$

$$KE = E - U(3) = 25 - 9 = 16.0 \text{ J} \qquad \text{the rest is motion}$$

Answer

16.0 J

KE from energy diagram, $x = 1$

E6

A particle with total energy $E = 12$ J moves in $U(x) = 3x^2$ (J). Find its kinetic energy at $x = 1$ m.

$$E = KE + U \qquad \text{total energy is constant}$$

$$KE = E - U(1) = 12 - 3 = 9.0 \text{ J} \qquad \text{the rest is motion}$$

Answer

9.0 J

KE from energy diagram, $x = 2$

E6

A particle with total energy $E = 8$ J moves in $U(x) = \frac{1}{2}x^2$ (J). Find its kinetic energy at $x = 2$ m.

$$E = KE + U$$ total energy is constant

$$KE = E - U(2) = 8 - 2 = 6.0 \text{ J}$$ the rest is motion

Answer

6.0 J

Turning point, $k = 200$, $E = 4$

A mass on a spring ($U = \frac{1}{2}kx^2$, $k = 200$ N/m) has total energy $E = 4$ J. Find the turning point (amplitude).

$$U = E: \ \tfrac{1}{2}kx^2 = 4 \qquad \text{at a turning point KE} = 0$$

$$x_{\text{turn}} = \sqrt{2E/k} = 0.200 \text{ m} \qquad \text{where motion reverses}$$

Answer

0.200 m

Turning point, $k = 500$, $E = 10$

A mass on a spring ($U = \frac{1}{2}kx^2$, $k = 500$ N/m) has total energy $E = 10$ J. Find the turning point (amplitude).

$$U = E: \ \tfrac{1}{2}kx^2 = 10 \qquad \text{at a turning point KE} = 0$$

$$x_{\text{turn}} = \sqrt{2E/k} = 0.200 \text{ m} \qquad \text{where motion reverses}$$

Answer

0.200 m

Turning point, $k = 100$, $E = 8$

A mass on a spring ($U = \frac{1}{2}kx^2$, $k = 100$ N/m) has total energy $E = 8$ J. Find the turning point (amplitude).

$$U = E: \ \tfrac{1}{2}kx^2 = 8 \qquad \text{at a turning point KE= 0}$$

$$x_{\text{turn}} = \sqrt{2E/k} = 0.400 \text{ m} \qquad \text{where motion reverses}$$

Answer

0.400 m

Turning point, $k = 800$, $E = 16$

A mass on a spring ($U = \frac{1}{2}kx^2$, $k = 800$ N/m) has total energy $E = 16$ J. Find the turning point (amplitude).

$$U = E: \ \tfrac{1}{2}kx^2 = 16 \qquad \text{at a turning point KE} = 0$$

$$x_{\text{turn}} = \sqrt{2E/k} = 0.200 \text{ m} \qquad \text{where motion reverses}$$

Answer

0.200 m

Spring work $\int kx\,dx$, $k = 200$

E7

How much work stretches a 200 N/m spring by 0.10 m from natural length?

$$W = \int_0^{0.10} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(200)(0.10)^2 = 1.000\ \text{J} \qquad \text{stored as PE}$$

Answer

1.000 J

Spring work $\int kx\,dx$, $k = 500$

How much work stretches a 500 N/m spring by 0.20 m from natural length?

$$W = \int_0^{0.20} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(500)(0.20)^2 = 10.000\ \text{J} \qquad \text{stored as PE}$$

Answer

10.000 J

Spring work $\int kx\,dx$, $k = 100$

E7

How much work stretches a 100 N/m spring by 0.15 m from natural length?

$$W = \int_0^{0.15} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(100)(0.15)^2 = 1.125 \text{ J} \qquad \text{stored as PE}$$

Answer

1.125 J

Spring work $\int kx\,dx$, $k = 800$

How much work stretches a 800 N/m spring by 0.05 m from natural length?

$$W = \int_0^{0.05} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(800)(0.05)^2 = 1.000\ \text{J} \qquad \text{stored as PE}$$

Answer

1.000 J

Spring work $\int kx\,dx$, $k = 300$

How much work stretches a 300 N/m spring by 0.25 m from natural length?

$$W = \int_0^{0.25} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(300)(0.25)^2 = 9.375 \text{ J} \qquad \text{stored as PE}$$

Answer

9.375 J

Spring work $\int kx\,dx$, $k = 400$

How much work stretches a 400 N/m spring by 0.10 m from natural length?

$$W = \int_0^{0.10} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(400)(0.10)^2 = 2.000\ \text{J} \qquad \text{stored as PE}$$

Answer

2.000 J

Spring work $\int kx\,dx$, $k = 250$

How much work stretches a 250 N/m spring by 0.20 m from natural length?

$$W = \int_0^{0.20} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(250)(0.20)^2 = 5.000\ \text{J} \qquad \text{stored as PE}$$

Answer

5.000 J

Spring work $\int kx\,dx$, $k = 600$

How much work stretches a 600 N/m spring by 0.15 m from natural length?

$$W = \int_0^{0.15} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(600)(0.15)^2 = 6.750 \text{ J} \qquad \text{stored as PE}$$

Answer

6.750 J

Spring work $\int kx\,dx$, $k = 150$

E7

How much work stretches a 150 N/m spring by 0.30 m from natural length?

$$W = \int_0^{0.30} kx'\,dx' = frac12kx^2 \qquad \text{integrate Hooke's force}$$

$$W = \tfrac{1}{2}(150)(0.30)^2 = 6.750\ \text{J} \qquad \text{stored as PE}$$

Answer

6.750 J

Conservation, drop 5 m

E8

A 2 kg object falls from rest through 5 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(5)} = 9.90 \text{ m/s} \qquad \text{mass cancels}$$

Answer

9.90 m/s

Conservation, drop 20 m

A 1 kg object falls from rest through 20 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(20)} = 19.80 \text{ m/s} \qquad \text{mass cancels}$$

Answer

19.80 m/s

Conservation, drop 45 m

E8

A 0.5 kg object falls from rest through 45 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(45)} = 29.70 \text{ m/s} \qquad \text{mass cancels}$$

Answer

29.70 m/s

Conservation, drop 10 m

A 5 kg object falls from rest through 10 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(10)} = 14.00 \text{ m/s} \qquad \text{mass cancels}$$

Answer

14.00 m/s

Conservation, drop 8 m

A 3 kg object falls from rest through 8 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(8)} = 12.52 \text{ m/s} \qquad \text{mass cancels}$$

Answer

12.52 m/s

Conservation, drop 30 m

A 2 kg object falls from rest through 30 m (only gravity). Find the speed at the bottom ($g = 9.8$).

$$KE + U = \text{const}: \ \tfrac{1}{2}mv^2 = mgh \qquad \text{mechanical energy conserved}$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(30)} = 24.25 \text{ m/s} \qquad \text{mass cancels}$$

Answer

24.25 m/s

Spring-energy speed, $A = 0.20$

A 1.0 kg mass on a 200 N/m spring, amplitude 0.20 m, passes equilibrium. Find its speed there (energy conservation).

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 \qquad \text{all PE} \to \text{all KE}$$

$$v = A\sqrt{k/m} = 2.828 \text{ m/s}$$

Answer

2.828 m/s

Spring-energy speed, $A = 0.10$

A 0.5 kg mass on a 500 N/m spring, amplitude 0.10 m, passes equilibrium. Find its speed there (energy conservation).

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 \qquad \text{all PE} \to \text{all KE}$$

$$v = A\sqrt{k/m} = 3.162 \text{ m/s}$$

Answer

3.162 m/s

Spring-energy speed, $A = 0.20$

A 2.0 kg mass on a 800 N/m spring, amplitude 0.20 m, passes equilibrium. Find its speed there (energy conservation).

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 \qquad \text{all PE} \to \text{all KE}$$

$$v = A\sqrt{k/m} = 4.000 \text{ m/s}$$

Answer

4.000 m/s

Spring-energy speed, $A = 0.30$

E8

A 0.5 kg mass on a 100 N/m spring, amplitude 0.30 m, passes equilibrium. Find its speed there (energy conservation).

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 \qquad \text{all PE} \to \text{all KE}$$

$$v = A\sqrt{k/m} = 4.243 \text{ m/s}$$

Answer

4.243 m/s

Power $P = Fv$, 200 N at 5 m/s

E2

A force of 200 *N acts on an object moving* 5 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 200(5) \qquad \text{rate of doing work}$$

$$P = 1000.0 \text{ W}$$

Answer

1000.0 W

Power $P = Fv$, 50 N at 20 m/s

E2

A force of 50 *N acts on an object moving* 20 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 50(20) \qquad \text{rate of doing work}$$

$$P = 1000.0 \text{ W}$$

Answer

1000.0 W

Power $P = Fv$, 800 N at 2 m/s

E2

A force of 800 *N acts on an object moving* 2 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 800(2) \qquad \text{rate of doing work}$$

$$P = 1600.0 \text{ W}$$

Answer

1600.0 W

Power $P = Fv$, 30 N at 12 m/s

E2

A force of 30 *N acts on an object moving* 12 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 30(12) \qquad \text{rate of doing work}$$

$$P = 360.0 \text{ W}$$

Answer

360.0 W

Power $P = Fv$, 500 N at 6 m/s

E2

A force of 500 *N acts on an object moving* 6 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 500(6) \qquad \text{rate of doing work}$$

$$P = 3000.0 \text{ W}$$

Answer

3000.0 W

Power $P = Fv$, 120 N at 10 m/s

E2

A force of 120 *N acts on an object moving* 10 *m/s. Find the instantaneous power.*

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = 120(10) \qquad \text{rate of doing work}$$

$$P = 1200.0 \text{ W}$$

Answer

1200.0 W

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Linear Momentum (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Set up → solve → answer
79 problems

Lesson `apc.momentum.session`

Momentum, 1000.000 kg, 10 m/s

Find the momentum of a 1000.000 *kg object moving* 10 *m/s.*

$$p = mv = 1000.000(10) \qquad \text{vector along velocity}$$

$$p = 10000.0\ \text{kg} \cdot \text{m/s}$$

Answer

10000.0 kg · m/s

Momentum, 2.000 kg, 15 m/s

Find the momentum of a 2.000 *kg object moving* 15 *m/s.*

$$p = mv = 2.000(15) \qquad \text{vector along velocity}$$

$$p = 30.0\ \text{kg} \cdot \text{m/s}$$

Answer

30.0 kg · m/s

Momentum, 0.145 kg, 40 m/s

P1

Find the momentum of a 0.145 *kg object moving* 40 *m/s.*

$$p = mv = 0.145(40) \qquad \text{vector along velocity}$$
$$p = 5.8\ \text{kg} \cdot \text{m/s}$$

Answer

5.8 kg · m/s

Momentum, 70.000 kg, 5 m/s

P1

Find the momentum of a 70.000 *kg object moving* 5 *m/s.*

$$p = mv = 70.000(5) \qquad \text{vector along velocity}$$
$$p = 350.0\ \text{kg} \cdot \text{m/s}$$

Answer

350.0 kg · m/s

Momentum, 0.500 kg, 20 m/s

Find the momentum of a 0.500 *kg object moving* 20 *m/s.*

$$p = mv = 0.500(20) \qquad \text{vector along velocity}$$

$$p = 10.0\ \text{kg} \cdot \text{m/s}$$

Answer

10.0 kg · m/s

Momentum, 5000.000 kg, 2 m/s

P1

Find the momentum of a 5000.000 *kg object moving* 2 *m/s.*

$$p = mv = 5000.000(2) \qquad \text{vector along velocity}$$

$$p = 10000.0\ \text{kg} \cdot \text{m/s}$$

Answer

10000.0 kg · m/s

Momentum, 0.020 kg, 300 m/s

P1

Find the momentum of a 0.020 *kg object moving* 300 *m/s.*

$$p = mv = 0.020(300) \qquad \text{vector along velocity}$$

$$p = 6.0\ \text{kg} \cdot \text{m/s}$$

Answer

6.0 kg · m/s

Momentum, 8.000 kg, 12 m/s

Find the momentum of a 8.000 *kg object moving* 12 *m/s.*

$$p = mv = 8.000(12) \qquad \text{vector along velocity}$$

$$p = 96.0\ \text{kg} \cdot \text{m/s}$$

Answer

96.0 kg · m/s

Momentum, 3.000 kg, 7 m/s

P1

Find the momentum of a 3.000 *kg object moving* 7 *m/s.*

$$p = mv = 3.000(7) \qquad \text{vector along velocity}$$

$$p = 21.0\ \text{kg} \cdot \text{m/s}$$

Answer

21.0 kg · m/s

$F = dp/dt$, 2.00 kg

A 2.00 kg body changes velocity from 0 to 10 m/s in 4.00 s. Find the net force.

$$F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t} = \frac{2.00(10-0)}{4.00}$$

force = rate of momentum change

$$F = 5.0 \text{ N}$$

Answer

5.0 N

$F = dp/dt$, 0.50 kg

P1

A 0.50 kg body changes velocity from 20 to 0 m/s in 0.50 s. Find the net force.

$$F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t} = \frac{0.50(0-20)}{0.50}$$

force = rate of momentum change

$$F = -20.0 \text{ N}$$

Answer

-20.0 N

$F = dp/dt$, 1000.00 kg

A 1000.00 kg body changes velocity from 30 to 20 m/s in 5.00 s. Find the net force.

$$F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t} = \frac{1000.00(20 - 30)}{5.00}$$

force = rate of momentum change

$$F = -2000.0 \text{ N}$$

Answer

-2000.0 N

$F = dp/dt$, 0.14 kg

A 0.14 kg body changes velocity from 40 to −40 m/s in 0.00 s. Find the net force.

$$F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t} = \frac{0.14(-40-40)}{0.00}$$

force = rate of momentum change

$$F = -5800.0 \text{ N}$$

Answer

−5800.0 N

$F = dp/dt$, 5.00 kg

A 5.00 kg body changes velocity from 2 to 8 m/s in 3.00 s. Find the net force.

$$F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t} = \frac{5.00(8-2)}{3.00}$$

force = rate of momentum change

$$F = 10.0 \text{ N}$$

Answer

10.0 N

Impulse $\int F\,dt$, $[0, 3]$

A force $F(t) = 6t$ (N) acts from $t = 0$ to $t = 3$ s. Find the impulse.

$$J = \int_0^3 (6t)\,dt \qquad \text{impulse = area under } F\text{-}t$$

$$J = 27\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

27 N · s

Impulse $\int F\,dt$, $[0, 4]$

P2

A force $F(t) = 2t + 4$ (N) acts from $t = 0$ to $t = 4$ s. Find the impulse.

$$J = \int_0^4 (2t + 4)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 32\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

32 N · s

Impulse $\int F\,dt$, $[0, 2]$

A force $F(t) = 3t^2$ (N) acts from $t = 0$ to $t = 2$ s. Find the impulse.

$$J = \int_0^2 (3t^2)\,dt \qquad \text{impulse = area under } F\text{-}t$$

$$J = 8\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

8 N · s

Impulse $\int F\,dt$, $[0, 4]$

A force $F(t) = -2t + 10$ (N) acts from $t = 0$ to $t = 4$ s. Find the impulse.

$$J = \int_0^4 (-2t + 10)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 24\ \text{N} \cdot \text{s} \qquad = \text{change in momentum}$$

Answer

24 N · s

Impulse $\int F\,dt$, $[0, 3]$

A force $F(t) = 12t$ (N) acts from $t = 0$ to $t = 3$ s. Find the impulse.

$$J = \int_0^3 (12t)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 54\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

54 N · s

Impulse $\int F\,dt$, $[1, 3]$

A force $F(t) = 3t^2 + 5$ (N) acts from $t = 1$ to $t = 3$ s. Find the impulse.

$$J = \int_1^3 (3t^2 + 5)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 36\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

36 N · s

Impulse $\int F\,dt$, $[0, 2]$

A force $F(t) = 4t + 8$ (N) acts from $t = 0$ to $t = 2$ s. Find the impulse.

$$J = \int_0^2 (4t + 8)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 24\ \text{N} \cdot \text{s} \qquad = \text{change in momentum}$$

Answer

24 N · s

Impulse $\int F\,dt$, $[0, 2]$

A force $F(t) = 6t^2$ (N) acts from $t = 0$ to $t = 2$ s. Find the impulse.

$$J = \int_0^2 (6t^2)\,dt \qquad \text{impulse = area under } F\text{-}t$$

$$J = 16\ \text{N}\cdot\text{s} \qquad = \text{change in momentum}$$

Answer

16 N · s

Impulse $\int F\,dt$, $[0, 3]$

A force $F(t) = -5t + 20$ (N) acts from $t = 0$ to $t = 3$ s. Find the impulse.

$$J = \int_0^3 (-5t + 20)\,dt \qquad \text{impulse} = \text{area under } F\text{-}t$$

$$J = 37.500 \text{ N} \cdot \text{s} \qquad = \text{change in momentum}$$

Answer

37.500 N · s

Final speed from impulse

P2

A force $F(t) = 6t$ (N) acts on a 3 kg body ($v_i = 0$ m/s) from $t = 0$ to $t = 2$ s. Find the final speed.

$$J = \int F\,dt = 12.0 \text{ N}\cdot\text{s} \qquad \text{impulse}$$

$$v_f = v_i + \frac{J}{m} = 0 + \frac{12.0}{3} = 4.00 \text{ m/s} \qquad J = m\Delta v$$

Answer

4.00 m/s

Final speed from impulse

A force $F(t) = -2t + 10$ (N) acts on a 2 kg body ($v_i = 5$ m/s) from $t = 0$ to $t = 4$ s. Find the final speed.

$$J = \int F\,dt = 24.0\ \text{N}\cdot\text{s} \qquad \text{impulse}$$

$$v_f = v_i + \frac{J}{m} = 5 + \frac{24.0}{2} = 17.00\ \text{m/s} \qquad J = m\Delta v$$

Answer

17.00 m/s

Recoil, 0.020 & 4 kg

P3

From rest, a 0.020 kg mass moves off at 400 m/s. How fast does the 4 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 2.00\ \text{m/s} \qquad \text{opposite direction}$$

Answer

2.00 m/s

Recoil, 60.000 & 40 kg

From rest, a 60.000 kg mass moves off at 3 m/s. How fast does the 40 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 4.50 \text{ m/s} \qquad \text{opposite direction}$$

Answer

4.50 m/s

Recoil, 2.000 & 8 kg

From rest, a 2.000 kg mass moves off at 5 m/s. How fast does the 8 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 1.25 \text{ m/s} \qquad \text{opposite direction}$$

Answer

1.25 m/s

Recoil, 1000.000 & 250 kg

From rest, a 1000.000 *kg mass moves off at* 1 *m/s. How fast does the* 250 *kg mass recoil (no external force)?*

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 4.00\ \text{m/s} \qquad \text{opposite direction}$$

Answer

4.00 m/s

Recoil, 5.000 & 20 kg

From rest, a 5.000 kg mass moves off at 4 m/s. How fast does the 20 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 1.00 \text{ m/s} \qquad \text{opposite direction}$$

Answer

1.00 m/s

Recoil, 0.500 & 3 kg

From rest, a 0.500 kg mass moves off at 12 m/s. How fast does the 3 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 2.00 \text{ m/s} \qquad \text{opposite direction}$$

Answer

2.00 m/s

Recoil, 80.000 & 60 kg

From rest, a 80.000 kg mass moves off at 2 m/s. How fast does the 60 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 2.67 \text{ m/s} \qquad \text{opposite direction}$$

Answer

2.67 m/s

Recoil, 1.000 & 5 kg

From rest, a 1.000 kg mass moves off at 10 m/s. How fast does the 5 kg mass recoil (no external force)?

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{total momentum conserved}$$

$$v_2 = \frac{m_1 v_1}{m_2} = 2.00 \text{ m/s} \qquad \text{opposite direction}$$

Answer

2.00 m/s

Perfectly inelastic, 2.00&3 kg

A 2.00 kg object at 6 m/s and a 3 kg object at 0 m/s collide and stick. Find the common velocity.

$$(m_1+m_2)v_f = m_1u_1 + m_2u_2 = 12.0 \text{ kg}\cdot\text{m/s} \quad \text{momentum conserved}$$

$$v_f = \frac{12.0}{5} = 2.40 \text{ m/s} \quad \text{move as one}$$

Answer

2.40 m/s

Perfectly inelastic, 1.00&4 kg

A 1.00 kg object at 10 m/s and a 4 kg object at 0 m/s collide and stick. Find the common velocity.

$$(m_1+m_2)v_f = m_1u_1 + m_2u_2 = 10.0\ \text{kg}\cdot\text{m/s} \qquad \text{momentum conserved}$$

$$v_f = \frac{10.0}{5} = 2.00\ \text{m/s} \qquad \text{move as one}$$

Answer

2.00 m/s

Perfectly inelastic, 1000.00&1500 kg

A 1000.00 kg object at 20 m/s and a 1500 kg object at 0 m/s collide and stick. Find the common velocity.

$$(m_1+m_2)v_f = m_1u_1 + m_2u_2 = 20000.0 \text{ kg}\cdot\text{m/s} \quad \text{momentum conserved}$$

$$v_f = \frac{20000.0}{2500} = 8.00 \text{ m/s} \quad \text{move as one}$$

Answer

8.00 m/s

Perfectly inelastic, 2.00&2 kg

A 2.00 kg object at 5 m/s and a 2 kg object at −3 m/s collide and stick. Find the common velocity.

$$(m_1+m_2)v_f = m_1u_1 + m_2u_2 = 4.0 \text{ kg}\cdot\text{m/s} \quad \text{momentum conserved}$$

$$v_f = \frac{4.0}{4} = 1.00 \text{ m/s} \quad \text{move as one}$$

Answer

1.00 m/s

Perfectly inelastic, 60.00&40 kg

A 60.00 kg object at 4 m/s and a 40 kg object at 0 m/s collide and stick. Find the common velocity.

$$(m_1+m_2)v_f = m_1u_1 + m_2u_2 = 240.0 \text{ kg}\cdot\text{m/s} \quad \text{momentum conserved}$$

$$v_f = \frac{240.0}{100} = 2.40 \text{ m/s} \quad \text{move as one}$$

Answer

2.40 m/s

1D elastic, 3→1 kg

Elastic collision: 3 *kg at* 8 *m/s hits* 1 *kg at rest. Find both final velocities.*

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} u_1 = 4.00 \text{ m/s}$$

elastic: p and KE conserved

$$v_2' = \frac{2m_1}{m_1 + m_2} u_1 = 12.00 \text{ m/s}$$

Answer

$$v_1' = 4.00, \; v_2' = 12.00 \text{ m/s}$$

1D elastic, 1→3 kg

Elastic collision: 1 *kg at* 8 *m/s hits* 3 *kg at rest. Find both final velocities.*

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} u_1 = -4.00 \text{ m/s}$$

elastic: p and KE conserved

$$v_2' = \frac{2m_1}{m_1 + m_2} u_1 = 4.00 \text{ m/s}$$

Answer

$$v_1' = -4.00, \; v_2' = 4.00 \text{ m/s}$$

1D elastic, 4→2 kg

Elastic collision: 4 kg at 6 m/s hits 2 kg at rest. Find both final velocities.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} u_1 = 2.00 \text{ m/s}$$

elastic: p and KE conserved

$$v_2' = \frac{2m_1}{m_1 + m_2} u_1 = 8.00 \text{ m/s}$$

Answer

$$v_1' = 2.00, \; v_2' = 8.00 \text{ m/s}$$

1D elastic, 2→6 kg

Elastic collision: 2 kg at 9 m/s hits 6 kg at rest. Find both final velocities.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} u_1 = -4.50 \text{ m/s}$$

elastic: p and KE conserved

$$v_2' = \frac{2m_1}{m_1 + m_2} u_1 = 4.50 \text{ m/s}$$

Answer

$$v_1' = -4.50, \; v_2' = 4.50 \text{ m/s}$$

1D elastic, 5→1 kg

Elastic collision: 5 *kg at* 10 *m/s hits* 1 *kg at rest. Find both final velocities.*

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} u_1 = 6.67 \text{ m/s}$$

elastic: p and KE conserved

$$v_2' = \frac{2m_1}{m_1 + m_2} u_1 = 16.67 \text{ m/s}$$

Answer

$$v_1' = 6.67, \; v_2' = 16.67 \text{ m/s}$$

Center of mass, 2 masses

Masses $2, 3$ kg sit at $x = 0,\ 5$ m. Find the center of mass.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{2(0) + 3(5)}{5} \qquad \textit{mass-weighted average}$$

$$x_{cm} = 3.00\ m$$

Answer

$3.00\ m$

Center of mass, 3 masses

Masses $1, 1, 2$ *kg sit at* $x = 0,\ 4,\ 8$ *m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1(0) + 1(4) + 2(8)}{4}$$

mass-weighted average

$$x_{cm} = 5.00\ m$$

Answer

5.00 m

Center of mass, 2 masses

Masses $4, 6$ *kg sit at* $x = 2,\ 12$ *m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{4(2) + 6(12)}{10}$$

mass-weighted average

$$x_{cm} = 8.00\ m$$

Answer

$8.00\ m$

Center of mass, 3 masses

Masses $3, 3, 3$ *kg sit at* $x =$*0, 3, 9 m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{3(0) + 3(3) + 3(9)}{9}$$

mass-weighted average

$$x_{cm} = 4.00 \ m$$

Answer

4.00 m

Center of mass, 2 masses

Masses $5, 1$ *kg sit at* $x = 0,\ 6$ *m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{5(0) + 1(6)}{6} \qquad \textit{mass-weighted average}$$

$$x_{cm} = 1.00\ m$$

Answer

$1.00\ m$

Center of mass, 3 masses

Masses $2, 2, 4$ kg sit at $x = 1,\ 5,\ 10$ m. Find the center of mass.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{2(1) + 2(5) + 4(10)}{8}$$

mass-weighted average

$$x_{cm} = 6.50\ m$$

Answer

$6.50\ m$

Center of mass, 3 masses

Masses $1, 2, 3$ *kg sit at* $x =$*0, 2, 4 m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1(0) + 2(2) + 3(4)}{6}$$

mass-weighted average

$$x_{cm} = 2.67 \ m$$

Answer

2.67 *m*

Center of mass, 2 masses

Masses $6, 2$ *kg sit at* $x = 0,\ 8$ *m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{6(0) + 2(8)}{8}$$

mass-weighted average

$$x_{cm} = 2.00\ m$$

Answer

$2.00\ m$

Center of mass, 2 masses

Masses $4, 4$ *kg sit at* $x = 3,\ 9$ *m. Find the center of mass.*

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{4(3) + 4(9)}{8}$$

mass-weighted average

$$x_{cm} = 6.00\ m$$

Answer

$6.00\ m$

CM velocity, 2 bodies

Masses $2, 3$ kg move at $4, -1$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{2(4) + 3(-1)}{5}$$ total p / total m

$$v_{cm} = 1.00 \text{ m/s}$$ external force sets its accel

Answer

1.00 m/s

CM velocity, 2 bodies

Masses $1,4$ kg move at $10,0$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{1(10)+4(0)}{5}$$ total p / total m

$$v_{cm} = 2.00 \text{ m/s}$$ external force sets its accel

Answer

2.00 m/s

CM velocity, 2 bodies

Masses $3,3$ kg move at $2,6$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{3(2)+3(6)}{6} \qquad \text{total } p \;/\; \text{total } m$$

$$v_{cm} = 4.00 \text{ m/s} \qquad \text{external force sets its accel}$$

Answer

4.00 m/s

CM velocity, 2 bodies

Masses $5, 5$ kg move at $3, -3$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{5(3) + 5(-3)}{10}$$ total p / total m

$$v_{cm} = 0.00 \text{ m/s}$$ external force sets its accel

Answer

0.00 m/s

CM velocity, 2 bodies

Masses $2,6$ kg move at $9,1$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{2(9)+6(1)}{8} \qquad \text{total } p \text{ / total } m$$

$$v_{cm} = 3.00 \text{ m/s} \qquad \text{external force sets its accel}$$

Answer

3.00 m/s

CM velocity, 3 bodies

P6

Masses $1, 1, 2$ kg move at $6, 0, -3$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{1(6) + 1(0) + 2(-3)}{4}$$ total p / total m

$$v_{cm} = 0.00 \text{ m/s}$$ external force sets its accel

Answer

0.00 m/s

CM velocity, 2 bodies

Masses $4, 2$ kg move at $5, 2$ m/s. Find the velocity of the center of mass.

$$v_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{4(5) + 2(2)}{6} \qquad \text{total } p \text{ / total } m$$

$$v_{cm} = 4.00 \text{ m/s} \qquad \text{external force sets its accel}$$

Answer

4.00 m/s

Rocket thrust, u=2000, rate=5

A rocket ejects exhaust at 2000 m/s (relative), burning 5 kg/s. Find the thrust.

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 2000(5) \qquad \text{momentum carried by exhaust}$$

$$F_{\text{thrust}} = 10000 \text{ N}$$

Answer

10000 N

Rocket thrust, u=3000, rate=10

A rocket ejects exhaust at 3000 m/s (relative), burning 10 kg/s. Find the thrust.

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 3000(10) \qquad \text{momentum carried by exhaust}$$

$$F_{\text{thrust}} = 30000 \text{ N}$$

Answer

30000 N

Rocket thrust, u=2500, rate=8

A rocket ejects exhaust at 2500 m/s (relative), burning 8 kg/s. Find the thrust.

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 2500(8) \qquad \text{momentum carried by exhaust}$$

$$F_{\text{thrust}} = 20000\ \text{N}$$

Answer

20000 N

Rocket thrust, u=4000, rate=20

A rocket ejects exhaust at 4000 m/s (relative), burning 20 kg/s. Find the thrust.

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 4000(20) \qquad \text{momentum carried by exhaust}$$

$$F_{\text{thrust}} = 80000 \text{ N}$$

Answer

80000 N

Rocket thrust, u=3500, rate=15

A rocket ejects exhaust at 3500 *m/s (relative), burning* 15 *kg/s. Find the thrust.*

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 3500(15) \qquad \text{momentum carried by exhaust}$$

$$F_{\text{thrust}} = 52500 \text{ N}$$

Answer

52500 N

Rocket thrust, u=2000, rate=25

A rocket ejects exhaust at 2000 m/s (relative), burning 25 kg/s. Find the thrust.

$$F_{\text{thrust}} = u\left|\frac{dm}{dt}\right| = 2000(25)$$ momentum carried by exhaust

$$F_{\text{thrust}} = 50000 \text{ N}$$

Answer

50000 N

Tsiolkovsky, 1000→400 kg

A rocket ($u = 2500$ m/s) burns from 1000 kg to 400 kg. Find the speed gained (Tsiolkovsky equation).

$$\Delta v = u \ln \frac{m_0}{m_f} = 2500 \ln \frac{1000}{400}$$ integrate the rocket equation

$$\Delta v = 2291 \text{ m/s}$$

Answer

2291 m/s

Tsiolkovsky, 5000→1000 kg

A rocket ($u = 3000$ m/s) burns from 5000 kg to 1000 kg. Find the speed gained (Tsiolkovsky equation).

$$\Delta v = u \ln \frac{m_0}{m_f} = 3000 \ln \frac{5000}{1000}$$ integrate the rocket equation

$$\Delta v = 4828 \text{ m/s}$$

Answer

4828 m/s

Tsiolkovsky, 800→200 kg

A rocket ($u = 2000$ m/s) burns from 800 kg to 200 kg. Find the speed gained (Tsiolkovsky equation).

$$\Delta v = u \ln \frac{m_0}{m_f} = 2000 \ln \frac{800}{200}$$

integrate the rocket equation

$$\Delta v = 2773 \text{ m/s}$$

Answer

2773 m/s

Tsiolkovsky, 2000→500 kg

A rocket ($u = 2800$ m/s) burns from 2000 kg to 500 kg. Find the speed gained (Tsiolkovsky equation).

$$\Delta v = u \ln \frac{m_0}{m_f} = 2800 \ln \frac{2000}{500}$$

integrate the rocket equation

$$\Delta v = 3882 \text{ m/s}$$

Answer

3882 m/s

Tsiolkovsky, 10000→2000 kg

A rocket ($u = 3200$ m/s) burns from 10000 kg to 2000 kg. Find the speed gained (Tsiolkovsky equation).

$$\Delta v = u \ln \frac{m_0}{m_f} = 3200 \ln \frac{10000}{2000}$$ integrate the rocket equation

$$\Delta v = 5150 \text{ m/s}$$

Answer

5150 m/s

CM of a rod, uniform, L=4

A rod of length 4 m has linear density uniform. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^4 x\lambda\, dx}{\int_0^4 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 2 \text{ m} \qquad \text{toward the dense end}$$

Answer

2 m

CM of a rod, $\lambda \propto x$, L=3

A rod of length 3 m has linear density $\lambda \propto x$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^3 x\lambda\, dx}{\int_0^3 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 2 \text{ m} \qquad \text{toward the dense end}$$

Answer

2 m

CM of a rod, $\lambda \propto x^2$, L=2

A rod of length 2 m has linear density $\lambda \propto x^2$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^2 x\lambda\, dx}{\int_0^2 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 1.500 \text{ m} \qquad \text{toward the dense end}$$

Answer

1.500 m

CM of a rod, $\lambda \propto x$, L=6

A rod of length 6 m has linear density $\lambda \propto x$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^6 x\lambda\, dx}{\int_0^6 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 4 \text{ m} \qquad \text{toward the dense end}$$

Answer

4 m

CM of a rod, uniform, L=5

P8

A rod of length 5 m has linear density uniform. Find its center of mass using $x_{cm} = \dfrac{\int x\,dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^5 x\lambda\,dx}{\int_0^5 \lambda\,dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 2.500\ \text{m} \qquad \text{toward the dense end}$$

Answer

2.500 m

CM of a rod, $\lambda \propto x^2$, L=3

A rod of length 3 m has linear density $\lambda \propto x^2$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^3 x\lambda\, dx}{\int_0^3 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 2.250 \text{ m} \qquad \text{toward the dense end}$$

Answer

2.250 m

CM of a rod, $\lambda \propto x$, L=4

A rod of length 4 m has linear density $\lambda \propto x$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^4 x\lambda\, dx}{\int_0^4 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 2.667 \text{ m} \qquad \text{toward the dense end}$$

Answer

2.667 m

CM of a rod, $\lambda \propto x^2$, L=4

A rod of length 4 m has linear density $\lambda \propto x^2$. Find its center of mass using $x_{cm} = \dfrac{\int x\, dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^4 x\lambda\, dx}{\int_0^4 \lambda\, dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 3 \text{ m} \qquad \text{toward the dense end}$$

Answer

3 m

CM of a rod, uniform, L=8

P8

A rod of length 8 m has linear density uniform. Find its center of mass using $x_{cm} = \dfrac{\int x\,dm}{\int dm}$.

$$x_{cm} = \frac{\int_0^8 x\lambda\,dx}{\int_0^8 \lambda\,dx} \qquad \text{continuous mass average}$$

$$x_{cm} = 4\ \text{m} \qquad \text{toward the dense end}$$

Answer

4 m

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Rotation (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Set up → solve → answer
82 problems

Lesson `apc.rotation.session`

Explore this live | Watch the walk-through

$\omega = d\theta/dt$ **at** $t = 2$

A body has $\theta(t) = 2t^2 + 3t$ (rad). Find its angular velocity at $t = 2$ s.

$$\omega(t) = \frac{d\theta}{dt} = 4t + 3 \qquad \text{differentiate angle}$$

$$\omega(2) = 11 \text{ rad/s}$$

Answer

11 rad/s

$\omega = d\theta/dt$ **at** $t = 1$

R1

A body has $\theta(t) = -5t^2 + 2$ (rad). Find its angular velocity at $t = 1$ s.

$$\omega(t) = \frac{d\theta}{dt} = -10t \qquad \text{differentiate angle}$$

$$\omega(1) = -10 \text{ rad/s}$$

Answer

-10 rad/s

$\omega = d\theta/dt$ **at** $t = 2$

A body has $\theta(t) = t^3 + 4t$ (rad). Find its angular velocity at $t = 2$ s.

$$\omega(t) = \frac{d\theta}{dt} = 3t^2 + 4 \qquad \text{differentiate angle}$$

$$\omega(2) = 16 \text{ rad/s}$$

Answer

16 rad/s

$\omega = d\theta/dt$ **at** $t = 3$

R1

A body has $\theta(t) = 3t^2 - 2t + 1$ (rad). Find its angular velocity at $t = 3$ s.

$$\omega(t) = \frac{d\theta}{dt} = 6t - 2 \qquad \text{differentiate angle}$$

$$\omega(3) = 16 \text{ rad/s}$$

Answer

16 rad/s

$\omega = d\theta/dt$ **at** $t = 1$

R1

A body has $\theta(t) = 6t^2$ (rad). Find its angular velocity at $t = 1$ s.

$$\omega(t) = \frac{d\theta}{dt} = 12t \qquad \text{differentiate angle}$$

$$\omega(1) = 12 \text{ rad/s}$$

Answer

12 rad/s

$\omega = d\theta/dt$ **at** $t = 1$

R1

A body has $\theta(t) = -4t^2 + 8t$ (rad). Find its angular velocity at $t = 1$ s.

$$\omega(t) = \frac{d\theta}{dt} = -8t + 8 \qquad \text{differentiate angle}$$

$$\omega(1) = 0 \text{ rad/s}$$

Answer

0 rad/s

$\alpha = d^2\theta/dt^2$ **at** $t = 2$ R1

For $\theta(t) = 3t^2$ (rad), find the angular acceleration at $t = 2$ s.

$$\alpha(t) = \frac{d^2\theta}{dt^2} = 6 \qquad \text{second derivative}$$

$$\alpha(2) = 6 \text{ rad/s}^2$$

Answer

6 rad/s^2

$\alpha = d^2\theta/dt^2$ **at** $t = 1$

R1

For $\theta(t) = t^3 - 3t^2 + 2t + 1$ (rad), find the angular acceleration at $t = 1$ s.

$$\alpha(t) = \frac{d^2\theta}{dt^2} = 6t - 6 \qquad \text{second derivative}$$

$$\alpha(1) = 0 \text{ rad/s}^2$$

Answer

0 rad/s^2

$\alpha = d^2\theta/dt^2$ **at** $t = 2$ R1

For $\theta(t) = -t^3 + 5t$ (rad), find the angular acceleration at $t = 2$ s.

$$\alpha(t) = \frac{d^2\theta}{dt^2} = -6t \qquad \text{second derivative}$$

$$\alpha(2) = -12 \text{ rad/s}^2$$

Answer

-12 rad/s^2

ω from $\alpha(t)$ at $t = 3$

A wheel has $\alpha(t) = 6$ (rad/s^2) and $\omega_0 = 0$ rad/s. Find ω at $t = 3$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = 6t \qquad \text{integrate acceleration}$$

$$\omega(3) = 18 \text{ rad/s}$$

Answer

18 rad/s

ω from $\alpha(t)$ at $t = 2$

A wheel has $\alpha(t) = 4t$ (rad/s^2) and $\omega_0 = 2$ rad/s. Find ω at $t = 2$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = 2t^2 + 2 \qquad \text{integrate acceleration}$$

$$\omega(2) = 10 \text{ rad/s}$$

Answer

10 rad/s

ω from $\alpha(t)$ at $t = 2$

A wheel has $\alpha(t) = 6t + 2$ (rad/s^2) and $\omega_0 = 1$ rad/s. Find ω at $t = 2$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = 3t^2 + 2t + 1 \qquad \text{integrate acceleration}$$

$$\omega(2) = 17 \text{ rad/s}$$

Answer

17 rad/s

ω from $\alpha(t)$ at $t = 2$

A wheel has $\alpha(t) = 3t^2$ (rad/s^2) and $\omega_0 = 0$ rad/s. Find ω at $t = 2$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = t^3 \qquad \text{integrate acceleration}$$
$$\omega(2) = 8 \text{ rad/s}$$

Answer

8 rad/s

ω from $\alpha(t)$ at $t = 3$

R2

A wheel has $\alpha(t) = -2t + 4$ (rad/s^2) and $\omega_0 = 5$ rad/s. Find ω at $t = 3$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = -t^2 + 4t + 5 \qquad \text{integrate acceleration}$$

$$\omega(3) = 8 \text{ rad/s}$$

Answer

8 rad/s

ω from $\alpha(t)$ at $t = 2$

A wheel has $\alpha(t) = 8t$ (rad/s^2) and $\omega_0 = 0$ rad/s. Find ω at $t = 2$ s.

$$\omega(t) = \omega_0 + \int_0^t \alpha\, dt' = 4t^2 \qquad \text{integrate acceleration}$$

$$\omega(2) = 16 \text{ rad/s}$$

Answer

16 rad/s

θ from $\omega(t)$ at $t = 2$

A disk has $\omega(t) = 6t$ (rad/s), $\theta_0 = 0$ rad. Find the angle at $t = 2$ s.

$$\theta(t) = \theta_0 + \int_0^t \omega\, dt' = 3t^2 \qquad \text{integrate angular velocity}$$

$$\theta(2) = 12 \text{ rad}$$

Answer

12 rad

θ from $\omega(t)$ at $t = 2$

R2

A disk has $\omega(t) = 2t + 3$ (rad/s), $\theta_0 = 1$ rad. Find the angle at $t = 2$ s.

$$\theta(t) = \theta_0 + \int_0^t \omega\, dt' = t^2 + 3t + 1 \qquad \text{integrate angular velocity}$$

$$\theta(2) = 11 \text{ rad}$$

Answer

11 rad

θ from $\omega(t)$ at $t = 2$

A disk has $\omega(t) = 3t^2$ (rad/s), $\theta_0 = 0$ rad. Find the angle at $t = 2$ s.

$$\theta(t) = \theta_0 + \int_0^t \omega\, dt' = t^3 \qquad \text{integrate angular velocity}$$
$$\theta(2) = 8 \text{ rad}$$

Answer

8 rad

Torque, r=0.80 m, 90°

A 50 N force acts 0.80 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.80(50)\sin 90^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 40.00\ \text{N} \cdot \text{m}$$

Answer

40.00 N · m

Torque, r=1.00 m, 90°

A 20 N force acts 1.00 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 1.00(20)\sin 90^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 20.00\ \text{N} \cdot \text{m}$$

Answer

20.00 N · m

Torque, r=0.50 m, 60°

A 100 N force acts 0.50 m from the axis at 60° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.50(100)\sin 60^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 43.30\ \text{N} \cdot \text{m}$$

Answer

43.30 N · m

Torque, r=0.05 m, 90°

A 200 N force acts 0.05 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.05(200)\sin 90° \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 10.00\ \text{N} \cdot \text{m}$$

Answer

10.00 N · m

Torque, r=0.60 m, 30°

A 40 N force acts 0.60 m from the axis at 30° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.60(40)\sin 30^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 12.00\ \text{N} \cdot \text{m}$$

Answer

12.00 N · m

Torque, r=0.80 m, 0°

A 50 N force acts 0.80 m from the axis at 0° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.80(50)\sin 0^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 0.00\ \text{N}\cdot\text{m}(\textit{alongthearm} : \textit{notorque})$$

Answer

0.00 N · m

Torque, r=1.20 m, 90°

A 25 N force acts 1.20 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 1.20(25)\sin 90^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 30.00\ \text{N} \cdot \text{m}$$

Answer

30.00 N · m

Torque, r=0.40 m, 45°

A 60 N force acts 0.40 m from the axis at 45° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.40(60)\sin 45° \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 16.97 \text{ N} \cdot \text{m}$$

Answer

16.97 N · m

Torque, r=0.70 m, 90°

A 80 N force acts 0.70 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.70(80)\sin 90° \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 56.00\ \text{N} \cdot \text{m}$$

Answer

56.00 N · m

Torque, r=0.90 m, 60°

A 30 N force acts 0.90 m from the axis at 60° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.90(30)\sin 60^\circ \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 23.38\ \text{N} \cdot \text{m}$$

Answer

23.38 N · m

Torque, r=0.30 m, 90°

A 150 N force acts 0.30 m from the axis at 90° to the arm. Find the torque.

$$\tau = rF\sin\theta = 0.30(150)\sin 90° \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 45.00\ \text{N} \cdot \text{m}$$

Answer

45.00 N · m

Rod $I = \int r^2\,dm$, M=3, L=2.0

R4

Derive the moment of inertia of a uniform rod ($M = 3$ kg, $L = 2.0$ m) about one end via $I = \int_0^L x^2\,dm$, $dm = \frac{M}{L}dx$.

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L} \cdot \frac{L^3}{3} = \frac{1}{3}ML^2 \qquad \text{integrate over the length}$$

$$I = 4.000\ \text{kg} \cdot \text{m}^2 \qquad \text{about the end}$$

Answer

4.000 kg · m^2

Rod $I = \int r^2\,dm$, M=6, L=1.0

R4

Derive the moment of inertia of a uniform rod ($M = 6$ kg, $L = 1.0$ m) about one end via $I = \int_0^L x^2\,dm$, $dm = \frac{M}{L}dx$.

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L} \cdot \frac{L^3}{3} = \frac{1}{3}ML^2 \qquad \text{integrate over the length}$$

$$I = 2.000\ \text{kg} \cdot \text{m}^2 \qquad \text{about the end}$$

Answer

2.000 kg · m^2

Rod $I = \int r^2\,dm$, M=2, L=3.0

Derive the moment of inertia of a uniform rod ($M = 2$ kg, $L = 3.0$ m) about one end via $I = \int_0^L x^2\,dm$, $dm = \frac{M}{L}dx$.

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L} \cdot \frac{L^3}{3} = \frac{1}{3}ML^2 \qquad \text{integrate over the length}$$

$$I = 6.000\ \text{kg} \cdot \text{m}^2 \qquad \text{about the end}$$

Answer

6.000 kg · m^2

Rod $I = \int r^2\,dm$, M=4, L=1.5

R4

Derive the moment of inertia of a uniform rod ($M = 4$ kg, $L = 1.5$ m) about one end via $I = \int_0^L x^2\,dm$, $dm = \frac{M}{L}dx$.

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L} \cdot \frac{L^3}{3} = \frac{1}{3}ML^2 \qquad \text{integrate over the length}$$

$$I = 3.000\ \text{kg} \cdot \text{m}^2 \qquad \text{about the end}$$

Answer

3.000 kg · m^2

Rod $I = \int r^2\,dm$, M=5, L=2.0

R4

Derive the moment of inertia of a uniform rod ($M = 5$ kg, $L = 2.0$ m) about one end via $I = \int_0^L x^2\,dm$, $dm = \frac{M}{L}dx$.

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L} \cdot \frac{L^3}{3} = \frac{1}{3}ML^2$$ integrate over the length

$$I = 6.667\ \text{kg} \cdot \text{m}^2$$ about the end

Answer

6.667 kg · m^2

Inertia of a solid disk

R4

Find the moment of inertia of a solid disk: $M = 2$ kg, $R = 0.50$ m ($I = \frac{1}{2}MR^2$).

$$I = \frac{1}{2}MR^2 = \frac{1}{2}2(0.50)^2 \qquad \text{standard result}$$

$$I = 0.250\ \text{kg} \cdot \text{m}^2$$

Answer

0.250 kg $\cdot$ m^2

Inertia of a hoop

Find the moment of inertia of a hoop: $M = 3$ kg, $R = 0.40$ m ($I = MR^2$).

$$I = MR^2 = 3(0.40)^2 \qquad \text{standard result}$$

$$I = 0.480 \text{ kg} \cdot \text{m}^2$$

Answer

0.480 kg $\cdot$ m^2

Inertia of a solid sphere

R4

Find the moment of inertia of a solid sphere: $M = 5$ kg, $R = 0.20$ m ($I = \frac{2}{5}MR^2$).

$$I = \frac{2}{5}MR^2 = \frac{2}{5}5(0.20)^2 \qquad \text{standard result}$$

$$I = 0.080 \text{ kg} \cdot \text{m}^2$$

Answer

0.080 kg $\cdot$ m^2

Inertia of a solid disk

R4

Find the moment of inertia of a solid disk: $M = 1$ kg, $R = 1.00$ m ($I = \frac{1}{2}MR^2$).

$$I = \frac{1}{2}MR^2 = \frac{1}{2}1(1.00)^2 \qquad \text{standard result}$$

$$I = 0.500\ \text{kg} \cdot \text{m}^2$$

Answer

0.500 kg · m^2

Inertia of a hoop

Find the moment of inertia of a hoop: $M = 4$ kg, $R = 0.30$ m ($I = MR^2$).

$$I = MR^2 = 4(0.30)^2 \qquad \text{standard result}$$
$$I = 0.360 \text{ kg} \cdot \text{m}^2$$

Answer

0.360 kg $\cdot$ m^2

Inertia of a solid sphere

R4

Find the moment of inertia of a solid sphere: $M = 2$ kg, $R = 0.60$ m ($I = \frac{2}{5}MR^2$).

$$I = \frac{2}{5}MR^2 = \frac{2}{5}2(0.60)^2 \qquad \text{standard result}$$

$$I = 0.288 \text{ kg} \cdot \text{m}^2$$

Answer

0.288 kg $\cdot$ m^2

Parallel axis, disk

A disk has $I_{cm} = 0.090$ kg·m^2 and mass $M = 2$ kg. Find I about a parallel axis $d = 0.3$ m away.

$$I = I_{cm} + Md^2 = 0.090 + 2(0.3)^2 \qquad \text{parallel-axis theorem}$$

$$I = 0.270 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

0.270 kg · m^2

Parallel axis, sphere

R5

A sphere has $I_{cm} = 0.048$ kg·m^2 and mass $M = 3$ kg. Find I about a parallel axis $d = 0.5$ m away.

$$I = I_{cm} + Md^2 = 0.048 + 3(0.5)^2 \qquad \text{parallel-axis theorem}$$

$$I = 0.798 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

0.798 kg · m^2

Parallel axis, rod (center)

A rod (center) has $I_{cm} = 1.333$ kg·m^2 and mass $M = 4$ kg. Find I about a parallel axis $d = 1.0$ m away.

$$I = I_{cm} + Md^2 = 1.333 + 4(1.0)^2 \qquad \text{parallel-axis theorem}$$

$$I = 5.333 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

5.333 kg · m^2

Parallel axis, hoop

A hoop has $I_{cm} = 0.160$ kg·m^2 and mass $M = 1$ kg. Find I about a parallel axis $d = 0.4$ m away.

$$I = I_{cm} + Md^2 = 0.160 + 1(0.4)^2 \qquad \text{parallel-axis theorem}$$

$$I = 0.320 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

0.320 kg · m^2

Parallel axis, disk

R5

A disk has $I_{cm} = 0.400$ kg·m^2 and mass $M = 5$ kg. Find I about a parallel axis $d = 0.6$ m away.

$$I = I_{cm} + Md^2 = 0.400 + 5(0.6)^2 \qquad \text{parallel-axis theorem}$$

$$I = 2.200 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

2.200 kg · m^2

Parallel axis, sphere

A sphere has $I_{cm} = 0.200$ kg·m^2 and mass $M = 2$ kg. Find I about a parallel axis $d = 1.0$ m away.

$$I = I_{cm} + Md^2 = 0.200 + 2(1.0)^2 \qquad \text{parallel-axis theorem}$$

$$I = 2.200 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

2.200 kg · m^2

Parallel axis, rod (center)

A rod (center) has $I_{cm} = 2.250$ kg·m^2 and mass $M = 3$ kg. Find I about a parallel axis $d = 1.5$ m away.

$$I = I_{cm} + Md^2 = 2.250 + 3(1.5)^2 \qquad \text{parallel-axis theorem}$$

$$I = 9.000 \text{ kg} \cdot \text{m}^2 \qquad \text{always} \geq I_{cm}$$

Answer

9.000 kg · m^2

$\tau = I\alpha$, I=2.00

A body with $I = 2.00$ kg·m^2 has $\alpha = 3.0$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 2.00(3.0) \qquad \text{rotational } F = ma$$
$$\tau = 6.00 \text{ N} \cdot \text{m}$$

Answer

6.00 N · m

$\tau = I\alpha$, I=0.50

A body with $I = 0.50$ kg·m^2 has $\alpha = 8.0$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 0.50(8.0) \qquad \text{rotational } F = ma$$

$$\tau = 4.00 \text{ N} \cdot \text{m}$$

Answer

4.00 N · m

$\tau = I\alpha$, I=4.00

A body with $I = 4.00$ kg·m^2 has $\alpha = 1.5$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 4.00(1.5) \qquad \text{rotational } F = ma$$
$$\tau = 6.00 \text{ N} \cdot \text{m}$$

Answer

6.00 N · m

$\tau = I\alpha$, I=10.00

A body with $I = 10.00$ kg·m^2 has $\alpha = 0.5$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 10.00(0.5) \qquad \text{rotational } F = ma$$

$$\tau = 5.00 \text{ N} \cdot \text{m}$$

Answer

5.00 N · m

$\tau = I\alpha$, I=0.20

A body with $I = 0.20$ kg·m^2 has $\alpha = 20.0$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 0.20(20.0) \qquad \text{rotational } F = ma$$

$$\tau = 4.00 \text{ N} \cdot \text{m}$$

Answer

4.00 N · m

$\tau = I\alpha$, I=1.50

A body with $I = 1.50$ kg·m^2 has $\alpha = 4.0$ rad/s^2. Find the net torque.

$$\tau = I\alpha = 1.50(4.0) \qquad \text{rotational } F = ma$$

$$\tau = 6.00 \text{ N} \cdot \text{m}$$

Answer

6.00 N · m

Angular accel, 12 N·m

R6

A 12 N·m torque acts on $I = 3.00$ kg·m^2. Find the angular acceleration.

$$\alpha = \frac{\tau}{I} = \frac{12}{3.00} \qquad \text{solve } \tau = I\alpha$$

$$\alpha = 4.00 \text{ rad/s}^2$$

Answer

4.00 rad/s^2

Angular accel, 5 N·m

R6

A 5 N·m torque acts on $I = 0.50$ kg·m^2. Find the angular acceleration.

$$\alpha = \frac{\tau}{I} = \frac{5}{0.50} \qquad \text{solve } \tau = I\alpha$$

$$\alpha = 10.00 \text{ rad/s}^2$$

Answer

10.00 rad/s^2

Angular accel, 40 N·m

R6

A 40 N·m torque acts on $I = 8.00$ kg·m^2. Find the angular acceleration.

$$\alpha = \frac{\tau}{I} = \frac{40}{8.00} \qquad \text{solve } \tau = I\alpha$$

$$\alpha = 5.00 \text{ rad/s}^2$$

Answer

5.00 rad/s^2

Angular accel, 20 N·m

R6

A 20 N·m torque acts on $I = 4.00$ kg·m^2. Find the angular acceleration.

$$\alpha = \frac{\tau}{I} = \frac{20}{4.00} \qquad \text{solve } \tau = I\alpha$$

$$\alpha = 5.00 \text{ rad/s}^2$$

Answer

5.00 rad/s^2

Rotational KE, I=2.00

Find the rotational kinetic energy of $I = 2.00$ kg·m^2 at $\omega = 5$ rad/s.

$$KE_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(2.00)(5)^2 \qquad \text{spin energy}$$

$$KE_{\text{rot}} = 25.0 \text{ J}$$

Answer

25.0 J

Rotational KE, I=0.50

Find the rotational kinetic energy of $I = 0.50$ kg·m^2 at $\omega = 10$ rad/s.

$$KE_{\text{rot}} = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}(0.50)(10)^2 \qquad \text{spin energy}$$

$$KE_{\text{rot}} = 25.0 \text{ J}$$

Answer

25.0 J

Rotational KE, I=4.00

Find the rotational kinetic energy of $I = 4.00$ kg·m^2 at $\omega = 3$ rad/s.

$$KE_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(4.00)(3)^2 \quad \text{spin energy}$$

$$KE_{\text{rot}} = 18.0 \text{ J}$$

Answer

18.0 J

Rotational KE, I=0.20

Find the rotational kinetic energy of $I = 0.20$ kg·m^2 at $\omega = 20$ rad/s.

$$KE_{\text{rot}} = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}(0.20)(20)^2 \qquad \text{spin energy}$$

$$KE_{\text{rot}} = 40.0 \text{ J}$$

Answer

40.0 J

Rotational KE, I=10.00

Find the rotational kinetic energy of $I = 10.00$ kg·m^2 at $\omega = 2$ rad/s.

$$KE_{\text{rot}} = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}(10.00)(2)^2 \qquad \text{spin energy}$$

$$KE_{\text{rot}} = 20.0 \text{ J}$$

Answer

20.0 J

Rolling solid sphere, 4 m/s

A solid sphere ($m = 2$ kg) rolls without slipping at 4 m/s. Find its total kinetic energy ($I = 0.4mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 0.4) \qquad \text{translation + rotation}$$

$$KE = 22.40 \text{ J}$$

Answer

22.40 J

Rolling solid disk, 3 m/s

R7

A solid disk ($m = 3$ kg) rolls without slipping at 3 m/s. Find its total kinetic energy ($I = 0.5mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 0.5) \qquad \text{translation + rotation}$$

$$KE = 20.25 \text{ J}$$

Answer

20.25 J

Rolling hoop, 6 m/s

A hoop ($m = 1$ kg) rolls without slipping at 6 m/s. Find its total kinetic energy ($I = 1.0mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 1.0) \qquad \text{translation + rotation}$$

$$KE = 36.00 \text{ J}$$

Answer

36.00 J

Rolling solid sphere, 2 m/s

R7

A solid sphere ($m = 5$ kg) rolls without slipping at 2 m/s. Find its total kinetic energy ($I = 0.4mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 0.4) \qquad \text{translation + rotation}$$

$$KE = 14.00 \text{ J}$$

Answer

14.00 J

Rolling solid disk, 5 m/s

A solid disk ($m = 4$ kg) rolls without slipping at 5 m/s. Find its total kinetic energy ($I = 0.5mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 0.5) \qquad \text{translation + rotation}$$

$$KE = 75.00 \text{ J}$$

Answer

75.00 J

Rolling hoop, 3 m/s

A hoop ($m = 2$ kg) rolls without slipping at 3 m/s. Find its total kinetic energy ($I = 1.0mR^2$, $\omega = v/R$).

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2(1 + 1.0) \qquad \text{translation + rotation}$$

$$KE = 18.00 \text{ J}$$

Answer

18.00 J

Angular momentum, I=2.00

Find the angular momentum of $I = 2.00$ kg·m^2 at $\omega = 5$ rad/s.

$$L = I\omega = 2.00(5) \qquad \text{rotational momentum}$$
$$L = 10.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

10.00 kg · m^2/s

Angular momentum, I=0.50

R8

Find the angular momentum of $I = 0.50$ kg·m^2 at $\omega = 12$ rad/s.

$$L = I\omega = 0.50(12) \qquad \text{rotational momentum}$$

$$L = 6.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

6.00 kg · m^2/s

Angular momentum, I=4.00

Find the angular momentum of $I = 4.00$ kg·m^2 at $\omega = 3$ rad/s.

$$L = I\omega = 4.00(3) \qquad \text{rotational momentum}$$
$$L = 12.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

12.00 kg · m^2/s

Angular momentum, I=10.00

Find the angular momentum of $I = 10.00$ kg·m^2 at $\omega = 1.5$ rad/s.

$$L = I\omega = 10.00(1.5) \qquad \text{rotational momentum}$$
$$L = 15.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

15.00 kg · m^2/s

Angular momentum, I=0.20

R8

Find the angular momentum of $I = 0.20$ kg·m^2 at $\omega = 30$ rad/s.

$$L = I\omega = 0.20(30) \qquad \text{rotational momentum}$$

$$L = 6.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

6.00 kg · m^2/s

Point-mass $L = mvr$

A 0.50 kg mass moves 4 m/s in a circle of radius 2.0 m. Find its angular momentum ($\vec{L} = \vec{r} \times \vec{p}$).

$$L = mvr = 0.50(4)(2.0) \qquad L = rp \text{ for tangential } v$$

$$L = 4.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

4.00 kg · m^2/s

Point-mass $L = mvr$

A 2.00 kg mass moves 3 m/s in a circle of radius 1.5 m. Find its angular momentum ($\vec{L} = \vec{r} \times \vec{p}$).

$$L = mvr = 2.00(3)(1.5) \qquad L = rp \text{ for tangential } v$$

$$L = 9.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

9.00 kg · m^2/s

Point-mass $L = mvr$

A 0.20 kg mass moves 10 m/s in a circle of radius 0.5 m. Find its angular momentum ($\vec{L} = \vec{r} \times \vec{p}$).

$$L = mvr = 0.20(10)(0.5) \qquad L = rp \text{ for tangential } v$$

$$L = 1.00 \text{ kg} \cdot \text{m}^2/\text{s}$$

Answer

$1.00 \text{ kg} \cdot \text{m}^2/\text{s}$

Conservation, I:6→2

A skater with $I_1 = 6$ kg·m^2 spins at 2 rad/s, then pulls in to $I_2 = 2$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 6.00 \text{ rad/s} \qquad \text{spins faster}$$

Answer

6.00 rad/s

Conservation, I:4→1

A skater with $I_1 = 4$ kg·m^2 spins at 3 rad/s, then pulls in to $I_2 = 1$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 12.00 \text{ rad/s} \qquad \text{spins faster}$$

Answer

12.00 rad/s

Conservation, I:8→3

A skater with $I_1 = 8$ kg·m^2 spins at 1.5 rad/s, then pulls in to $I_2 = 3$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 4.00 \text{ rad/s} \qquad \text{spins faster}$$

Answer

4.00 rad/s

Conservation, I:5→2

A skater with $I_1 = 5$ kg·m^2 spins at 4 rad/s, then pulls in to $I_2 = 2$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 10.00 \text{ rad/s} \qquad \text{spins faster}$$

Answer

10.00 rad/s

Conservation, I:10→4

A skater with $I_1 = 10$ kg·m^2 spins at 1 rad/s, then pulls in to $I_2 = 4$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 2.50 \text{ rad/s} \qquad \text{spins faster}$$

Answer

2.50 rad/s

Conservation, I:12→4

A skater with $I_1 = 12$ kg·m^2 spins at 2 rad/s, then pulls in to $I_2 = 4$ kg·m^2. Find the new angular speed.

$$I_1\omega_1 = I_2\omega_2 \qquad \text{no external torque: } L \text{ conserved}$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 6.00 \text{ rad/s} \qquad \text{spins faster}$$

Answer

6.00 rad/s

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Oscillations (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Recognize $\ddot{x} = -\omega^2 x$ $\rightarrow$ **solve**

71 problems

Lesson `apc.oscillations.session`

Restoring force, k=200, x=0.10

A spring ($k = 200$ N/m) is displaced 0.10 m. Find the restoring force and note why the motion is SHM.

$$F = -kx = -200(0.10) = -20.0 \text{ N}$$

$$|F| = 20.0 \text{ N}$$

$F \propto -x$ gives SHM toward equilibrium

Answer

20.0 N

Restoring force, k=500, x=0.05

S1

A spring ($k = 500$ N/m) is displaced 0.05 m. Find the restoring force and note why the motion is SHM.

$$F = -kx = -500(0.05) = -25.0 \text{ N}$$

$$|F| = 25.0 \text{ N}$$

$F \propto -x$ gives SHM toward equilibrium

Answer

25.0 N

Restoring force, k=50, x=0.20

S1

A spring ($k = 50$ N/m) is displaced 0.20 m. Find the restoring force and note why the motion is SHM.

$$F = -kx = -50(0.20) = -10.0 \text{ N}$$

$$|F| = 10.0 \text{ N}$$

$F \propto -x$ gives SHM toward equilibrium

Answer

10.0 N

Restoring force, k=800, x=0.15

A spring ($k = 800$ N/m) is displaced 0.15 m. Find the restoring force and note why the motion is SHM.

$$F = -kx = -800(0.15) = -120.0 \text{ N}$$

$$|F| = 120.0 \text{ N}$$

$F \propto -x$ gives SHM toward equilibrium

Answer

120.0 N

Restoring force, k=300, x=0.25

A spring ($k = 300$ N/m) is displaced 0.25 m. Find the restoring force and note why the motion is SHM.

$$F = -kx = -300(0.25) = -75.0 \text{ N}$$

$$|F| = 75.0 \text{ N}$$

$F \propto -x$ gives SHM toward equilibrium

Answer

75.0 N

$\omega = \sqrt{k/m}$, 200 N/m, 2.0 kg

Find the angular frequency of a 2.0 kg mass on a 200 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{200/2.0} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

$\omega = \sqrt{k/m}$, 100 N/m, 1.0 kg

S1

Find the angular frequency of a 1.0 kg mass on a 100 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{100/1.0} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

$\omega = \sqrt{k/m}$, 800 N/m, 0.5 kg

S1

Find the angular frequency of a 0.5 kg mass on a 800 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{800/0.5} \qquad \text{from } m\ddot{x} = -kx$$
$$\omega = 40.00 \text{ rad/s}$$

Answer

40.00 rad/s

$\omega = \sqrt{k/m}$, 50 N/m, 0.5 kg

Find the angular frequency of a 0.5 kg mass on a 50 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{50/0.5} \qquad \text{from } m\ddot{x} = -kx$$
$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

$\omega = \sqrt{k/m}$, 500 N/m, 5.0 kg

Find the angular frequency of a 5.0 *kg mass on a* 500 *N/m spring.*

$$\omega = \sqrt{k/m} = \sqrt{500/5.0} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

$\omega = \sqrt{k/m}$, 300 N/m, 3.0 kg

Find the angular frequency of a 3.0 kg mass on a 300 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{300/3.0} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

$\omega = \sqrt{k/m}$, 400 N/m, 2.0 kg

S1

Find the angular frequency of a 2.0 kg mass on a 400 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{400/2.0} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 14.14 \text{ rad/s}$$

Answer

14.14 rad/s

$\omega = \sqrt{k/m}$, 150 N/m, 1.5 kg

Find the angular frequency of a 1.5 kg mass on a 150 N/m spring.

$$\omega = \sqrt{k/m} = \sqrt{150/1.5} \qquad \text{from } m\ddot{x} = -kx$$

$$\omega = 10.00 \text{ rad/s}$$

Answer

10.00 rad/s

Period from ODE, 200 N/m, 2.0 kg

A 2.0 kg mass on a 200 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 10.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.628 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.628 s

Period from ODE, 100 N/m, 1.0 kg

A 1.0 kg mass on a 100 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 10.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.628 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.628 s

Period from ODE, 400 N/m, 4.0 kg

A 4.0 kg mass on a 400 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 10.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.628 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.628 s

Period from ODE, 800 N/m, 0.5 kg

A 0.5 kg mass on a 800 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 40.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.157 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.157 s

Period from ODE, 300 N/m, 3.0 kg

A 3.0 kg mass on a 300 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 10.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.628 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.628 s

Period from ODE, 50 N/m, 2.0 kg

A 2.0 kg mass on a 50 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 5.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 1.257 \text{ s} \qquad \text{amplitude-independent}$$

Answer

1.257 s

Period from ODE, 250 N/m, 1.0 kg

A 1.0 kg mass on a 250 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 15.81 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.397 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.397 s

Period from ODE, 600 N/m, 6.0 kg

A 6.0 kg mass on a 600 N/m spring obeys $\ddot{x} = -\omega^2 x$. Find ω and the period.

$$\omega = \sqrt{k/m} = 10.00 \text{ rad/s} \qquad \text{recognize } x = A\cos\omega t$$

$$T = \frac{2\pi}{\omega} = 0.628 \text{ s} \qquad \text{amplitude-independent}$$

Answer

0.628 s

$v_{\max}$, $a_{\max}$, A=0.20, $\omega = 5$

For $x(t) = 0.20\cos(5t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{\max} = A\omega = 1.00 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{\max} = A\omega^2 = 5.00 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{\max} = 1.00 \text{ m/s}, \; a_{\max} = 5.00 \text{ m/s}^2$$

v_{max}, a_{max}, **A=0.10,** $\omega = 10$

For $x(t) = 0.10\cos(10t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{max} = A\omega = 1.00 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{max} = A\omega^2 = 10.00 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{max} = 1.00 \text{ m/s}, \; a_{max} = 10.00 \text{ m/s}^2$$

v_{max}, a_{max}, A=0.30, $\omega = 4$

For $x(t) = 0.30\cos(4t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{max} = A\omega = 1.20 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{max} = A\omega^2 = 4.80 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{max} = 1.20 \text{ m/s}, \; a_{max} = 4.80 \text{ m/s}^2$$

$v_{\max}$, $a_{\max}$, **A=0.05,** $\omega = 20$

For $x(t) = 0.05\cos(20t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{\max} = A\omega = 1.00 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{\max} = A\omega^2 = 20.00 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{\max} = 1.00 \text{ m/s}, \; a_{\max} = 20.00 \text{ m/s}^2$$

$v_{\max}$, $a_{\max}$, A=0.15, $\omega = 8$

For $x(t) = 0.15\cos(8t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{\max} = A\omega = 1.20 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{\max} = A\omega^2 = 9.60 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{\max} = 1.20 \text{ m/s}, \; a_{\max} = 9.60 \text{ m/s}^2$$

v_{max}, a_{max}, A=0.25, $\omega = 6$

For $x(t) = 0.25\cos(6t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{max} = A\omega = 1.50 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{max} = A\omega^2 = 9.00 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{max} = 1.50 \text{ m/s}, \; a_{max} = 9.00 \text{ m/s}^2$$

v_{max}, a_{max}, A=0.08, $\omega = 12$

For $x(t) = 0.08\cos(12t)$ (m), find the maximum speed and maximum acceleration by differentiating.

$$v(t) = -A\omega \sin \omega t \Rightarrow v_{max} = A\omega = 0.96 \text{ m/s} \quad \text{first derivative}$$

$$a(t) = -A\omega^2 \cos \omega t \Rightarrow a_{max} = A\omega^2 = 11.52 \text{ m/s}^2 \quad \text{second derivative}$$

Answer

$$v_{max} = 0.96 \text{ m/s}, \; a_{max} = 11.52 \text{ m/s}^2$$

x, v, a at $t = 0.10$ s

For $x(t) = 0.20\cos(5t)$ (m), find x, v, and a at $t = 0.10$ s.

$$x = 0.20\cos(5 \cdot 0.10) = 0.176 \text{ m} \quad \text{position}$$

$$v = -A\omega \sin \omega t = -0.479 \text{ m/s} \quad \text{velocity}$$

$$a = -\omega^2 x = -4.388 \text{ m/s}^2 \quad a = -\omega^2 x$$

Answer

$$x = 0.176,\ v = -0.479,\ a = -4.388$$

x, v, a at $t = 0.05$ s

For $x(t) = 0.10\cos(10t)$ (m), find x, v, and a at $t = 0.05$ s.

$$x = 0.10\cos(10 \cdot 0.05) = 0.088 \text{ m} \quad \text{position}$$

$$v = -A\omega \sin \omega t = -0.479 \text{ m/s} \quad \text{velocity}$$

$$a = -\omega^2 x = -8.776 \text{ m/s}^2 \quad a = -\omega^2 x$$

Answer

$$x = 0.088,\ v = -0.479,\ a = -8.776$$

x, v, a at $t = 0.20$ s

For $x(t) = 0.30\cos(4t)$ (m), find x, v, and a at $t = 0.20$ s.

$$x = 0.30\cos(4 \cdot 0.20) = 0.209 \text{ m} \quad \text{position}$$

$$v = -A\omega \sin \omega t = -0.861 \text{ m/s} \quad \text{velocity}$$

$$a = -\omega^2 x = -3.344 \text{ m/s}^2 \quad a = -\omega^2 x$$

Answer

$$x = 0.209,\ v = -0.861,\ a = -3.344$$

x, v, a at $t = 0.10$ s

For $x(t) = 0.15\cos(8t)$ (m), find x, v, and a at $t = 0.10$ s.

$$x = 0.15\cos(8 \cdot 0.10) = 0.105 \text{ m} \quad \text{position}$$

$$v = -A\omega \sin \omega t = -0.861 \text{ m/s} \quad \text{velocity}$$

$$a = -\omega^2 x = -6.688 \text{ m/s}^2 \quad a = -\omega^2 x$$

Answer

$$x = 0.105,\ v = -0.861,\ a = -6.688$$

Total energy, k=200, A=0.10

A spring oscillator ($k = 200$ N/m, amplitude 0.10 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200)(0.10)^2 \qquad \text{constant total energy}$$

$$E = 1.000 \text{ J}$$

Answer

1.000 J

Total energy, k=500, A=0.20

A spring oscillator ($k = 500$ N/m, amplitude 0.20 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(500)(0.20)^2 \qquad \text{constant total energy}$$

$$E = 10.000 \text{ J}$$

Answer

10.000 J

Total energy, k=100, A=0.15

A spring oscillator ($k = 100$ N/m, amplitude 0.15 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(100)(0.15)^2 \qquad \text{constant total energy}$$

$$E = 1.125 \text{ J}$$

Answer

1.125 J

Total energy, k=800, A=0.05

A spring oscillator ($k = 800$ N/m, amplitude 0.05 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(800)(0.05)^2 \qquad \text{constant total energy}$$

$$E = 1.000 \text{ J}$$

Answer

1.000 J

Total energy, k=300, A=0.25

A spring oscillator ($k = 300$ N/m, amplitude 0.25 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(300)(0.25)^2 \qquad \text{constant total energy}$$

$$E = 9.375 \text{ J}$$

Answer

9.375 J

Total energy, k=250, A=0.18

A spring oscillator ($k = 250$ N/m, amplitude 0.18 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(250)(0.18)^2 \qquad \text{constant total energy}$$

$$E = 4.050 \text{ J}$$

Answer

4.050 J

Total energy, k=600, A=0.12

A spring oscillator ($k = 600$ N/m, amplitude 0.12 m) – find the total mechanical energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(600)(0.12)^2 \qquad \text{constant total energy}$$

$$E = 4.320 \text{ J}$$

Answer

4.320 J

$v_{\max}$ from energy, 0.50 kg

A 0.50 kg mass on a 200 N/m spring oscillates with amplitude 0.10 m. Find the maximum speed.

$$\begin{aligned} \tfrac{1}{2}kA^2 &= \tfrac{1}{2}mv_{\max}^2 && \text{all PE} \to \text{all KE} \\ v_{\max} &= A\sqrt{k/m} = 2.000 \text{ m/s} \end{aligned}$$

Answer

2.000 m/s

$v_{\max}$ from energy, 2.00 kg

A 2.00 kg mass on a 400 N/m spring oscillates with amplitude 0.20 m. Find the maximum speed.

$$\begin{aligned} \tfrac{1}{2}kA^2 &= \tfrac{1}{2}mv_{\max}^2 && \text{all PE} \to \text{all KE} \\ v_{\max} &= A\sqrt{k/m} = 2.828 \text{ m/s} \end{aligned}$$

Answer

2.828 m/s

$v_{\max}$ from energy, 1.00 kg

A 1.00 kg mass on a 100 N/m spring oscillates with amplitude 0.15 m. Find the maximum speed.

$$\begin{aligned} \tfrac{1}{2}kA^2 &= \tfrac{1}{2}mv_{\max}^2 && \text{all PE} \to \text{all KE} \\ v_{\max} &= A\sqrt{k/m} = 1.500 \text{ m/s} \end{aligned}$$

Answer

1.500 m/s

$v_{\max}$ from energy, 0.25 kg

A 0.25 kg mass on a 100 N/m spring oscillates with amplitude 0.05 m. Find the maximum speed.

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2 \qquad \text{all PE} \to \text{all KE}$$

$$v_{\max} = A\sqrt{k/m} = 1.000 \text{ m/s}$$

Answer

1.000 m/s

Simple pendulum, L=1.00 m

Find the period of a simple pendulum 1.00 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{1.00/9.8} \qquad \text{small-angle SHM}$$

$$T = 2.007 \text{ s}$$

Answer

2.007 s

Simple pendulum, L=0.50 m

Find the period of a simple pendulum 0.50 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.50/9.8} \qquad \text{small-angle SHM}$$

$$T = 1.419 \text{ s}$$

Answer

1.419 s

Simple pendulum, L=2.00 m

Find the period of a simple pendulum 2.00 m long ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{2.00/9.8} \qquad \text{small-angle SHM}$$

$$T = 2.838 \text{ s}$$

Answer

2.838 s

Simple pendulum, L=0.25 m

Find the period of a simple pendulum 0.25 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.25/9.8} \qquad \text{small-angle SHM}$$

$$T = 1.004 \text{ s}$$

Answer

1.004 s

Simple pendulum, L=1.50 m

Find the period of a simple pendulum 1.50 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{1.50/9.8} \qquad \text{small-angle SHM}$$

$$T = 2.458 \text{ s}$$

Answer

2.458 s

Simple pendulum, L=0.80 m

Find the period of a simple pendulum 0.80 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.80/9.8} \qquad \text{small-angle SHM}$$

$$T = 1.795 \text{ s}$$

Answer

1.795 s

Simple pendulum, L=3.00 m

Find the period of a simple pendulum 3.00 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{3.00/9.8} \qquad \text{small-angle SHM}$$

$$T = 3.476 \text{ s}$$

Answer

3.476 s

Simple pendulum, L=0.40 m

Find the period of a simple pendulum 0.40 *m long* ($g = 9.8$).

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.40/9.8} \qquad \text{small-angle SHM}$$

$$T = 1.269 \text{ s}$$

Answer

1.269 s

Physical pendulum: rod

A physical pendulum (a rod pivoted at its end) has $I = 0.667$ kg·m^2, $m = 2$ kg, pivot-to-CM $d = 0.5$ m. Find the period.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{0.667}{2(9.8)(0.5)}}$$

$$T = 1.639 \text{ s}$$

restoring torque $= -mgd\sin\theta$

Answer

1.639 s

Physical pendulum: disk

A physical pendulum (a disk pivoted at its rim) has $I = 0.180$ kg·m^2, $m = 3$ kg, pivot-to-CM $d = 0.2$ m. Find the period.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{0.180}{3(9.8)(0.2)}}$$

restoring torque $= -mgd\sin\theta$

$$T = 1.099 \text{ s}$$

Answer

1.099 s

Physical pendulum: hoop

A physical pendulum (a hoop on a nail) has $I = 0.180$ kg·m^2, $m = 1$ kg, pivot-to-CM $d = 0.3$ m. Find the period.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{0.180}{1(9.8)(0.3)}}$$

restoring torque $= -mgd\sin\theta$

$$T = 1.555 \text{ s}$$

Answer

1.555 s

Damped amplitude at t=4 s

A damped oscillator ($m = 2$ kg, damping $b = 0.5$ kg/s, initial amplitude 0.20 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 4$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.20 e^{-0.5(4)/(2\cdot 2)}$$ velocity-dependent drag

$$A(4) = 0.1213 \text{ m}$$ envelope decays

Answer

0.1213 m

Damped amplitude at t=5 s

A damped oscillator ($m = 1$ kg, damping $b = 0.2$ kg/s, initial amplitude 0.10 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 5$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.10 e^{-0.2(5)/(2\cdot 1)}$$ velocity-dependent drag

$$A(5) = 0.0607 \text{ m}$$ envelope decays

Answer

0.0607 m

Damped amplitude at t=3 s

A damped oscillator ($m = 5$ kg, damping $b = 1.0$ kg/s, initial amplitude 0.30 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 3$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.30 e^{-1.0(3)/(2\cdot 5)}$$ velocity-dependent drag

$$A(3) = 0.2222 \text{ m}$$ envelope decays

Answer

0.2222 m

Damped amplitude at t=6 s

A damped oscillator ($m = 2$ kg, damping $b = 0.4$ kg/s, initial amplitude 0.15 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 6$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.15 e^{-0.4(6)/(2\cdot 2)}$$ velocity-dependent drag

$$A(6) = 0.0823 \text{ m}$$ envelope decays

Answer

0.0823 m

Damped amplitude at t=5 s

A damped oscillator ($m = 3$ kg, damping $b = 0.6$ kg/s, initial amplitude 0.25 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 5$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.25 e^{-0.6(5)/(2\cdot 3)}$$ velocity-dependent drag

$$A(5) = 0.1516 \text{ m}$$ envelope decays

Answer

0.1516 m

Damped amplitude at t=8 s

A damped oscillator ($m = 1$ kg, damping $b = 0.3$ kg/s, initial amplitude 0.10 m) decays as $A(t) = A_0 e^{-bt/2m}$. Find the amplitude at $t = 8$ s.

$$A(t) = A_0 e^{-bt/2m} = 0.10 e^{-0.3(8)/(2\cdot 1)}$$ velocity-dependent drag

$$A(8) = 0.0301 \text{ m}$$ envelope decays

Answer

0.0301 m

Damping concept

What does the damping term add to the SHM equation of motion?

idea $=$ $-b\dot{x}$ (drag) energy lost to drag

Answer

$-b\dot{x}$ (drag)

Damping concept

In light damping, what happens to the oscillation over time?

idea = amplitude decays, keeps oscillating energy lost to drag

Answer

amplitude decays, keeps oscillating

Resonance frequency, 200 N/m, 2.0 kg

At what driving frequency does a 2.0 kg mass on a 200 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 10.00 \text{ rad/s}$$ resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 1.59 \text{ Hz}$$

Answer

10.00 rad/s

Resonance frequency, 800 N/m, 0.5 kg

At what driving frequency does a 0.5 kg mass on a 800 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 40.00 \text{ rad/s}$$ resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 6.37 \text{ Hz}$$

Answer

40.00 rad/s

Resonance frequency, 100 N/m, 1.0 kg

At what driving frequency does a 1.0 kg mass on a 100 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 10.00 \text{ rad/s}$$ resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 1.59 \text{ Hz}$$

Answer

10.00 rad/s

Resonance frequency, 500 N/m, 5.0 kg

At what driving frequency does a 5.0 kg mass on a 500 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 10.00 \text{ rad/s}$$

resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 1.59 \text{ Hz}$$

Answer

10.00 rad/s

Resonance frequency, 300 N/m, 3.0 kg

At what driving frequency does a 3.0 kg mass on a 300 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 10.00 \text{ rad/s}$$ resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 1.59 \text{ Hz}$$

Answer

10.00 rad/s

Resonance frequency, 150 N/m, 1.5 kg

At what driving frequency does a 1.5 kg mass on a 150 N/m spring resonate (light damping)?

$$\omega_0 = \sqrt{k/m} = 10.00 \text{ rad/s}$$ resonance near natural frequency

$$f_0 = \frac{\omega_0}{2\pi} = 1.59 \text{ Hz}$$

Answer

10.00 rad/s

Driven/resonance concept

What happens to a driven oscillator's amplitude when the drive frequency matches its natural frequency?

idea = amplitude peaks (resonance) steady-state response

Answer

amplitude peaks (resonance)

Driven/resonance concept

After transients die out, at what frequency does a driven oscillator oscillate?

idea = the drive frequency ω_d steady-state response

Answer

the drive frequency ω_d

Driven/resonance concept

How does more damping affect the resonance peak?

idea = lower, broader peak steady-state response

Answer

lower, broader peak

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Gravitation (Calculus-Based) — Full Worked Session

Every skill in the unit, worked one problem at a time

Set up → solve → answer
68 problems

Lesson `apc.gravitation.session`

Gravitation, point masses

Find the gravitational force between 1.00×10^3 *kg and* 2.00×10^3 *kg a distance* 1.00×10^1 *m apart* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(1.00 \times 10^3)(2.00 \times 10^3)}{(1.00 \times 10^1)^2}$$ inverse-square law

$$F = 1.33 \times 10^{-6}\ \text{N}$$ along the line

Answer

1.33×10^{-6} N

Gravitation (Earth-satellite)

Find the gravitational force between 5.97×10^{24} *kg and* 1.00×10^{3} *kg a distance* 6.37×10^{6} *m apart (Earth-satellite) (*$G = 6.67 \times 10^{-11}$*).*

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(1.00 \times 10^{3})}{(6.37 \times 10^{6})^2}$$

inverse-square law

$$F = 9.81 \times 10^{3}\ \text{N}$$

along the line

Answer

9.81×10^{3} N

Gravitation, point masses

Find the gravitational force between 1.00×10^2 *kg and* 1.00×10^2 *kg a distance* 1.00×10^0 *m apart* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(1.00 \times 10^2)(1.00 \times 10^2)}{(1.00 \times 10^0)^2}$$

inverse-square law

$$F = 6.67 \times 10^{-7}\ \text{N}$$

along the line

Answer

6.67×10^{-7} N

Gravitation (Earth-Moon)

Find the gravitational force between 5.97×10^{24} *kg and* 7.35×10^{22} *kg a distance* 3.84×10^{8} *m apart (Earth-Moon) (*$G = 6.67 \times 10^{-11}$*).*

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(7.35 \times 10^{22})}{(3.84 \times 10^{8})^2}$$ inverse-square law

$$F = 1.98 \times 10^{20}\ \text{N}$$ along the line

Answer

1.98×10^{20} N

Gravitation, point masses

Find the gravitational force between 5.00×10^3 *kg and* 5.00×10^3 *kg a distance* 2.00×10^1 *m apart* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(5.00 \times 10^3)(5.00 \times 10^3)}{(2.00 \times 10^1)^2}$$

inverse-square law

$$F = 4.17 \times 10^{-6}\ \text{N}$$

along the line

Answer

4.17×10^{-6} N

Gravitation (Sun-Earth)

Find the gravitational force between 1.99×10^{30} *kg and* 5.97×10^{24} *kg a distance* 1.50×10^{11} *m apart (Sun-Earth) (*$G = 6.67 \times 10^{-11}$*).*

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(1.99 \times 10^{30})(5.97 \times 10^{24})}{(1.50 \times 10^{11})^2}$$

inverse-square law

$$F = 3.52 \times 10^{22}\ \text{N}$$

along the line

Answer

3.52×10^{22} N

Gravitation, point masses

Find the gravitational force between 2.00×10^3 *kg and* 3.00×10^3 *kg a distance* 5.00×10^0 *m apart* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(2.00 \times 10^3)(3.00 \times 10^3)}{(5.00 \times 10^0)^2}$$

inverse-square law

$$F = 1.60 \times 10^{-5}\ \text{N}$$

along the line

Answer

1.60×10^{-5} N

Gravitation (Earth, one radius up)

Find the gravitational force between 5.97×10^{24} *kg and* 5.00×10^{2} *kg a distance* 1.27×10^{7} *m apart (Earth, one radius up)* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(5.00 \times 10^{2})}{(1.27 \times 10^{7})^2}$$

inverse-square law

$$F = 1.23 \times 10^{3}\ \text{N}$$

along the line

Answer

1.23×10^{3} N

Gravitation, point masses

Find the gravitational force between 8.00×10^{1} *kg and* 6.00×10^{1} *kg a distance* 2.00×10^{0} *m apart* ($G = 6.67 \times 10^{-11}$).

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(8.00 \times 10^{1})(6.00 \times 10^{1})}{(2.00 \times 10^{0})^2}$$

inverse-square law

$$F = 8.00 \times 10^{-8}\ \text{N}$$

along the line

Answer

8.00×10^{-8} N

Gravitation (Sun-Mars)

Find the gravitational force between 1.99×10^{30} *kg and* 6.42×10^{23} *kg a distance* 2.28×10^{11} *m apart (Sun-Mars) (*$G = 6.67 \times 10^{-11}$*).*

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11}(1.99 \times 10^{30})(6.42 \times 10^{23})}{(2.28 \times 10^{11})^2}$$

inverse-square law

$$F = 1.64 \times 10^{21}\ \text{N}$$

along the line

Answer

1.64×10^{21} N

Field at Earth's surface

Find the gravitational field Earth'ssurface: $M = 5.97 \times 10^{24}$ *kg,* $r = 6.37 \times 10^{6}$ *m* $(G = 6.67 \times 10^{-11})$.

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})}{(6.37 \times 10^{6})^2}$$

field = force per mass

$$g = 9.81 \text{ N/kg}$$

Answer

9.81 N/kg

Field at the Moon's surface

Find the gravitational field the Moon's surface: $M = 7.35 \times 10^{22}$ *kg,* $r = 1.74 \times 10^{6}$ *m* ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(7.35 \times 10^{22})}{(1.74 \times 10^{6})^2}$$

field = force per mass

$$g = 1.62 \text{ N/kg}$$

Answer

1.62 N/kg

Field at Mars' surface

Find the gravitational field Mars' surface: $M = 6.42 \times 10^{23}$ *kg,* $r = 3.39 \times 10^{6}$ *m* $(G = 6.67 \times 10^{-11})$.

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(6.42 \times 10^{23})}{(3.39 \times 10^{6})^2}$$

field = force per mass

$$g = 3.73 \text{ N/kg}$$

Answer

3.73 N/kg

Field at low Earth orbit

Find the gravitational field lowEarthorbit: $M = 5.97 \times 10^{24}$ kg, $r = 6.77 \times 10^{6}$ m ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})}{(6.77 \times 10^{6})^2}$$

field = force per mass

$$g = 8.69 \text{ N/kg}$$

Answer

8.69 N/kg

Field at Jupiter's surface

Find the gravitational field Jupiter'ssurface: $M = 1.90 \times 10^{27}$ kg, $r = 7.15 \times 10^{7}$ m ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(1.90 \times 10^{27})}{(7.15 \times 10^{7})^2}$$

field = force per mass

$$g = 24.79 \text{ N/kg}$$

Answer

24.79 N/kg

Field at one Earth-radius up

Find the gravitational field oneEarth − radiusup: $M = 5.97 \times 10^{24}$ kg, $r = 1.27 \times 10^{7}$ m ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})}{(1.27 \times 10^{7})^2}$$

field = force per mass

$$g = 2.45 \text{ N/kg}$$

Answer

2.45 N/kg

Field at Mercury's surface

Find the gravitational field Mercury'ssurface: $M = 3.30 \times 10^{23}$ kg, $r = 2.44 \times 10^{6}$ m ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(3.30 \times 10^{23})}{(2.44 \times 10^{6})^2}$$

field = force per mass

$$g = 3.70 \text{ N/kg}$$

Answer

3.70 N/kg

Field at Uranus' surface

Find the gravitational field Uranus' surface: $M = 8.68 \times 10^{25}$ *kg,* $r = 2.54 \times 10^{7}$ *m* $(G = 6.67 \times 10^{-11})$.

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(8.68 \times 10^{25})}{(2.54 \times 10^{7})^2}$$

field = force per mass

$$g = 8.97 \text{ N/kg}$$

Answer

8.97 N/kg

Field at two Earth-radii up

Find the gravitational field twoEarth − radiiup: $M = 5.97 \times 10^{24}$ *kg,* $r = 1.91 \times 10^{7}$ *m* ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(5.97 \times 10^{24})}{(1.91 \times 10^{7})^2}$$

field = force per mass

$$g = 1.09 \text{ N/kg}$$

Answer

1.09 N/kg

Field at Mars' surface again

Find the gravitational field Mars' surface again: $M = 6.39 \times 10^{23}$ *kg,* $r = 3.39 \times 10^{6}$ *m* ($G = 6.67 \times 10^{-11}$).

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11}(6.39 \times 10^{23})}{(3.39 \times 10^{6})^2}$$

field = force per mass

$$g = 3.71 \text{ N/kg}$$

Answer

3.71 N/kg

Inside Earth at $r = 0.50R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.50R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.50) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 4.91 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

4.91 N/kg

Inside Earth at $r = 0.25R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.25R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.25) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 2.45 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

2.45 N/kg

Inside Earth at $r = 0.75R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.75R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.75) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 7.36 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

7.36 N/kg

Inside Earth at $r = 0.10R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.10R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.10) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 0.98 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

0.98 N/kg

Inside Earth at $r = 0.60R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.60R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.60) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 5.89 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

5.89 N/kg

Inside Earth at $r = 0.90R$

Inside a uniform sphere the field grows linearly: $g(r) = g_{surf} \cdot \frac{r}{R}$. Find g at $0.90R_E$ (surface $g = 9.8$).

$$g(r) = g_{\text{surf}}\frac{r}{R} = 9.8(0.90) \qquad \text{only mass within } r \text{ contributes}$$

$$g = 8.83 \text{ N/kg} \qquad \text{linear in } r \text{ inside}$$

Answer

8.83 N/kg

Shell theorem

G3

What is the gravitational field at the exact center of a uniform sphere?

$$\text{result} = 0$$ Newton's shell theorem

Answer

0

Shell theorem

Outside a uniform sphere, how does its field compare to a point mass at its center?

result = identical Newton's shell theorem

Answer

identical

Grav. PE, at Earth's surface

G4

Find the gravitational potential energy of a 1.00×10^3 *kg mass at Earth's surface (*$M = 5.97 \times 10^{24}$ *kg,* $r = 6.37 \times 10^6$ *m).*

$$U = -\int_{\infty}^{r} F\, dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(1.00 \times 10^3)}{6.37 \times 10^6}$$

integrate th

$$U = -6.25 \times 10^{10} \text{ J}$$

negative: b

Answer

-6.25×10^{10} J

Grav. PE, in low orbit

Find the gravitational potential energy of a 5.00×10^2 *kg mass inloworbit* ($M = 5.97 \times 10^{24}$ *kg*, $r = 6.77 \times 10^6$ *m*).

$$U = -\int_{\infty}^{r} F\,dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(5.00 \times 10^2)}{6.77 \times 10^6}$$

integrate th

$$U = -2.94 \times 10^{10}\ \text{J}$$

negative: b

Answer

-2.94×10^{10} J

Grav. PE, Earth's orbit

Find the gravitational potential energy of a 5.97×10^{24} *kg mass Earth's orbit* ($M = 1.99 \times 10^{30}$ *kg*, $r = 1.50 \times 10^{11}$ *m*).

$$U = -\int_{\infty}^{r} F\,dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(1.99 \times 10^{30})(5.97 \times 10^{24})}{1.50 \times 10^{11}}$$

integrate

$$U = -5.28 \times 10^{33}\ \text{J}$$

negative:

Answer

-5.28×10^{33} J

Grav. PE, one radius up

Find the gravitational potential energy of a 2.00×10^3 *kg mass oneradiusup* ($M = 5.97 \times 10^{24}$ *kg*, $r = 1.27 \times 10^7$ *m*).

$$U = -\int_{\infty}^{r} F\,dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(2.00 \times 10^3)}{1.27 \times 10^7}$$

integrate th

$$U = -6.25 \times 10^{10}\ \text{J}$$

negative: b

Answer

-6.25×10^{10} J

Grav. PE, at the surface

Find the gravitational potential energy of a 1.00×10^2 *kg mass atthesurface* ($M = 5.97 \times 10^{24}$ *kg*, $r = 6.37 \times 10^6$ *m*).

$$U = -\int_{\infty}^{r} F\,dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(5.97 \times 10^{24})(1.00 \times 10^2)}{6.37 \times 10^6}$$

integrate th

$$U = -6.25 \times 10^9 \text{ J}$$

negative: b

Answer

-6.25×10^9 J

Grav. PE, on the Moon

Find the gravitational potential energy of a 5.00×10^2 *kg mass ontheMoon* ($M = 7.35 \times 10^{22}$ *kg*, $r = 1.74 \times 10^6$ *m*).

$$U = -\int_{\infty}^{r} F\,dr' = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11}(7.35 \times 10^{22})(5.00 \times 10^2)}{1.74 \times 10^6}$$

integrate th

$$U = -1.41 \times 10^9 \text{ J}$$

negative: b

Answer

-1.41×10^9 J

Change in PE, raising a satellite

A 1.00×10^3 *kg satellite is raised from* $r_1 = 6.37 \times 10^6$ *to* $r_2 = 7.37 \times 10^6$ *m around Earth. Find* ΔU.

$$\Delta U = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of potentials}$$

$$\Delta U = 8.48 \times 10^9 \text{ J} \qquad \text{positive: work done}$$

Answer

8.48×10^9 J

Change in PE, raising a satellite

A 5.00×10^2 *kg satellite is raised from* $r_1 = 6.37 \times 10^6$ *to* $r_2 = 1.91 \times 10^7$ *m around Earth. Find* ΔU.

$$\Delta U = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of potentials}$$

$$\Delta U = 2.08 \times 10^{10}\ \text{J} \qquad \text{positive: work done}$$

Answer

2.08×10^{10} J

Change in PE, raising a satellite

A 2.00×10^3 kg satellite is raised from $r_1 = 6.37 \times 10^6$ to $r_2 = 1.27 \times 10^7$ m around Earth. Find ΔU.

$$\Delta U = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of potentials}$$

$$\Delta U = 6.25 \times 10^{10}\ \text{J} \qquad \text{positive: work done}$$

Answer

6.25×10^{10} J

Change in PE, raising a satellite

A 8.00×10^2 *kg satellite is raised from* $r_1 = 6.77 \times 10^6$ *to* $r_2 = 2.64 \times 10^7$ *m around Earth. Find* ΔU.

$$\Delta U = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of potentials}$$

$$\Delta U = 3.50 \times 10^{10}\ \text{J} \qquad \text{positive: work done}$$

Answer

3.50×10^{10} J

Escape speed, Earth

Find the escape speed from Earth: $M = 5.97 \times 10^{24}$ kg, $R = 6.37 \times 10^{6}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 1.12 \times 10^{4}\ \text{m/s}$$

mass-independent

Answer

1.12×10^{4} m/s

Escape speed, the Moon

Find the escape speed from the Moon: $M = 7.35 \times 10^{22}$ *kg,* $R = 1.74 \times 10^{6}$ *m.*

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 2.37 \times 10^{3}\ \text{m/s}$$

mass-independent

Answer

2.37×10^{3} m/s

Escape speed, Mars

Find the escape speed from Mars: $M = 6.42 \times 10^{23}$ kg, $R = 3.39 \times 10^{6}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 5.03 \times 10^{3}\ \text{m/s}$$

mass-independent

Answer

5.03×10^{3} m/s

Escape speed, the Sun

Find the escape speed from the Sun: $M = 1.99 \times 10^{30}$ *kg,* $R = 6.96 \times 10^{8}$ *m.*

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 6.18 \times 10^5 \text{ m/s}$$

mass-independent

Answer

6.18×10^5 m/s

Escape speed, Jupiter

Find the escape speed from Jupiter: $M = 1.90 \times 10^{27}$ kg, $R = 7.15 \times 10^{7}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 5.95 \times 10^{4}\ \text{m/s}$$

mass-independent

Answer

5.95×10^{4} m/s

Escape speed, Mercury

Find the escape speed from Mercury: $M = 3.30 \times 10^{23}$ kg, $R = 2.44 \times 10^{6}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 4.25 \times 10^{3}\ \text{m/s}$$

mass-independent

Answer

4.25×10^{3} m/s

Escape speed, Saturn

Find the escape speed from Saturn: $M = 5.68 \times 10^{26}$ kg, $R = 6.03 \times 10^{7}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 3.54 \times 10^{4}\ \text{m/s}$$

mass-independent

Answer

3.54×10^{4} m/s

Escape speed, Pluto

Find the escape speed from Pluto: $M = 1.31 \times 10^{22}$ kg, $R = 1.19 \times 10^{6}$ m.

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R}$$

KE just cancels the negative PE

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 1.21 \times 10^{3}\ \text{m/s}$$

mass-independent

Answer

1.21×10^{3} m/s

Orbital speed, low Earth orbit

G6

Find the circular orbital speed around Earth at $r = 6.77 \times 10^6$ m (low Earth orbit).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 7.67 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

7.67×10^3 m/s

Orbital speed, a high orbit

G6

Find the circular orbital speed around Earth at $r = 2.64 \times 10^7$ m (a high orbit).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 3.89 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

3.89×10^3 m/s

Orbital speed, geostationary

G6

Find the circular orbital speed around Earth at $r = 4.22 \times 10^7$ m (geostationary).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 3.07 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

3.07×10^3 m/s

Orbital speed, the Moon's orbit

G6

Find the circular orbital speed around Earth at $r = 3.84 \times 10^8$ m (the Moon's orbit).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 1.02 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

1.02×10^3 m/s

Orbital speed, a 1000-km orbit

G6

Find the circular orbital speed around Earth at $r = 7.37 \times 10^6$ m (a 1000-km orbit).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 7.35 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

7.35×10^3 m/s

Orbital speed, a medium orbit

G6

Find the circular orbital speed around Earth at $r = 1.44 \times 10^7$ m (a medium orbit).

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

gravity = centripetal

$$v = \sqrt{GM/r} = 5.26 \times 10^3 \text{ m/s}$$

and $T = 2\pi r/v$

Answer

5.26×10^3 m/s

Kepler period, a low Earth orbit

Find the orbital period at $r = 6.77 \times 10^6$ m around Earth (Kepler's third law, $M = 5.97 \times 10^{24}$ kg).

$$T^2 = \frac{4\pi^2 r^3}{GM}$$ Kepler's third law

$$T = 2\pi\sqrt{r^3/GM} = 5.55 \times 10^3 \text{ s}$$

Answer

5.55×10^3 s

Kepler period, geostationary

G7

Find the orbital period at $r = 4.22 \times 10^7$ m around Earth (Kepler's third law, $M = 5.97 \times 10^{24}$ kg).

$$T^2 = \frac{4\pi^2 r^3}{GM}$$ Kepler's third law

$$T = 2\pi\sqrt{r^3/GM} = 8.63 \times 10^4 \text{ s}$$

Answer

8.63×10^4 s

Kepler period, the Moon

Find the orbital period at $r = 3.84 \times 10^8$ m around Earth (Kepler's third law, $M = 5.97 \times 10^{24}$ kg).

$$T^2 = \frac{4\pi^2 r^3}{GM}$$ Kepler's third law

$$T = 2\pi\sqrt{r^3/GM} = 2.37 \times 10^6 \text{ s}$$

Answer

2.37×10^6 s

Kepler period, a high orbit

Find the orbital period at $r = 2.64 \times 10^{7}$ m around Earth (Kepler's third law, $M = 5.97 \times 10^{24}$ kg).

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Kepler's third law

$$T = 2\pi\sqrt{r^3/GM} = 4.26 \times 10^{4}\ \text{s}$$

Answer

4.26×10^{4} s

Kepler period, a 1000-km orbit

Find the orbital period at $r = 7.37 \times 10^6$ m around Earth (Kepler's third law, $M = 5.97 \times 10^{24}$ kg).

$$T^2 = \frac{4\pi^2 r^3}{GM}$$ Kepler's third law

$$T = 2\pi\sqrt{r^3/GM} = 6.30 \times 10^3 \text{ s}$$

Answer

6.30×10^3 s

Kepler ratio, r×4

G7

A planet orbits at 4× Earth's orbital radius. How many years is its period ($T^2 \propto r^3$)?

$$\left(\frac{T}{T_E}\right)^2 = \left(\frac{r}{r_E}\right)^3 = 4^3 \qquad \text{Kepler's third law}$$

$$T = 4^{3/2} = 8.00 \text{ yr}$$

Answer

8.00 yr

Kepler ratio, r×9

G7

A planet orbits at 9× Earth's orbital radius. How many years is its period ($T^2 \propto r^3$)?

$$\left(\frac{T}{T_E}\right)^2 = \left(\frac{r}{r_E}\right)^3 = 9^3 \qquad \text{Kepler's third law}$$

$$T = 9^{3/2} = 27.00 \text{ yr}$$

Answer

27.00 yr

Kepler ratio, r×8

G7

A planet orbits at 8× Earth's orbital radius. How many years is its period ($T^2 \propto r^3$)?

$$\left(\frac{T}{T_E}\right)^2 = \left(\frac{r}{r_E}\right)^3 = 8^3 \qquad \text{Kepler's third law}$$

$$T = 8^{3/2} = 22.63 \text{ yr}$$

Answer

22.63 yr

Kepler ratio, r×2

G7

A planet orbits at 2× Earth's orbital radius. How many years is its period ($T^2 \propto r^3$)?

$$\left(\frac{T}{T_E}\right)^2 = \left(\frac{r}{r_E}\right)^3 = 2^3 \qquad \text{Kepler's third law}$$

$$T = 2^{3/2} = 2.83 \text{ yr}$$

Answer

2.83 yr

Orbital energy, low Earth orbit

Find the total energy of a 1.00×10^3 kg satellite in circular orbit at $r = 6.77 \times 10^6$ m ($M = 5.97 \times 10^{24}$ kg).

$$KE = \frac{GMm}{2r} = 2.94 \times 10^{10}\ \text{J}$$

half the well depth

$$E = KE + U = -\frac{GMm}{2r} = -2.94 \times 10^{10}\ \text{J}$$

negative: bound orbit

Answer

-2.94×10^{10} J

Orbital energy, geostationary

G8

Find the total energy of a 5.00×10^2 *kg satellite in circular orbit at* $r = 4.22 \times 10^7$ *m* ($M = 5.97 \times 10^{24}$ *kg*).

$$KE = \frac{GMm}{2r} = 2.36 \times 10^9 \text{ J}$$

half the well depth

$$E = KE + U = -\frac{GMm}{2r} = -2.36 \times 10^9 \text{ J}$$

negative: bound orbit

Answer

-2.36×10^9 J

Orbital energy, a 1000-km orbit

Find the total energy of a 2.00×10^3 *kg satellite in circular orbit at* $r = 7.37 \times 10^6$ *m* ($M = 5.97 \times 10^{24}$ *kg*).

$$KE = \frac{GMm}{2r} = 5.40 \times 10^{10}\ \text{J}$$

half the well depth

$$E = KE + U = -\frac{GMm}{2r} = -5.40 \times 10^{10}\ \text{J}$$

negative: bound orbit

Answer

-5.40×10^{10} J

Orbital energy, the Moon's orbit

Find the total energy of a 1.50×10^3 *kg satellite in circular orbit at* $r = 3.84 \times 10^8$ *m* ($M = 5.97 \times 10^{24}$ *kg*).

$$KE = \frac{GMm}{2r} = 7.78 \times 10^8 \text{ J}$$ half the well depth

$$E = KE + U = -\frac{GMm}{2r} = -7.78 \times 10^8 \text{ J}$$ negative: bound orbit

Answer

-7.78×10^8 J

Orbital energy, a high orbit

Find the total energy of a 8.00×10^2 *kg satellite in circular orbit at* $r = 2.64 \times 10^7$ *m* ($M = 5.97 \times 10^{24}$ *kg*).

$$KE = \frac{GMm}{2r} = 6.04 \times 10^9 \text{ J}$$

half the well depth

$$E = KE + U = -\frac{GMm}{2r} = -6.04 \times 10^9 \text{ J}$$

negative: bound orbit

Answer

-6.04×10^9 J

Energy to raise orbit

How much energy is needed to move a 1.00×10^3 *kg satellite from* $r_1 = 6.77 \times 10^6$ *to a higher* $r_2 = 2.64 \times 10^7$ *m orbit?*

$$\Delta E = -\frac{GMm}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of orbital energies}$$

$$\Delta E = 2.19 \times 10^{10}\ \text{J} \qquad \text{positive: energy added}$$

Answer

2.19×10^{10} J

Energy to raise orbit

How much energy is needed to move a 5.00×10^2 *kg satellite from* $r_1 = 7.37 \times 10^6$ *to a higher* $r_2 = 4.22 \times 10^7$ *m orbit?*

$$\Delta E = -\frac{GMm}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{difference of orbital energies}$$

$$\Delta E = 1.11 \times 10^{10}\ \text{J} \qquad \text{positive: energy added}$$

Answer

1.11×10^{10} J

Practice more, then watch the walk-through

Explore this live

Watch the walk-through

Coulomb's Law

Gravitation resigned, with two signs

$$F = \frac{kq_1q_2}{r^2}$$
$$k = 8.99 \times 10^9$$

Lesson `calcphys.efield.Q1`

Explore this live Watch the walk-through

The twin of Newton's gravitation

Electric charge comes in two kinds — positive and negative — with like charges repelling and unlike attracting. The force between two point charges obeys Coulomb's law.

$$F = \frac{kq_1 q_2}{r^2}, \qquad k = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

This is the structural twin of Newton's $F = Gm_1 m_2/r^2$ from Chapter 8: same inverse-square falloff, same action along the joining line. The one difference — charge has a sign, so the force attracts or repels.

Two facts anchor all of electrostatics

Beyond the force law, charge itself obeys two rules.

Quantized integer multiples of the elementary charge e

Conserved total charge of an isolated system never changes

Because the force law matches gravitation, everything you learned about fields, superposition, and shells carries straight over — with charge playing the role of mass, and the new freedom of a sign.

Watch every step: the inverse-square falloff

The distance between two point charges is doubled. What happens to the force?

$$F \propto 1/r^2 \qquad \text{inverse square}$$

$$F_{\text{new}} = \frac{F}{2^2} = \frac{F}{4} \qquad \text{double } r \Rightarrow \text{quarter } F$$

The force drops to one-quarter — exactly as gravity would weaken, the same $1/r^2$ law. Replace G with k and mass with charge, and every Chapter-8 result transfers, now with charge free to be negative.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q2` — Coulomb's law $F = kq_1q_2/r^2$ and its inverse-square falloff, without calculus.

The rule, in one line

$F = kq_1q_2/r^2$ — **inverse-square, along the joining line**

Like repel, unlike attract; charge quantized and conserved. The twin of gravitation, but signed.

The whole of Part III begins here, with Part I's last law resigned: Coulomb's law is Newton's gravitation with charge for mass and k for G — plus the pictures-changing, mathematics-preserving fact of two signs.

Watch out: two traps to dodge

⚠ Treating the electric force like gravity (always attractive).
Like charges *repel* — the sign of the product $q_1 q_2$ sets attract vs. repel.

⚠ Misremembering the falloff.
It is strictly $1/r^2$: triple the distance, one-ninth the force.

Product of charges over distance squared; signs decide direction; quantized and conserved.

Your turn: Coulomb's law

$F \propto q_1 q_2 / r^2$; signs set direction. Cover the right column.

1. Triple the separation: force $\times$? $\Rightarrow 1/9$
2. Two negative charges: attract or repel? $\Rightarrow$ repel
3. Double both charges: force $\times$? $\Rightarrow \times 4$

What single new feature separates Coulomb's law from Newton's gravitation?

Your turn: electric charge

Cover the answers.

1. Force at r; move to $2r$: new force _____

2. Unlike charges: attract or repel? _____

3. Is total charge conserved? _____

4. Which constant replaces G? _____

Why does everything from Chapter 8's gravitation transfer to electrostatics?

Answers, with the reason

1. $F/2^2$
$= F/4$

2. opposite signs
$\Rightarrow$ attract

3. isolated system
$\Rightarrow$ yes

4. Coulomb constant
$\Rightarrow k$

The two laws share the same $1/r^2$ form, so the field, superposition, and shell results all carry over — with charge for mass.

In one breath

$F = kq_1q_2/r^2$ — **inverse-square, signed**

Like repel, unlike attract; quantized, conserved; gravitation's twin.

Rather than charges reaching across empty space, we say a charge fills the space around it with a field — and other charges respond to the field where they sit.

Next up: **Q2** The electric field

Explore this live | Watch the walk-through

The Electric Field

Force per unit test charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}$$
$$\vec{F} = q\vec{E}$$

Lesson `calcphys.efield.Q2`

Explore this live Watch the walk-through

A charge fills space with a field

Rather than charges reaching across empty space, a charge sets up an electric field, and other charges respond to the field where they sit. The field is the force per unit positive test charge.

$$\vec{E} = \frac{\vec{F}}{q_0}, \qquad E = \frac{kq}{r^2} \text{ (point charge)}$$

A vector in newtons per coulomb, pointing radially away from a positive charge and toward a negative one. This is the direct descendant of the gravitational field $\vec{g} = \vec{F}/m$ from Chapter 8 — the test mass becomes a test charge.

Away from $+$, toward $-$

The field of a point charge radiates outward (positive) or inward (negative), thinning with distance as $1/r^2$.



The field exists in space *independently* of any test charge — it is a property of the source and the location, not of whatever probes it.

Watch every step: force from the field

To find the force on a charge q placed in a field $\vec{E}$, just multiply.

$$\vec{F} = q\vec{E} \qquad \text{field} \times \text{charge}$$

$$q < 0 \;\Rightarrow\; \vec{F} \text{ opposite to } \vec{E} \qquad \text{sign flips direction}$$

Positive charges feel a force along the field; negative charges feel it against the field. The field maps the force a positive test charge would feel — multiply by the actual charge (sign and all) to get the real force.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q3` — the electric field $E = F/q$ and $E = kq/r^2$, without calculus.

The rule, in one line

$$\vec{E} = \vec{F}/q_0 = kq/r^2; \vec{F} = q\vec{E}$$

Away from $+$, toward $-$; units N/C. The field exists with or without a test charge.

Because charge has a sign, the field can point toward or away from its source — unlike gravity, which only ever pulls inward. The field concept lets us describe the source once and read off the force on any charge placed anywhere.

Watch out: two traps to dodge

⚠ The field depends on the test charge.
$\vec{E} = \vec{F}/q_0$ is *per unit charge* — independent of the probe.

⚠ No charge present, no field.
The field is present regardless — a property of the source and location.

Field per unit charge; away from $+$, toward $-$; force is $q\vec{E}$ with the sign flipping direction.

Your turn: the field

$E = kq/r^2$; $\vec{F} = q\vec{E}$. Cover the right column.

1. $E = 200$ N/C, $q = 0.5$ C: force? ⇒ 100 N
2. Negative charge in a field: force direction? ⇒ against $\vec{E}$
3. Field of a point charge at $2r$ vs. r? ⇒ $E/4$

Does the electric field at a point change if you remove the test charge you used to measure it?

Your turn: electric field

Cover the answers.

1. $E = 300$ N/C, $q = 2$ C: force _____

2. Field direction near a $-$ charge _____

3. Units of E _____

4. Which Ch.8 quantity is $\vec{E}$ the twin of? _____

? *Why do we describe a source by its field rather than its force on one particular charge?*

Answers, with the reason

1. $F = qE = 2(300)$
$= 600$ N

2. inward
$\Rightarrow$ toward $-$

3. per unit charge
$\Rightarrow$ N/C

4. gravitational field
$\Rightarrow \vec{g} = \vec{F}/m$

The field is a property of space; describe it once and the force on any charge follows from $\vec{F} = q\vec{E}$.

In one breath

$$\vec{E} \;=\; \vec{F}/q_0 \;=\; kq/r^2\text{; } \vec{F} \;=\; q\vec{E}$$

Away from $+$, toward $-$; independent of the probe; gravity's field, signed.

Real objects aren't points. To find the field of a charged rod, ring, or disk, we chop it into pieces and integrate — the shell integral of Chapter 8, now with charge.

Next up: **Q3** The field of continuous charge distributions

Explore this live Watch the walk-through

Continuous Charge

The shell integral, with charge

$$E = \int \frac{k\,dq}{r^2}$$
$$dq = \lambda\,dl,\ \sigma\,dA,\ \rho\,dV$$

Lesson `calcphys.efield.Q3`

Explore this live Watch the walk-through

Superposition in the limit

Real charged objects — rods, rings, disks — aren't points but continuous distributions. Divide the object into infinitesimal pieces dq, each a point charge, and integrate.

$$E = \int \frac{k\,dq}{r^2}$$

This is exactly the shell-integration idea from Chapter 8's gravitation, now with charge instead of mass. Each piece produces $dE = k\,dq/r^2$; sum them over the object. This integral is the calculus heart of electrostatics.

Write dq with a charge density

The element dq is expressed through a density matched to the geometry.

Line	$dq = \lambda\, dl$	$\lambda = \mathrm{C/m}$
Surface	$dq = \sigma\, dA$	$\sigma = \mathrm{C/m^2}$
Volume	$dq = \rho\, dV$	$\rho = \mathrm{C/m^3}$

The density tells you how to convert dq into a length, area, or volume element so the integral runs over the object's actual geometry — the same setup as the mass element dm in Chapter 5's center of mass.

Watch every step: let symmetry cancel components

The field is a vector, so direction matters. On a symmetric object, opposite pieces cancel one component.

$$\text{each } dq: \; dE = k\,dq/r^2 \text{ (a vector)} \qquad \text{point-charge field}$$

$$\text{symmetry}: \; \perp \text{ components cancel} \qquad \text{only one survives}$$

On the axis of a ring, every dq has a partner across the ring; their sideways parts cancel, leaving only the axial component to integrate. Exploiting symmetry turns a hard vector integral into a simple scalar one.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q7` — common field patterns of rods, rings, and sheets, without the integral.

The rule, in one line

$$E \;=\; \int k\,dq/r^2 \text{, with } dq \;=\; \lambda\,dl,\ \sigma\,dA,\ \rho\,dV$$

Add fields as *vectors*; symmetry cancels components.
The gravitational shell integral, with charge for mass.

In Chapter 8 we integrated $dg = Gm_{enc}/r^2$ over shells of mass; here we integrate $dE = k\,dq/r^2$ over a charge distribution — the identical calculus, one law standing in for the other.

Watch out: two traps to dodge

⚠ Adding the dE magnitudes as scalars.
The field is a *vector* — add components, and let symmetry cancel what it can.

⚠ Reusing $\lambda\, dl$ for a surface.
Match density to dimension: $\sigma\, dA$ for an area, $\rho\, dV$ for a volume.

Chop into dq, write the density, integrate $k\, dq/r^2$ as a vector, and lean on symmetry.

Your turn: continuous charge

dq = density × element; add as vectors. Cover the right column.

1. *dq* for a charged rod? ⇒ $\lambda\, dl$
2. *dq* for a charged sheet? ⇒ $\sigma\, dA$
3. On a ring's axis, which components cancel? ⇒ perpendicular

? *Which earlier integral — center of mass or the gravitational shell — is this the twin of?*

Your turn: field by integration

Cover the answers.

1. Field of each piece dq _____

2. dq for a charged volume _____

3. How are the dE's combined? _____

4. What simplifies a hard vector integral? _____

Why must the density match the geometry's dimension?

Answers, with the reason

1. point charge
$\Rightarrow k\,dq/r^2$

2. volume element
$\Rightarrow \rho\,dV$

3. as vectors
$\Rightarrow$ by components

4. symmetry
$\Rightarrow$ cancels components

A line needs charge-per-length, a surface charge-per-area, a volume charge-per-volume — or the integral would count the wrong amount of charge.

In one breath

$$E = \int k\,dq/r^2\text{; } dq = \lambda\,dl,\ \sigma\,dA,\ \rho\,dV$$

Add as vectors; symmetry cancels components; the shell integral, with charge.

Before computing more fields, let's learn to picture them. Field lines map direction and strength at a glance — a faithful portrait of the field, with strict rules.

Next up: **Q4** Electric field lines

Explore this live | Watch the walk-through

Electric Field Lines

A faithful map of the field

tangent = direction, density = strength

start on $+$, end on $-$

Lesson `calcphys.efield.Q4`

Explore this live

Watch the walk-through

Curves that picture the field

Field lines are continuous curves whose tangent at each point is the field direction there — a faithful map you can read without calculation.

- **Tangent** gives the field *direction*; **density** (how tightly packed) gives the *strength*.
- Lines **start** on positive charges, **end** on negative ones (or run to infinity).
- Lines **never cross** — the field has one direction at each point.

The number of lines from a charge is proportional to its magnitude.

The dipole: out of +, into −

Lines leave the positive charge, curve through space, and land on the negative one — bunched where the field is strong.



Bunched lines mean a strong field, spread lines a weak one. Reading line density and direction gives the field everywhere, with no computation.

Watch every step: does a charge ride the line?

A positive charge is released from rest on a curved field line. Does it travel along the line?

line shows the force direction here	$\vec{F} = q\vec{E}$, tangent
but it accelerates and gains sideways velocity	curves off the line

A field line shows the force a positive charge feels, but a released charge does not generally follow the line — it accelerates along the local field, and its curved path need not match the curve.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q4` — field lines, superposition, and reading field strength from line density, without calculus.

The rule, in one line

Tangent = direction; density = strength; start on +, end on −
Never cross; count $\propto$ charge. A released charge accelerates along $\vec{E}$, not necessarily along a line.

The picture is a faithful map: line density and direction give the field's strength and heading everywhere. But it maps the field, not trajectories — a charge's path curves off the line as it picks up sideways speed.

Watch out: two traps to dodge

⚠ Letting lines cross.
Crossing would give two field directions at one point — impossible. Lines never intersect.

⚠ A charge rides along a curved line.
It follows the *local force* and curves off the line as it accelerates.

Tangent for direction, density for strength; out of $+$, into $-$; never crossing; not a trajectory map.

Your turn: field lines

Read the rules of the map. Cover the right column.

1. Where do lines start and end?	$\Rightarrow + \rightarrow -$
2. Bunched lines mean what?	$\Rightarrow$ strong field
3. Can two field lines cross?	$\Rightarrow$ no

? *Why doesn't a charge released from rest travel along the field line it starts on?*

Your turn: field lines

Cover the answers.

1. What does the tangent give? _____

2. What does line density give? _____

3. Number of lines is proportional to _____

4. Do lines map field or trajectory? _____

Why would two crossing field lines be a contradiction?

Answers, with the reason

1. field direction
$\Rightarrow$ tangent

2. field strength
$\Rightarrow$ density

3. charge magnitude
$\Rightarrow \propto q$

4. the field
$\Rightarrow$ not the path

Two lines crossing would mean two field directions at one point — but the field has a single, unique direction everywhere.

In one breath

Tangent = direction, density = strength; out of +, into −, never cross
Count $\propto$ charge; maps the field, not the trajectory.

Count the lines piercing a surface and you have a new quantity: electric flux. It is the field-through-area that Gauss's law is built on.

Next up: **Q5** Electric flux

Explore this live | Watch the walk-through

Electric Flux

Field through an area

$$\Phi = EA\cos\theta = \int \vec{E} \cdot d\vec{A}$$

θ from the *normal*

Lesson `calcphys.efield.Q5`

Explore this live Watch the walk-through

Counting field lines through a surface

Electric flux measures how much field flows through a surface — the field lines piercing an area. For a uniform field through a flat surface:

$$\Phi = EA\cos\theta, \qquad \Phi = \int \vec{E}\cdot d\vec{A} \ \text{(general)}$$

Here θ is the angle between the field and the surface normal (the perpendicular) — not the surface itself. The general form sums the perpendicular field component over every patch via a dot product.

Max perpendicular, zero parallel

Flux is largest when the field pierces straight through, and zero when it skims along the surface.



Flux carries a *sign*: positive where the field exits, negative where it enters. It's a scalar built from a dot product — only the field passing *through* counts.

Watch every step: field parallel to a sheet

A uniform field runs parallel to a flat sheet of paper. What is the flux through the paper?

$$\Phi = EA\cos\theta \qquad \theta \text{ from the normal}$$

$$\theta = 90^\circ \Rightarrow \cos 90^\circ = 0 \Rightarrow \Phi = 0 \qquad \text{no lines pierce}$$

The field slides along the surface, passing through nothing — zero flux. The flux is maximum when the field is perpendicular ($\theta = 0$, along the normal) and zero when parallel.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q6` — electric flux $\Phi = EA\cos\theta$ and its role in Gauss's law, without the integral.

The rule, in one line

$$\Phi \;=\; EA\cos\theta \;=\; \int \vec{E}\cdot d\vec{A},\ \theta \textbf{ from the } \textbf{\textit{normal}}$$

Max perpendicular, zero parallel. Signed:
positive exiting, negative entering.

Flux is the quantity Gauss's law is built on. The dot product captures only the field passing through the area, not the part sliding along it — which is why a parallel field, however strong, contributes nothing.

Watch out: two traps to dodge

⚠ Measuring θ from the surface.
θ is from the *normal* (the perpendicular). Perpendicular field: $\theta = 0$, max flux.

⚠ Dropping $\cos\theta$.
Only the component along the normal pierces. A parallel field gives zero, not full, flux.

Field dotted with area; angle from the normal; max through, zero along, signed by direction.

Your turn: flux

$\Phi = EA\cos\theta$, θ *from the normal. Cover the right column.*

1. $E = 100$, $A = 2$, $\theta = 0$: flux? $\Rightarrow 200$

2. Same, $\theta = 90°$: flux? $\Rightarrow 0$

3. Field exiting vs. entering: sign? $\Rightarrow +/-$

? *Why does a field parallel to a surface, no matter how strong, produce zero flux?*

Your turn: electric flux

Cover the answers.

1. $E = 50$, $A = 4$, $\theta = 0$: flux _____

2. θ measured from what? _____

3. Flux when field is perpendicular _____

4. Sign of flux for entering field _____

What does the dot product $\vec{E} \cdot d\vec{A}$ physically select?

Answers, with the reason

1. $50(4)(1)$
$= 200$

2. perpendicular
$\Rightarrow$ the normal

3. $\cos 0 = 1$
$\Rightarrow$ maximum

4. into surface
$\Rightarrow$ negative

The dot product keeps only the field component along the normal — the part actually passing through the area.

In one breath

$$\Phi = EA\cos\theta = \int \vec{E} \cdot d\vec{A}\text{, angle from the normal}$$

Max perpendicular, zero parallel, signed by direction.

Now close the surface completely. The total flux through any closed surface counts only the charge inside — Gauss's law, the flux form of the inverse-square law.

Next up: **Q6** Gauss's law

Explore this live | Watch the walk-through

Gauss's Law

Flux counts only the charge inside

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

the flux form of $1/r^2$

Lesson `calcphys.efield.Q6`

Explore this live Watch the walk-through

One of the four Maxwell equations

Gauss's law makes a remarkable claim: the total flux through any closed surface equals the charge enclosed, divided by ϵ_0.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

The power lies in what it does not depend on: not the charges outside the surface, not where inside the charge sits, and not the surface's shape. Only the enclosed charge sets the net flux.

The flux restatement of the inverse-square law

Gauss's law works because two geometric facts exactly cancel.

$$E \propto \frac{1}{r^2} \quad \text{while} \quad \text{sphere area} \propto r^2 \quad \Rightarrow \quad \Phi \text{ independent of } r$$

The field falls as $1/r^2$ while a sphere's area grows as r^2, so their product — the flux — is fixed by the charge alone. This is exactly the gravitational shell theorem of Chapter 8, now stated as flux. An outside charge gives zero net flux: every line it sends in one side comes back out another.

Watch every step: an outside charge

A point charge sits outside a closed surface. What net flux does it produce through that surface?

$$Q_{enc} = 0 \qquad \text{charge is outside}$$

$$\Phi = \frac{Q_{enc}}{\epsilon_0} = 0 \qquad \text{lines in} = \text{lines out}$$

Every field line the outside charge sends into the surface exits somewhere else — net flux zero. Gauss's law is always true, but only useful for finding $\vec{E}$ when the distribution is symmetric enough to pull E out of the integral.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q6` — Gauss's law and flux counting enclosed charge, without the closed-surface integral.

The rule, in one line

$$\oint \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0$$

Flux set by enclosed charge alone — independent of outside charges, position, and surface shape. Always true; the flux form of $1/r^2$.

The shell theorem of Chapter 8 — external field as a point at the center, zero field inside — is Gauss's law in disguise. Same inverse-square origin, now written as flux. This is one of the four equations that govern all of electromagnetism.

Watch out: two traps to dodge

⚠ Counting outside charges.
Only Q_{enc} matters. An outside charge contributes exactly zero net flux.

⚠ A bigger/odder surface changes the flux.
Shape and size are irrelevant — the net flux depends only on the charge inside.

Enclosed charge over ϵ_0; outside charges and surface shape don't matter; always true.

Your turn: Gauss's law

$\Phi = Q_{enc}/\epsilon_0$; *only inside charge counts. Cover the right column.*

1. Charge outside the surface: net flux? $\Rightarrow 0$
2. Double Q_{enc}: flux $\times$? $\Rightarrow \times 2$
3. Change the surface shape (same Q_{enc}): flux? $\Rightarrow$ unchanged

? *Why does the $1/r^2$ falloff make the flux through a sphere independent of its radius?*

Your turn: Gauss's law

Cover the answers.

1. What sets the net flux? _____

2. Flux from an outside charge _____

3. Does surface shape matter? _____

4. Which Ch.8 theorem is this in disguise? _____

When is Gauss's law merely true, and when is it actually useful for finding $\vec{E}$?

Answers, with the reason

1. enclosed charge
$\Rightarrow Q_{enc}/\epsilon_0$

2. in $=$ out
$\Rightarrow 0$

3. irrelevant
$\Rightarrow$ no

4. shell theorem
$\Rightarrow$ Gauss in disguise

It's useful only when symmetry lets you pull E out of the integral — otherwise it's true but doesn't isolate the field.

In one breath

$\oint \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0$ — **flux from enclosed charge alone**
Shape and outside charges irrelevant; the flux form of $1/r^2$; a Maxwell equation.

Where the charge is symmetric enough, Gauss's law hands back the field instantly. Match a Gaussian surface to the symmetry — sphere, line, or plane — and read E off.

Next up: **Q7** Applying Gauss's law with symmetry

Explore this live | Watch the walk-through

Gauss with Symmetry

Match the surface, read off the field

sphere $1/r^2$, line $1/r$, plane const
choose the Gaussian surface

Lesson `calcphys.efield.Q7`

Explore this live | Watch the walk-through

From always-true to actually-useful

Gauss's law becomes a tool only when the distribution is symmetric enough that E is constant over a well-chosen ***Gaussian surface****.*

$$\text{match the surface so } E \text{ is constant } \Rightarrow \ \Phi = E \times \text{Area}$$

The trick: pick a surface where E is either constant and perpendicular (pulling out of the integral) or parallel (no flux). Then $\oint \vec{E} \cdot d\vec{A}$ becomes $E \times$ Area, and you solve for E.

Three symmetries, three falloffs

The distance dependence follows the geometry of spreading.

Sphere	concentric sphere	$E = kQ/r^2$ (0 inside a shell)
Line	coaxial cylinder	$E = \lambda/2\pi\epsilon_0 r$ $(1/r)$
Plane	pillbox	$E = \sigma/2\epsilon_0$ (uniform)

Point $\to 1/r^2$, line $\to 1/r$, plane $\to$ constant — because the geometry of spreading differs. Choosing the right Gaussian surface is the whole art.

Watch every step: inside a charged shell

A test charge sits at the center of a uniformly charged spherical shell. What field does it feel?

$$\text{Gaussian sphere inside} \quad : \quad Q_{enc} = 0 \qquad \text{no charge enclosed}$$

$$\Phi = E \cdot A = 0 \;\Rightarrow\; E = 0 \qquad \text{shell theorem, for charge}$$

No enclosed charge means zero flux, so the field inside a uniform shell is zero — exactly the gravitational shell theorem of Chapter 8, restated for charge. Outside, the same shell acts as a point kQ/r^2.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q7` — the field patterns of spheres, lines, and sheets, without the Gaussian surface.

The rule, in one line

Sphere: kQ/r^2 **out,** 0 **in a shell; Line:** $\lambda/2\pi\epsilon_0 r$**; Plane:** $\sigma/2\epsilon_0$
Match the Gaussian surface to the symmetry; falloff follows the geometry.

An infinite sheet's field is uniform — independent of distance — because the lines spread out over a fixed area no matter how far you go. A line spreads over a growing cylinder ($1/r$); a point over a growing sphere ($1/r^2$).

Watch out: two traps to dodge

⚠ Expecting a field inside a shell.
A uniform shell has $E = 0$ inside — the shell theorem, restated for charge.

⚠ A distance falloff for an infinite sheet.
The sheet's field is *uniform*, $\sigma/2\epsilon_0$ — it doesn't weaken with distance.

Sphere $1/r^2$, line $1/r$, plane constant; match the surface, and inside a shell is zero.

Your turn: Gauss with symmetry

Match falloff to symmetry. Cover the right column.

1. Field of a point/sphere outside: falloff? $\Rightarrow 1/r^2$
2. Field of an infinite line: falloff? $\Rightarrow 1/r$
3. Field of an infinite sheet: falloff? $\Rightarrow$ constant

? *Why does an infinite charged sheet produce a field that doesn't weaken with distance?*

Your turn: symmetry & Gauss

Cover the answers.

1. Field inside a uniform shell _____

2. Gaussian surface for a line charge _____

3. Field of a sphere, outside _____

4. What makes Gauss useful, not just true? _____

Why does the choice of Gaussian surface make or break the calculation?

Answers, with the reason

1. no enclosed charge
$\Rightarrow 0$

2. coaxial cylinder
$\Rightarrow E = \lambda/2\pi\epsilon_0 r$

3. point at center
$\Rightarrow kQ/r^2$

4. symmetry
$\Rightarrow E$ pulls out

A wrong surface leaves E varying over it, and it can't come out of the integral — the whole trick collapses.

In one breath

Sphere kQ/r^2, line $\lambda/2\pi\epsilon_0 r$, plane $\sigma/2\epsilon_0$
Match the Gaussian surface to the symmetry; falloff follows geometry.

One symmetric object matters most of all — the conductor. Its free charges rearrange until the field inside vanishes, with striking consequences.

Next up: **Q8** Conductors in electrostatic equilibrium

Explore this live | Watch the walk-through

Conductors at Equilibrium

Free charges kill the inside field

interior $E = 0$

charge on surface; cavity shielded

Lesson `calcphys.efield.Q8`

Explore this live | Watch the walk-through

Equilibrium demands zero interior field

A conductor has charges free to move. In electrostatic equilibrium they arrange themselves by one requirement: the field inside the material must be zero.

$$E_{\text{inside}} = 0$$

If the interior field were nonzero, the free charges would keep moving — so equilibrium demands $E = 0$ inside. This is not an assumption but a consequence of charges being free to move until they cancel any inside field.

Four consequences follow

From the single fact $E_{inside} = 0$, everything about conductors flows.

- The field in the **bulk** is zero.
- Excess charge resides entirely on the **surface** (Gauss: no interior field $\Rightarrow$ no enclosed net charge).
- Just outside, the field is **perpendicular**, $E = \sigma/\epsilon_0$ (a parallel component would push surface charge around).
- A hollow **cavity is shielded** — field-free (a Faraday cage).

Watch every step: the Faraday cage

Why does a metal box (or a car) protect you from external electric fields?

$$\text{free charges} \;\rightarrow\; \text{rearrange on the surface} \qquad \text{fast, automatic}$$

$$E_{\text{cavity}} \;=\; 0 \qquad \text{external field cancelled inside}$$

Place charge on a conductor and it flees to the surface, arranging itself to kill the interior field — and the field within any enclosed cavity. The redistribution is automatic and fast, which is why a metal enclosure shields whatever is inside.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.charge.Q5` — conductors, insulators, and how charge redistributes, without calculus.

The rule, in one line

Equilibrium $\Rightarrow$ interior $E = 0$; charge on the surface;
exterior $E = \sigma/\epsilon_0$, perpendicular; cavities shielded
Free charges redistribute to cancel any inside field.

The zero interior field is a consequence, not an assumption: charges move until they cancel any inside field, then stop. That single result — proved with Gauss's law — explains grounding, shielding, and why lightning spares those inside a car.

Watch out: two traps to dodge

⚠ Charge or field inside the metal.
At equilibrium the interior field is zero and excess charge lives on the *surface*.

⚠ External field leaks into a cavity.
Cavities are *shielded* — the field-free Faraday cage.

Interior field zero, charge on the surface, exterior field perpendicular, cavities shielded.

Your turn: conductors

$E_{inside} = 0$; *charge on the surface. Cover the right column.*

1. Field inside a conductor at equilibrium? $\Rightarrow 0$
2. Where does excess charge sit? $\Rightarrow$ surface
3. Field direction just outside the surface? $\Rightarrow \perp$

? *Why are you safe from lightning inside a metal car?*

Your turn: conductors

Cover the answers.

1. Interior field at equilibrium _____

2. Exterior field magnitude _____

3. What is a Faraday cage? _____

4. Why must the outside field be perpendicular? _____

Why is the zero interior field a consequence rather than an assumption?

Answers, with the reason

1. free charges cancel it
$\Rightarrow 0$

2. surface charge
$\Rightarrow \sigma/\epsilon_0$

3. shielded cavity
$\Rightarrow$ field-free box

4. else charge moves
$\Rightarrow$ no parallel E

> *Any interior field would push the free charges, so they keep moving until the field is gone — equilibrium is exactly where that field is zero.*

In one breath

Interior $E = 0$; charge on the surface; exterior $\perp$, σ/ϵ_0; cavities shielded
Free charges redistribute to cancel any inside field.

The So-What of Chapter 13: ***electrostatics is gravitation resigned*** *— Coulomb's $1/r^2$, its field, its shell integral, and Gauss's flux law all descend from Part I's last chapter. Next we build the energy side: electric potential, the voltage that drives every circuit.*

Chapter 13 complete. Next chapter: **Electric Potential & Capacitance**

Explore this live Watch the walk-through

Electric Potential Energy

The electric force stores energy

$$U = \frac{kq_1q_2}{r}$$
$$\propto 1/r, \text{ signed}$$

Lesson `calcphys.potential.C1`

Explore this live Watch the walk-through

A conservative force, so a potential energy exists

Just as gravity stores energy when you lift a mass, the electric force stores potential energy when charges move. Because it is conservative — like gravity, unlike friction — its work depends only on the endpoints.

$$U = \frac{kq_1 q_2}{r}$$

It goes as $1/r$ — not the $1/r^2$ of the force — because energy is the integral of force over distance. This is the exact sibling of gravitational PE from Chapters 4 and 8, with charge for mass and a new freedom: a sign.

The sign carries real meaning

Two like charges vs. two unlike charges store energy of opposite sign.

like charges	$U > 0$	unbound — fly apart if released
unlike charges	$U < 0$	bound — held together

You do positive work to push two like charges together (raising U), and they spring apart when let go. Unlike charges sit in an energy well, exactly as a planet does around the Sun.

Watch every step: energy is the force integral

How does U depend on r compared with Coulomb's force? Integrate the $1/r^2$ force over distance.

$$U = -\int F\,dr = -\int \frac{kq_1q_2}{r^2}\,dr \qquad F \propto 1/r^2$$

$$= \frac{kq_1q_2}{r} \qquad \int r^{-2}dr \propto r^{-1}$$

The same integral that built gravitational PE in Chapter 8, now signed. And the work done by the electric force is $W = -\Delta U$ — exactly the conservative-force rule of Chapter 4.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I1` — electric potential energy and voltage, without the calculus.

The rule, in one line

$$U = kq_1q_2/r \ (\propto 1/r, \textbf{ the force integral})$$

Like charges $U > 0$ (unbound), unlike $U < 0$ (bound). $W = -\Delta U$.

This chapter does for the electric force what Chapter 4 did for mechanics: it trades force for energy. Tracking the vector field everywhere is powerful but laborious — energy is often far easier, and it starts here.

Watch out: two traps to dodge

⚠ Using $1/r^2$ for the energy.
Energy goes as $1/r$; the *force* is what goes as $1/r^2$. Integrating raised the power.

⚠ Calling like charges bound.
Like charges have *positive* PE — unbound, ready to fly apart. Only unlike charges are bound.

$U = kq_1q_2/r$: goes as $1/r$, positive for like charges, negative for unlike.

Your turn: electric PE

$U = kq_1q_2/r$; sign follows the charges. Cover the right column.

1. U at r; move to $2r$: new U? $\Rightarrow U/2$
2. Two positive charges: sign of U? $\Rightarrow +$
3. A positive and a negative charge: sign of U? $\Rightarrow -$

Why does the electric PE go as $1/r$ when the force goes as $1/r^2$?

Your turn: electric potential energy

Cover the answers.

1. U at r; move to $3r$: new U _____

2. Which power of r for U? For F? _____

3. Work by the electric force in terms of ΔU _____

4. Which earlier PE is this the sibling of? _____

What new feature does electric PE have that gravitational PE never did?

Answers, with the reason

1. $U \propto 1/r$
$\Rightarrow U/3$

2. $U: 1/r$, $F: 1/r^2$
$\Rightarrow$ differ by one

3. conservative force
$\Rightarrow W = -\Delta U$

4. gravitational PE
$\Rightarrow -GMm/r$

Electric PE can be positive or negative — like charges repel (positive U), unlike attract (negative U). Gravity, always attractive, had only negative PE.

In one breath

$U = kq_1q_2/r$ — **the signed force integral**
$\propto 1/r$; like charges positive, unlike negative; $W = -\Delta U$.

Divide this energy by the charge and you strip away the test charge entirely — leaving a scalar property of location itself: the electric potential, or voltage.

Next up: **C2** Electric potential (voltage)

Explore this live Watch the walk-through

Electric Potential

Energy per unit charge — a scalar

$$V = \frac{U}{q} = \frac{kq}{r}$$

scalars just add

Lesson `calcphys.potential.C2`

Explore this live

Watch the walk-through

Strip away the test charge

Electric potential, or voltage, is the potential energy per unit charge — a property of a location, independent of any test charge.

$$V = \frac{U}{q} \text{ (volts, } 1\text{ V} = 1\text{ J/C)}, \qquad V = \frac{kq}{r} \text{ for a point charge}$$

*Just as the field is force per unit charge, the potential is energy per unit charge. It falls as $1/r$ — and, crucially, it is a **scalar**: magnitude and sign, but no direction.*

The scalar makes life easy

The crucial contrast with the field: potential has no direction, so potentials from several charges just add as numbers.

$$V_{\text{total}} = \sum \frac{kq_i}{r_i} \qquad \text{(no components, no angles)}$$

No vector decomposition, no triangles — just signed arithmetic. This makes potential far easier to compute than the field. And the energy of a charge q placed at a point is simply $U = qV$: the same spot is high-energy for a $+$ charge, low-energy for a $-$ one.

Watch every step: potential just adds

Two charges, $+2q$ at distance r and $-q$ at distance r from a point P. Find the potential at P.

$$V_P = \frac{k(2q)}{r} + \frac{k(-q)}{r} \qquad \text{add signed scalars}$$

$$= \frac{kq}{r} = \text{one clean number} \qquad \text{no components}$$

The field at P would need vector components and angles; the potential needs only the distances and signs. What usually matters is the difference ΔV between two points — it drives charge and is what a voltmeter reads.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I1` — potential (voltage) as energy per charge, without calculus.

The rule, in one line

$V = U/q$ **(volts),** $V = kq/r$ **for a point charge**
A *scalar* — contributions add as signed numbers. $U = qV$; ΔV drives charge.

Potential is to the electric force what potential energy was to gravity, divided by charge. Because V is a scalar, the potential of a group of charges is just the algebraic sum of their kq/r contributions — a genuine computational gift.

Watch out: two traps to dodge

⚠ Treating potential as a vector.
It's a *scalar* — add signed numbers, no components or angles.

⚠ Conflating V with U, or using $1/r^2$.
$V = U/q$ is energy *per charge*, and goes as $1/r$ — not $1/r^2$.

Potential is a scalar going as $1/r$; $V = U/q$; multiply back by charge to get $U = qV$.

Your turn: potential

$V = kq/r$; add as scalars; $U = qV$. Cover the right column.

1. V at r; move to $2r$: new V? $\Rightarrow V/2$

2. Charge $q = 3$ C at $V = 4$ V: energy U? $\Rightarrow$ 12 J

3. Is potential a scalar or a vector? $\Rightarrow$ scalar

Why is adding potentials so much easier than adding fields?

Your turn: voltage

Cover the answers.

1. Units of potential _____

2. Potential of a point charge _____

3. Energy of charge q at potential V _____

4. Two $+q$ and $-q$ equidistant from P: net V? _____

❓ *Why does a voltmeter read a potential difference rather than an absolute potential?*

Answers, with the reason

1. joules per coulomb
$=$ volts

2. scalar, $1/r$
$\Rightarrow kq/r$

3. $U = qV$
$\Rightarrow qV$

4. equal $+$ and $-$
$\Rightarrow 0$

Only differences in potential do physical work; the zero point is a free choice, so a meter measures ΔV between its two leads.

In one breath

$V = U/q = kq/r$ — **a scalar**
Contributions add as signed numbers; $U = qV$; ΔV drives charge.

For an extended object we integrate — but because potential is a scalar, the integral drops all the components and angles the field integral demanded.

Next up: **C3** Potential of continuous charge distributions

Explore this live Watch the walk-through

Potential of a Distribution

A scalar integral — no components

$$V = \int \frac{k\,dq}{r}$$

only distances matter

Lesson `calcphys.potential.C3`

Explore this live

Watch the walk-through

Integrate the scalar contributions

To find the potential of an extended object, divide it into pieces dq, each contributing $dV = k\,dq/r$, and add.

$$V = \int \frac{k\,dq}{r}, \qquad dq = \lambda\,dl,\ \sigma\,dA,\ \rho\,dV$$

*It's the same shell-integration technique yet again (Chapters 8 and 13) — but with a wonderful simplification: because potential is a **scalar**, there are no components and no angles. You simply add the scalar contributions $k\,dq/r$.*

Find the easy potential first, then differentiate

The only geometry needed is the distance r from each dq to the field point — no vector decomposition at all.

$$V \text{ (scalar, easy)} \xrightarrow{E=-dV/dx} E \text{ (vector, harder)}$$

This often makes the potential far easier than the field, so a common strategy is to find the (easier) scalar potential first, then get the field by differentiation — the reverse of grinding out the vector field directly.

Watch every step: a charged ring on its axis

Why is a charged ring's potential on its axis easier to find than its field?

$$V = \int \frac{k\,dq}{r} = \frac{k}{r}\int dq \qquad \text{every } dq \text{ same distance } r$$

$$= \frac{kQ}{r} \qquad \text{no components to resolve}$$

Every dq on the ring is the same distance r from an axis point, so r comes out of the integral. The field would need the axial components of each $d\vec{E}$; the potential does not.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I1` — potential of charge configurations, without the integral.

The rule, in one line

$V = \int k\,dq/r$**, a scalar integral**

Only distances matter — no components. Often easier than the field; find V first, then differentiate.

Write dq as density times an element ($\lambda\,dl$, $\sigma\,dA$, $\rho\,dV$), match the geometry, and the integral is routine — with none of the angle-chasing the field integral of Chapter 13 required.

Watch out: two traps to dodge

⚠ Adding components to the potential integral.
Potential is a scalar — no components. The integrand is just $k\,dq/r$.

⚠ Using $1/r^2$ in the integrand.
The potential integrand is $k\,dq/r$ — distance only, first power. $1/r^2$ is the field.

Scalar integral $\int k\,dq/r$: distances only, first power, no angles.

Your turn: potential integrals

$V = \int k\,dq/r$; scalar. Cover the right column.

1. Ring charge Q, axis distance r: V? $\Rightarrow kQ/r$
2. dq for a charged rod? $\Rightarrow \lambda\,dl$
3. Does the potential integral need angles? $\Rightarrow$ no

Why is finding V first, then differentiating, often easier than finding the field directly?

Your turn: continuous distributions

Cover the answers.

1. The potential integrand _____

2. dq for a charged sheet _____

3. What geometry does V need? _____

4. How do you get the field from V? _____

What makes the scalar potential integral simpler than the vector field integral?

Answers, with the reason

1. scalar
$= k\,dq/r$

2. surface density
$\Rightarrow \sigma\,dA$

3. distances only
$\Rightarrow r$

4. differentiate
$\Rightarrow E = -dV/dx$

The field integral must resolve each $d\vec{E}$ into components; the scalar potential just sums numbers, so no direction ever enters.

In one breath

$V = \int k\,dq/r$ — **scalar, distances only**
No components; often easier than the field; find V first, then differentiate.

That last move — differentiate V to get E — is no accident. Field and potential are the derivative/integral pair from the start of the course, now applied to energy.

Next up: **C4** The relationship between field & potential

Explore this live Watch the walk-through

Field & Potential

The derivative/integral pair, again

$$E = -\frac{dV}{dx}$$
$$V = -\int \vec{E} \cdot d\vec{l}$$

Lesson `calcphys.potential.C4`

Explore this live

Watch the walk-through

Two views of the same thing, tied by calculus

Field and potential are the derivative/integral pair from the start of the course — now applied to energy. The field is the negative gradient of the potential.

$$E = -\frac{dV}{dx}, \qquad V = -\int \vec{E} \cdot d\vec{l}$$

The field points toward decreasing potential (downhill), and its magnitude is the steepness. This is the exact analog of $F = -dU/dx$ from Chapter 4 — field is to potential as force is to potential energy.

One derivative/integral pair, three chapters

You have met this exact relationship before, dressed differently each time.

Ch.4 (mechanics)	$F = -\dfrac{dU}{dx}$
Ch.9 (heat)	$\dfrac{dQ}{dt} = -kA\dfrac{dT}{dx}$
Ch.14 (electricity)	$E = -\dfrac{dV}{dx}$

Divide $F = -dU/dx$ by charge and you get exactly $E = -dV/dx$. A rate driven by a spatial gradient, pointing downhill — the same idea carries force, heat, and now the electric field.

Watch every step: parallel plates

For a uniform field (as between parallel plates), integrate $E = -dV/dx$ across the gap d.

$$V = E\,d \ \text{(uniform field)} \qquad \text{integrate constant } E$$

$$E = \frac{V}{d} = \text{volts per metre} \qquad \text{why field is quoted in V/m}$$

A constant field over a gap d gives $V = Ed$, so $E = V/d$ — which is exactly why field is often quoted in volts per metre. And because the field is a derivative of V, a region of constant potential has zero field: flat means no slope.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I2` — how potential and field relate, and $E = V/d$, without calculus.

The rule, in one line

$E = -dV/dx$; $V = -\int \vec{E} \cdot d\vec{l}$; **uniform:** $V = Ed$

Field points downhill (high→low V); constant V $\Rightarrow E = 0$. The analog of $F = -dU/dx$.

The field points from high to low potential, always downhill, because a positive charge moves toward lower potential energy — rolling down the voltage hill just as a ball rolls down a gravitational one.

Watch out: two traps to dodge

⚠ Pointing the field uphill.
The field points toward *lower* potential — downhill. The minus sign is the whole story.

⚠ Expecting a field where V is flat.
Constant potential means *zero* field — a derivative of a constant is zero.

Field is minus the slope of V: downhill, magnitude the steepness, zero where V is flat.

Your turn: field & potential

$E = -dV/dx$; uniform $V = Ed$. Cover the right column.

1. $V = 100$ V across $d = 0.05$ m: E? ⇒ 2000 V/m
2. Constant potential region: field? ⇒ 0
3. Which way does the field point? ⇒ downhill

❓ *Which Chapter 4 relationship becomes $E = -dV/dx$ when you divide by charge?*

Your turn: field from potential

Cover the answers.

1. $V = 60$ V across $d = 0.02$ m: E _____

2. Field where potential is constant _____

3. Units of field from $E = V/d$ _____

4. Direction of the field vs. potential _____

Why does the field vanish inside a region of constant potential?

Answers, with the reason

1. 60/0.02
$= 3000$ V/m

2. no slope
$\Rightarrow 0$

3. $V = Ed$
$\Rightarrow$ V/m

4. high $\rightarrow$ low
$\Rightarrow$ downhill

The field is the derivative $-dV/dx$; the derivative of a constant is zero, so a flat potential means no field.

In one breath

$$E = -dV/dx\text{; } V = -\int \vec{E} \cdot d\vec{l}\text{; uniform } V = Ed$$

Field points downhill; constant V gives zero field; the echo of $F = -dU/dx$.

If the field is always perpendicular to the direction of no change in V, then surfaces of constant potential form a grid the field lines must cross at right angles — equipotentials.

Next up: **C5** Equipotential surfaces

Explore this live | Watch the walk-through

Equipotential Surfaces

Surfaces of constant potential

field lines ⊥ equipotentials
no work to move along one

Lesson `calcphys.potential.C5`

Explore this live | Watch the walk-through

A geometric picture of potential

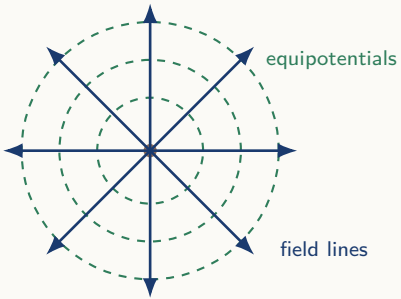
*An **equipotential** is a surface on which the potential is everywhere the same — a picture that complements field lines.*

point charge : concentric spheres • uniform field : parallel planes

Three rules govern them: field lines are perpendicular to equipotentials, moving a charge along one takes no work, and any conductor's surface in equilibrium is an equipotential.

Field lines cross equipotentials at right angles

The field points along the steepest change; equipotentials are the surfaces of no change — so they meet at right angles.



For a point charge the equipotentials are concentric spheres; the radial field lines cross every one at 90°.

Watch every step: no work along an equipotential

Why does moving a charge along an equipotential require no work?

$$V = \text{const} \;\Rightarrow\; \Delta V = 0 \qquad \text{same potential}$$

$$W \;=\; q\,\Delta V = 0 \qquad U = qV \text{ unchanged}$$

Since $U = qV$ and V doesn't change, the energy doesn't change, so no work is done. This is why charge slides freely across a conductor's surface — it's all at one potential, so nothing pushes charge along it.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I2` — equipotentials and how field relates to potential, without calculus.

The rule, in one line

Surfaces of constant V; field lines $\perp$; no work to move along one

A conductor's surface in equilibrium is an equipotential.

Equipotentials and field lines form a perpendicular grid — knowing one gives you the other. And a conductor's surface must be an equipotential: if it weren't, charges would flow until it evened out.

Watch out: two traps to dodge

⚠ Drawing field lines along equipotentials.
Lines cross equipotentials *perpendicularly* — never along them.

⚠ Claiming work is done moving along one.
Constant V means $\Delta V = 0$, so $W = q\,\Delta V = 0$ — no work.

Perpendicular grid; zero work along a surface; every conductor surface is one equipotential.

Your turn: equipotentials

Constant V; field $\perp$; no work along. Cover the right column.

1. Angle between field lines and equipotentials? $\Rightarrow 90^\circ$
2. Work to move a charge along one? $\Rightarrow 0$
3. Equipotentials of a point charge? $\Rightarrow$ spheres

? *Why must the surface of a conductor in equilibrium be an equipotential?*

Your turn: equipotential surfaces

Cover the answers.

1. Equipotentials of a uniform field _____

2. Field-line angle to equipotentials _____

3. Work moving charge along an equipotential _____

4. A conductor's surface is a(n) _____

If you know the equipotentials, how do you find the field lines?

Answers, with the reason

1. $\perp$ to field
$\Rightarrow$ parallel planes

2. perpendicular
$\Rightarrow 90^\circ$

3. $\Delta V = 0$
$\Rightarrow 0$

4. constant V
$\Rightarrow$ equipotential

Draw the field lines crossing every equipotential at right angles — the two form a perpendicular grid, each determining the other.

In one breath

Constant-V surfaces; field lines $\perp$; no work along one
A conductor's surface is an equipotential.

Now fix two conductors at a potential difference and let them hold equal and opposite charge — you have built a device that stores charge and energy: the capacitor.

Next up: **C6** Capacitance

Explore this live Watch the walk-through

Capacitance

Charge stored per volt

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

geometry only

Lesson `calcphys.potential.C6`

Explore this live

Watch the walk-through

A device that stores charge and energy

*A **capacitor** is two conductors with equal and opposite charges and a potential difference between them. Its capacitance is the charge stored per volt.*

$$C = \frac{Q}{V} \text{ (farads, } 1 \text{ F} = 1 \text{ C/V)}$$

The key insight: capacitance depends only on the geometry of the conductors and the material between them — not on the charge or voltage. It's a fixed property of the device.

Parallel plates: bigger area, smaller gap

For a parallel-plate capacitor with plate area A and separation d:

$$C = \frac{\epsilon_0 A}{d}$$

Larger plates or a smaller gap give more capacitance. Double the charge and the voltage doubles too, so the ratio $C = Q/V$ is unchanged — a fixed property. Because C is fixed, Q and V are proportional: $Q = CV$, they rise together.

Watch every step: double the charge

You double the charge on a capacitor. What happens to its capacitance?

$$Q \to 2Q \;\Rightarrow\; V \to 2V \;(\text{since } Q = CV) \qquad \text{both double}$$

$$C \;=\; \frac{Q}{V} = \frac{2Q}{2V} = \boxed{\text{unchanged}} \qquad \text{property of the device}$$

Capacitance is set by geometry alone. Only the stored charge and voltage change, in lockstep — C itself doesn't move. To store more charge at a given voltage, you enlarge the plates or narrow the gap.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I3` — capacitance $C = Q/V$ and parallel plates, without calculus.

The rule, in one line

$C = Q/V$ **(farads), fixed by *geometry*; parallel plates** $C = \epsilon_0 A/d$
$Q = CV$: charge and voltage rise together.

Real capacitors chase exactly this: large plate area (often rolled up into a tiny cylinder) and a thin gap of insulator, to pack maximum charge into minimum space. The physics is all in $C = \epsilon_0 A/d$.

Watch out: two traps to dodge

⚠ Thinking C changes with charge or voltage.
C is fixed by geometry. Q and V change together; their ratio stays put.

⚠ A bigger gap raises C.
$C = \epsilon_0 A/d$ *falls* as d grows — a wider gap stores *less*.

Capacitance is geometry only: $Q = CV$ rise together, and $C = \epsilon_0 A/d$ grows with area, shrinks with gap.

Your turn: capacitance

$C = Q/V = \epsilon_0 A/d$; $Q = CV$. *Cover the right column.*

1. $Q = 6$ C at $V = 3$ V: C? $\Rightarrow$ 2 F
2. Double the plate area: C changes by? $\Rightarrow \times 2$
3. Double the gap d: C changes by? $\Rightarrow \times \frac{1}{2}$

Why doesn't a capacitor's capacitance change when you charge it up?

Your turn: capacitance

Cover the answers.

1. $Q = 10$ C at $V = 5$ V: C _____

2. Formula for parallel plates _____

3. What does C depend on? _____

4. Halve the gap: C does what? _____

How do real capacitors pack a large capacitance into a small volume?

Answers, with the reason

1. $C = Q/V = 10/5$
$= 2$ F

2. area over gap
$\Rightarrow \epsilon_0 A/d$

3. geometry
$\Rightarrow A,\ d,$ material

4. $C \propto 1/d$
$\Rightarrow \times 2$

Large area rolled into a tiny cylinder, plus a razor-thin insulating gap — maximizing A and minimizing d in $C = \epsilon_0 A/d$.

In one breath

$$C = Q/V = \epsilon_0 A/d \text{ — } \textbf{geometry only}$$

$Q = CV$ rise together; bigger area or smaller gap gives more C.

Wire several capacitors together and they combine by rules that are the exact reverse of the resistor rules you'll meet next chapter.

Next up: **C7** Capacitors in series & parallel

Explore this live | Watch the walk-through

Combining Capacitors

The reverse of the resistor rules

parallel: $C_{eq} = C_1 + C_2$

series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Lesson `calcphys.potential.C7`

Explore this live

Watch the walk-through

Two ways to wire, two rules

Combining capacitors follows rules that are the opposite of the resistor rules of the next chapter.

Parallel	$C_{eq} = C_1 + C_2 + \cdots$	same voltage, charges add
Series	$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$	same charge, voltages add

Parallel plates enlarge the total area, so capacitances add. Series stacking widens the effective gap, so the equivalent is less than the smallest.

Why each rule holds

The physics behind the two combinations.

- **Parallel**: both share the same *voltage*, and their charges add — so $C = Q/V$ adds directly, like enlarging the plate area.
- **Series**: both carry the same *charge*, and their voltages add — so the *reciprocals* add, like widening the gap.

Getting series and parallel straight — and remembering they reverse the resistor rules — is the key skill.

Watch every step: two 6 μF capacitors

Combine two 6 μF capacitors in parallel, then in series.

$$\text{parallel} : \; C_{eq} = 6 + 6 = 12\ \mu\text{F} \qquad \text{add directly}$$

$$\text{series} : \; \frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \Rightarrow 3\ \mu\text{F} \qquad \text{less than the smallest}$$

Parallel gives more (12), series gives less than either (3). The series result is always below the smallest capacitor — exactly opposite to how resistors behave.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I7` — series and parallel combinations, without calculus.

The rule, in one line

Parallel: $C_{eq} = C_1 + C_2$ **(same V); series:** $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ **(same Q)**
Series is below the smallest — reversed from resistors.

The mirror image is worth memorizing: capacitors add in parallel and add reciprocals in series; resistors do the exact opposite. Whenever you recall one, flip it for the other.

Watch out: two traps to dodge

⚠ Swapping the rules.
Parallel adds directly; series adds reciprocals. (Resistors are the reverse.)

⚠ Thinking series raises capacitance.
Series gives *less* than the smallest — stacking widens the effective gap.

Parallel adds (same V); series adds reciprocals (same Q) and comes out below the smallest.

Your turn: combinations

Parallel adds; series adds reciprocals. Cover the right column.

1. 4 μF and 2 μF in parallel: C_{eq}? $\Rightarrow$ 6 μF

2. Two 4 μF in series: C_{eq}? $\Rightarrow$ 2 μF

3. Series result vs. smallest capacitor? $\Rightarrow$ less

❓ *Why do parallel capacitors share the same voltage while series ones share the same charge?*

Your turn: series & parallel

Cover the answers.

1. 3 μF and 6 μF in parallel: C_{eq} _____

2. 3 μF and 6 μF in series: C_{eq} _____

3. Which combination shares the charge? _____

4. How do resistor rules compare? _____

Why does adding a capacitor in series always lower the total capacitance?

Answers, with the steps

1. $3 + 6$
$= 9\ \mu\text{F}$

2. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$
$= 2\ \mu\text{F}$

3. series
$\Rightarrow$ same Q

4. reversed
$\Rightarrow$ opposite

Series stacking widens the effective gap, and $C = \epsilon_0 A/d$ falls as the gap grows — so the combination is always weaker than any single member.

In one breath

Parallel adds ($C_1 + C_2$); series adds reciprocals; below the smallest
Reversed from resistors; parallel shares V, series shares Q.

However you combine them, a charged capacitor holds energy — and the way that energy accumulates against a rising voltage brings back the spring's factor of one-half.

Next up: **C8** Energy stored in a capacitor

Explore this live | Watch the walk-through

Energy in a Capacitor

The spring's half-factor returns

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \frac{Q^2}{2C}$$

energy lives in the field

Lesson `calcphys.potential.C8`

Explore this live Watch the walk-through

Work done against a rising voltage

A charged capacitor stores energy — the work done moving charge against the growing potential difference. Because the voltage rises as charge accumulates, the total is not QV but half of it.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \frac{Q^2}{2C}$$

Three equivalent forms, related by $Q = CV$. The factor of one-half is the signature of accumulating against a linearly rising voltage — exactly like the $\tfrac{1}{2}kx^2$ of a spring, whose force rises linearly with stretch (Chapter 4).

Watch every step: where the half comes from

Integrate the work $dW = V\,dq$ as the voltage climbs from zero, using $V = q/C$.

$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq \qquad V \text{ rises with } q$$

$$= \frac{Q^2}{2C} = \tfrac{1}{2}QV \qquad \text{triangle under the line}$$

You integrate $V\,dq$ as V climbs from 0, giving $\frac{1}{2}QV$ — not QV. The triangle under the linearly rising voltage contributes exactly the factor of one-half, the very same half as the spring's $\frac{1}{2}kx^2$.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I3` — energy stored in a capacitor, $\frac{1}{2}CV^2$, without the integral.

The energy lives in the field

Where is the stored energy? Not on the plates — in the electric field itself, in the space between them.

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad \text{(energy per unit volume)}$$

The field is a repository of energy, with density $u = \frac{1}{2}\epsilon_0 E^2$. This picture — energy residing in the field, not the charges — is deep, and it is central to electromagnetic waves, which carry energy through empty space with no charges present at all.

The rule, in one line

$$U \;=\; \tfrac{1}{2}CV^2 \;=\; \tfrac{1}{2}QV \;=\; Q^2/2C$$

Half-factor from the rising voltage (like $\tfrac{1}{2}kx^2$).

Energy in the field: density $u \;=\; \tfrac{1}{2}\epsilon_0 E^2$.

The half in $\tfrac{1}{2}CV^2$ is the same half as the spring's $\tfrac{1}{2}kx^2$: both integrate a linearly rising quantity (voltage here, force there) from zero, and the triangle under the line gives the one-half.

Watch out: two traps to dodge

⚠ Dropping the half ($U = QV$).
The voltage rises as you charge, so you integrate $V\,dq$ to $\frac{1}{2}QV$, not QV.

⚠ Placing the energy on the plates.
The energy is stored in the *field* between the plates, with density $\frac{1}{2}\epsilon_0 E^2$.

Stored energy $\frac{1}{2}QV$ — the spring's half again — and it lives in the field, not on the charges.

Your turn: stored energy

$U = \frac{1}{2}CV^2$. *Cover the right column.*

1. $C = 2$ F, $V = 3$ V: U? ⇒ 9 J

2. Double the voltage: energy $\times$? ⇒ $\times 4$

3. Where does the energy reside? ⇒ the field

Why is the stored energy $\frac{1}{2}QV$ and not QV?

Your turn: capacitor energy

Cover the answers.

1. $C = 4$ F, $V = 2$ V: U _____

2. Three equivalent energy forms _____

3. Which Ch.4 quantity shares the half-factor? _____

4. Field energy density _____

Why is "energy lives in the field" the seed of electromagnetic waves?

Answers, with the reason

1. $\frac{1}{2}(4)(4)$
$= 8$ J

2. related by $Q = CV$
$\Rightarrow \frac{1}{2}CV^2 = \frac{1}{2}QV = Q^2/2C$

3. spring PE
$\Rightarrow \frac{1}{2}kx^2$

4. $\frac{1}{2}\epsilon_0 E^2$
$\Rightarrow$ per volume

If the field itself holds energy ($\frac{1}{2}\epsilon_0 E^2$), then a travelling field — a light wave — carries energy through empty space, with no charges needed.

In one breath

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = Q^2/2C;\ u = \tfrac{1}{2}\epsilon_0 E^2$$

Half-factor from the rising voltage (the spring's half); energy lives in the field.

*The So-What of Chapter 14: **trade force for energy**, and the vector field becomes the scalar potential, with $E = -dV/dx$ echoing $F = -dU/dx$. Potential is the bridge from the static field of Chapter 13 to moving charge — current, resistance, and circuits, next.*

Chapter 14 complete. Next chapter: **Current, Resistance & Circuits**

Explore this live | Watch the walk-through

Electric Current

Charge flowing — a rate, a derivative

$$I = \frac{dQ}{dt}$$
$$Q = \int I\,dt$$

Lesson `calcphys.circuits.I1`

Explore this live

Watch the walk-through

Current is the rate of charge flow

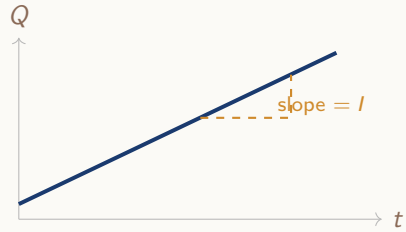
So far the charges stood still. Now we set them moving — and calculus returns to its original home, time.

$$I = \frac{dQ}{dt} \qquad (1\ \text{A} = 1\ \text{C/s})$$

Current is a derivative: current is to charge exactly what velocity was to position ($v = dx/dt$, Chapter 1). A steady current means charge flows at a constant rate; the total charge that has passed is the integral $Q = \int I\,dt$.

The slope of charge-versus-time

Because current is a rate, a graph of charge Q versus time has the current as its slope.



Conventional current is the direction *positive* charge would flow — opposite the actual electron drift in a metal. It runs from + to − in the external circuit.

Watch every step: charge from a steady current

A steady current of 2 A flows for 5 s. How much charge passes, and what is the current's relation to charge?

$$\begin{aligned} Q &= \int I\,dt = I\,\Delta t && \text{steady current} \\ &= 2 \times 5 = \boxed{10\ \text{C}} && \text{area under } I\text{–}t \end{aligned}$$

Current is the rate dQ/dt; charge is the accumulation $\int I\,dt$. The rate/accumulation pair from Chapter 1 returns, now for flowing charge.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I4` — electric current as charge per second, without the derivative.

The rule, in one line

$I = dQ/dt$ **(amperes), a derivative like** $v = dx/dt$
Total charge $Q = \int I\,dt$. Conventional current (+ flow)
is opposite the electron drift; same around a series loop.

Chapter 1 defined velocity as the derivative of position; current is the identical calculus for charge. And current is the same all the way around a simple series loop — charge is conserved and does not pile up, so what flows in flows out.

Watch out: two traps to dodge

⚠ Confusing current with charge.
Current is the *rate* dQ/dt — charge per second, not the charge itself.

⚠ Current "used up" around a loop.
It's the *same* everywhere in a series loop — charge is conserved, none piles up.

Current is a derivative; charge is its integral; the loop carries one current throughout.

Your turn: current

$I = dQ/dt$; $Q = I\,\Delta t$ if steady. Cover the right column.

1. 6 C in 3 s: current? ⇒ 2 A

2. 3 A for 4 s: charge? ⇒ 12 C

3. Series loop: same current everywhere? ⇒ yes

Which earlier quantity is current the exact analogue of?

Your turn: charge flow

Cover the answers.

1. 10 C in 2 s: current _____

2. 5 A for 6 s: charge _____

3. Units of current _____

4. Conventional current direction vs. electrons _____

On a charge-versus-time graph, what does the slope represent?

Answers, with the reason

1. $I = Q/t = 10/2$
$= 5$ A

2. $Q = It = 5(6)$
$= 30$ C

3. coulombs/second
$\Rightarrow$ amperes

4. opposite
$\Rightarrow +$ flow, not e^-

The slope of $Q(t)$ is $dQ/dt = I$ — the current, exactly as the slope of $x(t)$ was the velocity.

In one breath

$I = dQ/dt$; $Q = \int I\,dt$; **same around a series loop**
Current is to charge as velocity is to position (Ch.1).

Zoom into the wire and the current resolves into countless electrons drifting — astonishingly slowly, yet the light comes on at once.

Next up: **I2** Current density & drift velocity

Explore this live Watch the walk-through

Current Density & Drift

Slow electrons, instant light

$$J = \frac{I}{A}$$
$$I = nqv_dA$$

Lesson `calcphys.circuits.I2`

Explore this live

Watch the walk-through

Zoom into the wire

*The **current density** $J = I/A$ is the current per unit cross-sectional area. Microscopically, the current is carried by electrons drifting under the field.*

$$J = \frac{I}{A}, \qquad I = nqv_d A$$

*Here n is the number of carriers per volume, q the charge each carries, and v_d the average **drift velocity**. The striking fact: v_d is astonishingly slow — typically under a millimetre per second.*

Slow drift, yet the bulb lights at once

If electrons crawl, why does a lamp light the instant you flip the switch?

$$\text{drift } v_d \lesssim 1 \text{ mm/s} \qquad \text{vs.} \qquad \text{signal} \approx \text{light speed}$$

The resolution: the field that pushes the electrons is established through the wire at nearly light speed, so all electrons start drifting almost at once. You don't wait for one electron to travel from switch to bulb — the whole column of charge lurches forward together.

Watch every step: how a slow drift gives a big current

Drift is slow, yet currents are large. Reconcile this with $I = nqv_dA$.

$$I = nqv_dA \qquad \text{four factors}$$
$$\therefore \; n \text{ is } \textit{enormous} \qquad \text{huge } n \Rightarrow \text{large } I$$

There are astronomically many carriers (n huge), so even a tiny v_d sums to a substantial current. The drift is a slow average on top of the electrons' fast random thermal motion.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I4` — electric current and how charge carriers move, without the microscopic model.

The rule, in one line

$$J = I/A;\ I = nqv_dA$$

Drift velocity is very slow, yet the signal travels near light speed because the field sets all electrons moving at once.

Large n makes the current large despite the crawl. The mental picture: a pipe already full of water — open the tap and water leaves the far end immediately, even though any one molecule moves slowly.

Watch out: two traps to dodge

⚠ Electrons race at light speed.
The drift is slow (mm/s). It's the *field* that propagates near light speed.

⚠ The signal waits for one electron to cross.
No — all electrons start drifting almost at once when the field arrives.

Slow drift, fast field; large n makes $I = nqv_dA$ big anyway.

Your turn: drift

$J = I/A$; $I = nqv_dA$. Cover the right column.

1. $I = 6$ A, $A = 2$ mm^2: J? ⇒ 3 A/mm^2

2. What makes the current large despite slow drift? ⇒ huge n

3. What travels near light speed? ⇒ the field

? *Why does a lamp light instantly if electrons drift at under a millimetre per second?*

Your turn: current density

Cover the answers.

1. $I = 10$ A, $A = 5$ mm^2: J _____

2. The four factors in $I = nqv_dA$ _____

3. Is drift fast or slow? _____

4. Why does the bulb light at once?

_____

How does a slow drift velocity coexist with a large current?

Answers, with the reason

1. $J = I/A = 10/5$
$= 2\ \mathrm{A/mm^2}$

2. carriers, charge, drift, area
$\Rightarrow n, q, v_d, A$

3. mm/s
$\Rightarrow$ slow

4. field arrives fast
$\Rightarrow$ all e^- at once

The enormous carrier density n means even a crawl of drift, multiplied through, yields amperes.

In one breath

$J = I/A$; $I = nqv_dA$; **slow drift, fast field**

Huge n makes the current large; the field sets all electrons moving at once.

What sets how much current a given push produces? Resistance — and for many materials, current is simply proportional to voltage: Ohm's law.

Next up: **I3** Resistance & Ohm's law

Explore this live | Watch the walk-through

Ohm's Law

Voltage, current, resistance

$$V = IR$$
$$I = V/R,\ R = V/I$$

Lesson `calcphys.circuits.I3`

Explore this live

Watch the walk-through

Current proportional to voltage

Resistance *measures how strongly a component opposes current. For many materials the current is proportional to the voltage across them — Ohm's law.*

$$V = IR \qquad (1\ \Omega = 1\ \mathrm{V/A})$$

Read three ways: a larger voltage drives more current ($I = V/R$), a larger resistance chokes it, and $R = V/I$. Getting comfortable rearranging $V = IR$ three ways is the core circuit skill.

An empirical rule, not a fundamental law

Unlike Newton's or Gauss's laws, Ohm's law is empirical — an approximate description that many (not all) materials follow.

Ohmic: constant R Non-ohmic: R varies

*Materials with constant R are **ohmic**. A bulb filament, whose resistance rises as it heats, is famously non-ohmic. And resistance is what converts electrical energy to heat — the reason wires warm and toasters glow.*

Watch every step: find the resistance

A 12 V source drives 3 A through a resistor. What is its resistance?

$$R = \frac{V}{I} \qquad \text{rearrange } V = IR$$
$$= \frac{12}{3} = 4\ \Omega \qquad \text{ohms} = \text{volts/amp}$$

Doubling the resistance would halve the current at this voltage — more resistance means less current at fixed V. The three rearrangements of $V = IR$ handle every basic circuit question.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I5` — Ohm's law $V = IR$ and rearranging it three ways, without calculus.

The rule, in one line

$V = IR$, **so** $I = V/R$ **and** $R = V/I$

Ohmic = constant R. It is empirical, not fundamental; more resistance means *less* current at fixed V.

Ohm's law is the workhorse of the whole chapter. Whatever you're missing — voltage, current, or resistance — one rearrangement delivers it. But remember it's an approximation, not a guarantee.

Watch out: two traps to dodge

Mis-arranging to $V = I/R$.
It's $V = IR$. Rearrange to $I = V/R$ or $R = V/I$ — never I/R.

More resistance ⇒ more current.
The opposite — resistance *reduces* current at fixed voltage.

$V = IR$, three ways; ohmic means constant R; more R, less current.

Your turn: Ohm's law

$V = IR$. Cover the right column.

1. $V = 10$ V, $R = 5\ \Omega$: current? $\Rightarrow$ 2 A
2. $I = 2$ A, $R = 6\ \Omega$: voltage? $\Rightarrow$ 12 V
3. $V = 9$ V, $I = 3$ A: resistance? $\Rightarrow$ 3 Ω

Why is a bulb filament non-ohmic?

Your turn: resistance

Cover the answers.

1. $V = 24$ V, $R = 8\ \Omega$: I _____

2. $I = 4$ A, $R = 3\ \Omega$: V _____

3. Is Ohm's law fundamental? _____

4. Fixed V, double R: current does what? _____

What does a resistor convert electrical energy into?

Answers, with the reason

1. $I = V/R = 24/8$
$= 3$ A

2. $V = IR = 4(3)$
$= 12$ V

3. empirical
$\Rightarrow$ no

4. halves
$\Rightarrow I \to I/2$

> *A resistor turns electrical energy into heat — Joule heating, the subject two frames ahead.*

In one breath

$V = IR$, **three ways; ohmic = constant** R
Empirical, not fundamental; more resistance means less current at fixed V.

But where does resistance come from? From the material and the shape of the wire — resistivity times geometry.

Next up: **I4** Resistivity & the resistance of a wire

Explore this live | Watch the walk-through

Resistance of a Wire

Material times geometry

$$R = \frac{\rho L}{A}$$

longer ↑, thicker ↓

Lesson `calcphys.circuits.I4`

Explore this live

Watch the walk-through

What the wire is made of, and its shape

Resistance depends on both what the wire is made of and its shape. The material property is the ***resistivity*** ρ.

$$R = \frac{\rho L}{A}$$

Resistance grows with length L and shrinks with cross-sectional area A. Think of a water pipe: a longer pipe resists flow more, a wider one less. A long thin wire has high resistance; a short fat wire, low.

Separate the material from the shape

The whole idea is to split resistivity ρ (the material) from the geometric factor L/A (the shape).

$$R = \underbrace{\rho}_{\text{material}} \times \underbrace{\frac{L}{A}}_{\text{geometry}}$$

Resistivity ρ is intrinsic to the material and (for most conductors) rises with temperature — which is why a hot filament resists more. Resistance R combines that intrinsic ρ with the geometry L/A.

Watch every step: double the radius

A wire's radius is doubled (same material, same length). What happens to its resistance?

$$A = \pi r^2 \Rightarrow r \to 2r \text{ gives } A \to 4A \qquad \text{area scales as } r^2$$

$$R = \frac{\rho L}{A} \Rightarrow R \to \tfrac{1}{4}R \qquad \text{quarter the resistance}$$

Doubling the length doubles R; doubling the radius quadruples the area and cuts R to a quarter. The area's quadratic dependence on radius is easy to forget.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I5` — resistance and how it depends on the wire, without the $\rho L/A$ formula.

The rule, in one line

$R = \rho L/A$ — **material ρ times geometry L/A**

Longer $\Rightarrow$ more R; thicker $\Rightarrow$ less. Double the radius $\Rightarrow$ quarter the resistance.

Separating the material property ρ from the geometric factor L/A is the key idea. Change the metal, ρ changes; change the dimensions, L/A changes; the resistance follows from their product.

Watch out: two traps to dodge

⚠ Confusing resistivity with resistance.
ρ is the material *alone*; $R = \rho L/A$ folds in the geometry too.

⚠ Reversing length/area.
R grows with L and *falls* with A — longer resists more, thicker resists less.

Resistivity is the metal; resistance is metal times shape; area beats length via the r^2.

Your turn: wire resistance

$R = \rho L/A$. Cover the right column.

1. Double the length: R changes by? $\Rightarrow \times 2$
2. Double the area: R changes by? $\Rightarrow \times \frac{1}{2}$
3. Double the radius: R changes by? $\Rightarrow \times \frac{1}{4}$

? *Why does doubling the radius cut resistance more than doubling the area would suggest at first glance?*

Your turn: resistivity

Cover the answers.

1. Triple the length: R _____

2. Which factor is the material? _____

3. Does ρ rise or fall with temperature? _____

4. Long thin vs. short fat: which resists more? _____

What is the difference between resistivity and resistance?

Answers, with the reason

1. $R \propto L$
$\Rightarrow \times 3$

2. resistivity
$\Rightarrow \rho$

3. conductors
$\Rightarrow$ rises

4. long thin
$\Rightarrow$ more

> *Resistivity is intrinsic to the metal; resistance combines it with the wire's length and thickness.*

In one breath

$R = \rho L/A$; **material times geometry**
Longer more, thicker less; double the radius, quarter the resistance.

Push current through resistance and energy is spent — turned into heat. The rate of that conversion is electric power.

Next up: **I5** Electric power

Explore this live | Watch the walk-through

Electric Power

The rate of energy conversion

$$P = IV = I^2R = \frac{V^2}{R}$$

watts = joules/second

Lesson `calcphys.circuits.I5`

Explore this live

Watch the walk-through

Power is energy per second

When current flows through a component, electrical energy is converted to other forms, and the rate is the power.

$$P = IV \qquad (1\ \text{W} = 1\ \text{J/s})$$

Using Ohm's law, this has two other forms for a resistor: $P = I^2R = V^2/R$, all equivalent via $V = IR$. Power was defined in Chapter 4 as the rate of doing work ($P = Fv$) — here it's the electrical rate of delivering energy.

Pick the form that matches what's fixed

The three forms are algebraically equal, but which is useful depends on what is held constant.

fixed current	$P = I^2R$	more $R \Rightarrow$ more power
fixed voltage	$P = V^2/R$	more $R \Rightarrow$ less power

In a resistor the energy becomes heat (Joule heating, $P = I^2R$) — why wires warm, toasters glow, and power lines waste energy. The opposite trends warn you: always know which quantity is held constant.

Watch every step: one resistor, three routes

A 4-Ω resistor carries 3 A. Find the power three ways and confirm they agree.

$$
\begin{aligned}
V &= IR = 3(4) = 12\ \text{V} && \text{Ohm's law} \\
P &= IV = 3(12) = I^2R = 9(4) = \frac{V^2}{R} = \frac{144}{4} && \text{all three} \\
&= 36\ \text{W} && \text{they agree}
\end{aligned}
$$

All three forms give 36 W — they must, since $V = IR$ links them. This energy leaves as heat in the resistor.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I6` — electric power $P = IV$ and its resistor forms, without calculus.

The rule, in one line

$$P = IV = I^2R = V^2/R \textbf{ (watts)}$$

Joule heating in a resistor. Fixed V: more R $\Rightarrow$ less power. Fixed I: more $R \Rightarrow$ more power.

Same notion of power as Chapter 4 — energy per second — now for charge pushed through a potential difference. A watt is a rate, not a quantity of energy; energy is power times time.

Watch out: two traps to dodge

⚠ Treating watts as energy.
A watt is a joule per *second* — a rate. Energy is $P \times t$.

⚠ More resistance always means more power.
Depends what's fixed: V^2/R falls with R, I^2R rises with R.

$P = IV$, three forms; heat in a resistor; the trend flips with what's held constant.

Your turn: power

$P = IV = I^2R = V^2/R$. *Cover the right column.*

1. $I = 2$ A, $V = 6$ V: power? ⇒ 12 W
2. $I = 3$ A, $R = 2\ \Omega$: power? ⇒ 18 W
3. $V = 10$ V, $R = 5\ \Omega$: power? ⇒ 20 W

? *At fixed voltage, does a higher-resistance bulb draw more or less power?*

Your turn: electric power

Cover the answers.

1. $I = 4$ A, $V = 5$ V: P _____

2. $I = 2$ A, $R = 3\ \Omega$: P _____

3. Into what form does a resistor convert energy? _____

4. Units of power _____

Which power formula would you use if the current is fixed?

Answers, with the reason

1. $P = IV = 4(5)$
$= 20$ W

2. $I^2R = 4(3)$
$= 12$ W

3. Joule heating
$\Rightarrow$ heat

4. joules/second
$\Rightarrow$ watts

With current fixed, $P = I^2R$ makes the dependence direct — more resistance dissipates more power.

In one breath

$$P = IV = I^2R = V^2/R \textbf{ (watts)}$$

Joule heating; fixed V falls with R, fixed I rises with R.

Real batteries aren't perfect sources — some voltage is lost inside them. Under load, the terminal voltage sags below the EMF.

Next up: **I6** EMF & internal resistance

Explore this live Watch the walk-through

EMF & Internal Resistance

Why a real battery sags under load

$$V = \mathcal{E} - Ir$$

open: $V = \mathcal{E}$

Lesson `calcphys.circuits.I6`

Explore this live

Watch the walk-through

A real battery is not a perfect source

*The **EMF** $\mathcal{E}$ is the voltage a battery would supply with no current flowing — the ideal push. But every battery has some **internal resistance** r.*

$$V = \mathcal{E} - Ir$$

*When current I flows, some voltage is lost inside, so the **terminal voltage** delivered drops below the EMF by Ir. With no current (open circuit) the terminal voltage equals the full EMF.*

Headlights dim when the starter turns

A car's headlights dim the instant the starter motor (drawing huge current) engages.

$$\text{big } I \;\Rightarrow\; \text{big } Ir \;\Rightarrow\; V \text{ sags}$$

The large starter current makes Ir large, pulling the terminal voltage down — and the lights, sharing that terminal, dim. A nearly "dead" battery often has a normal EMF but a large internal resistance: it reads fine unloaded yet collapses under load.

Watch every step: terminal voltage under load

A battery has $\mathcal{E} = 12$ V and internal resistance $r = 0.5\ \Omega$. It delivers 4 A. Find the terminal voltage.

$$\begin{aligned} V &= \mathcal{E} - Ir && \text{terminal voltage} \\ &= 12 - (4)(0.5) = 12 - 2 = \boxed{10\ \text{V}} && \text{sags by } Ir \end{aligned}$$

The full 12 V appears only with no current; under a 4-A load, 2 V is lost inside, leaving 10 V at the terminals. The bigger the current, the bigger the sag.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I5` — voltage, current, and resistance — the ideas behind $V = \mathcal{E} - Ir$.

The rule, in one line

$$V = \mathcal{E} - Ir \text{; } \textbf{open circuit } V = \mathcal{E}$$

Under load V sags by Ir. A "dead" battery: normal EMF, large r.

EMF is the no-current, open-circuit voltage; terminal voltage is what you actually get under load. Internal resistance is usually small, but at high current it matters — which is exactly when things sag.

Watch out: two traps to dodge

⚠ Terminal voltage always equals EMF.
Only with no current. Under load $V = \mathcal{E} - Ir < \mathcal{E}$.

⚠ Ignoring r.
Small though it usually is, r dominates at high current — the sag grows with I.

EMF is the open-circuit push; terminal voltage sags by Ir under load; big current, big sag.

Your turn: EMF

$V = \mathcal{E} - Ir$. *Cover the right column.*

1. $\mathcal{E} = 9$ V, $I = 2$ A, $r = 0.5\ \Omega$: V? $\Rightarrow$ 8 V

2. Same battery, open circuit: V? $\Rightarrow$ 9 V

3. Larger current: terminal voltage does what? $\Rightarrow$ sags

? *Why do a car's headlights dim when the starter engages?*

Your turn: real batteries

Cover the answers.

1. $\mathcal{E} = 6$ V, $I = 3$ A, $r = 0.4\ \Omega$: V _____

2. EMF is the voltage at what current? _____

3. A "dead" battery has large _____

4. Under load, V vs. $\mathcal{E}$ _____

How can a battery read a full voltage unloaded yet fail under load?

Answers, with the reason

1. $6 - (3)(0.4)$
$= 4.8$ V

2. no current
$\Rightarrow$ open circuit

3. internal resistance
$\Rightarrow$ large r

4. sags
$\Rightarrow V < \mathcal{E}$

Unloaded, $I = 0$ so $V = \mathcal{E}$; under load the large r makes Ir big and the terminal voltage collapses.

In one breath

$V = \mathcal{E} - Ir$; **open circuit** $V = \mathcal{E}$

Terminal voltage sags by Ir under load; a dead battery has large r.

Circuits rarely hold one resistor. When several combine, the rules are the exact mirror of the capacitors from last chapter.

Next up: **I7** Resistors in series & parallel

Explore this live Watch the walk-through

Series & Parallel Resistors

The mirror image of capacitors

series: $R_{eq} = R_1 + R_2$

parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Lesson `calcphys.circuits.I7`

Explore this live | Watch the walk-through

Two ways to combine

Resistors combine by rules that are the mirror image of the capacitor rules from last chapter.

Series (same I) $\quad R_{eq} = R_1 + R_2 + \cdots$

Parallel (same V) $\quad \dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots$

Series (end to end): the equivalent is larger than any single one — current fights through them in turn. Parallel (side by side): the equivalent is smaller than the smallest — more paths for current.

Capacitors do the opposite

In Chapter 14 capacitors added in parallel and combined reciprocally in series. Resistors are exactly reversed.

	Series	**Parallel**
Resistors	add	reciprocal
Capacitors	reciprocal	add

The mnemonic: capacitance grows with area (parallel plates), while resistance grows with length (series wire). Same current in series, same voltage in parallel — the two conserved quantities.

Watch every step: parallel lowers resistance

Two 6-Ω resistors are placed in parallel. Find the equivalent resistance.

$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \qquad \text{reciprocals add}$$

$$R_{eq} = 3\ \Omega \qquad \text{below the smallest!}$$

Two 6-Ω resistors in parallel give 3 Ω — less than either. Adding a parallel resistor always lowers total resistance, because it opens another route for current. In series they'd give 12 Ω.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.circuits.I7` — combining resistors in series and parallel, without calculus.

The rule, in one line

Series: $R_{eq} = R_1 + R_2$ **(same I), larger than any; Parallel:** $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ **(same V), smaller than smallest**
The reverse of capacitors.

Series resistors carry the same current and their voltages add; parallel resistors share the same voltage and their currents add. Reduce a network step by step to one equivalent resistance.

Watch out: two traps to dodge

⚠ Swapping the rules.
Series *adds*; parallel adds *reciprocals*. Capacitors are the reverse — don't cross them.

⚠ A parallel resistor raises resistance.
It *lowers* the total below the smallest — another route always eases the flow.

Series-add, parallel-reciprocal; parallel always lowers; capacitors do the opposite.

Your turn: combinations

Series adds; parallel adds reciprocals. Cover the right column.

1. 3 Ω and 5 Ω in series: R_{eq}? ⇒ 8 Ω

2. Two 4 Ω in parallel: R_{eq}? ⇒ 2 Ω

3. Add a parallel resistor: total goes? ⇒ down

? *Why does adding another parallel path lower the total resistance?*

Your turn: series & parallel

Cover the answers.

1. 2 Ω and 6 Ω in series _____

2. Two 6 Ω in parallel _____

3. Series resistors share the same _____

4. How do capacitors combine vs. resistors? _____

What quantity is shared in series? In parallel?

Answers, with the reason

1. $2 + 6$
$= 8\ \Omega$

2. $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$
$= 3\ \Omega$

3. current
$\Rightarrow$ same I

4. reversed
$\Rightarrow$ opposite rules

Each extra parallel branch is another lane for current, so more charge flows for the same voltage — lower resistance.

In one breath

Series adds; parallel adds reciprocals; the reverse of capacitors

Same I in series, same V in parallel; parallel always lowers the total.

Add a capacitor to a resistor and time enters: the charge relaxes gradually, along an exponential curve — a first-order differential equation.

Next up: **I8** RC circuits

Explore this live Watch the walk-through

RC Circuits

Charge relaxes exponentially

$$Q = Q_0\, e^{-t/RC}$$
$$\tau = RC$$

Lesson `calcphys.circuits.I8`

Explore this live

Watch the walk-through

Add a capacitor, and time enters

*With a capacitor in the circuit, charge builds or drains gradually, along an **exponential** curve.*

$$Q(t) = Q_0\, e^{-t/RC} \text{ (discharge)}, \qquad Q(t) = Q_f\,(1 - e^{-t/RC}) \text{ (charge)}$$

*The product $\tau = RC$ is the **time constant**: the time to fall to about 37% $(1/e)$ of the initial charge, or rise to about 63% of the final. A larger R or C means a slower process.*

A first-order differential equation

This exponential solves a first-order differential equation: the rate of change of charge is proportional to the charge.

$$\frac{dQ}{dt} \propto -Q \quad \Longrightarrow \quad Q = Q_0\, e^{-t/RC}$$

This is the same mathematical form as Chapter 7's damped oscillations — and radioactive decay, and cooling. The damped oscillator decayed because a term proportional to velocity drained its energy; the RC circuit is the same decay for charge. Different physics, identical calculus.

Watch every step: one time constant

A capacitor discharges through a resistor with $R = 2\ k\Omega$, $C = 5\ \mu F$, starting at Q_0. What remains after one time constant?

$$\tau = RC = (2000)(5 \times 10^{-6}) = 0.01\ \text{s} \qquad \tau = RC$$

$$Q(\tau) = Q_0\, e^{-1} \approx 0.37\, Q_0 \qquad \text{37\% remains}$$

After one time constant, 37% of the charge remains; after a few τ it's essentially gone ($5\tau \Rightarrow$ over 99% drained). The approach is asymptotic — it never quite finishes.

🎓 **Math support** Shaky on this step? See **Algebra II** lesson `algebra2.exponential.B2` — exponential-decay graphs — reading the $e^{-t/RC}$ curve and its 37% mark.

The rule, in one line

$Q = Q_0 e^{-t/RC}$ **(discharge),** $Q = Q_f(1 - e^{-t/RC})$ **(charge);** $\tau = RC$
37%/63% at one τ. Larger R or $C \Rightarrow$ slower. A first-order decay ODE.

The timescale is RC: doubling either R or C doubles the time. After a few time constants the capacitor is essentially done — 5τ drains over 99%.

Watch out: two traps to dodge

⚠ The capacitor charges instantly.
It charges over *time*, along $e^{-t/RC}$ — asymptotically, never quite finishing.

⚠ Mis-forming τ as R/C or $R + C$.
The time constant is the *product* $\tau = RC$; larger R or C is slower.

Exponential relaxation, time constant RC, 37% at one τ — the same decay as damped motion.

Your turn: RC circuits

$\tau = RC$; 37% remains at one τ. Cover the right column.

1. $R = 1$ kΩ, $C = 2$ μF: τ? ⇒ 2 ms
2. Fraction left after one τ? ⇒ 37%
3. Double C: process faster or slower? ⇒ slower

? *Which Chapter 7 phenomenon shares this exact exponential decay?*

Your turn: RC decay

Cover the answers.

1. $R = 3\ \text{k}\Omega$, $C = 2\ \mu\text{F}$: τ _____

2. Charge remaining at one τ _____

3. Form of the time constant _____

4. What kind of equation is an RC circuit? _____

Why does a bigger R or C slow the charging, not speed it up?

Answers, with the reason

1. $\tau = RC = (3000)(2\mu)$
$= 6$ ms

2. e^{-1}
$= 37\%$

3. product
$\Rightarrow \tau = RC$

4. first-order
$\Rightarrow$ decay ODE

A bigger R chokes the current and a bigger C demands more charge — both stretch out the time $\tau = RC$.

In one breath

$$Q = Q_0 e^{-t/RC}; \tau = RC \text{ (37\%/63\%)}$$

Larger R or C is slower; a first-order decay ODE, like damped oscillations.

The So-What of Chapter 15: ***let charge flow and calculus returns to time*** *— current is dQ/dt (Ch.1's derivative), and the RC circuit is a differential equation (Ch.7's decay). Next, moving charge makes a new field of its own: magnetism.*

Chapter 15 complete. Next chapter: **Magnetic Field & Ampère's Law**

Explore this live | Watch the walk-through

The Magnetic Force

Acts only on moving charge, does no work

$$\vec{F} = q\vec{v} \times \vec{B}$$
$$F = qvB\sin\theta$$

Lesson `calcphys.magnetism.B1`

Explore this live | Watch the walk-through

A force unlike any so far

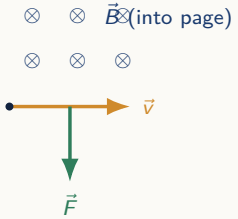
*A magnetic field exerts a force on a charge only when it is moving — and the force is a **cross product**.*

$$\vec{F} = q\vec{v} \times \vec{B}, \qquad F = qvB\sin\theta$$

Three features make it strange: the magnitude is largest when $v \perp B$ and zero when $v \parallel B$ (or the charge is at rest); the direction is perpendicular to both v and B; and because it's always perpendicular to the velocity, it does no work — it steers without speeding up.

Perpendicular to both, by the right-hand rule

Point fingers along v, curl toward B; the thumb gives $\vec{v} \times \vec{B}$ (flip for negative q).



A stationary charge feels *nothing* ($v = 0$) — unlike the electric force. And the force never changes the speed, only the direction.

Watch every step: when is the force zero?

A charge q moves at speed v through a field B. Find the force when $v \perp B$, and when $v \parallel B$.

$$v \perp B \ (\theta = 90^\circ) \ : \ F = qvB \sin 90^\circ = qvB \qquad \text{maximum}$$

$$v \parallel B \ (\theta = 0) \ : \ F = qvB \sin 0 = 0 \qquad \text{no force at all}$$

The $\sin\theta$ is the whole story: full force across the field, nothing along it. And since $\vec{F} \perp \vec{v}$ always, the work $\vec{F} \cdot \vec{v} = 0$ — magnetism steers charges without adding energy.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B2` — the force on a moving charge, $F = qvB$, without the cross product.

The rule, in one line

$$\vec{F} = q\vec{v} \times \vec{B}, \; F = qvB\sin\theta$$

Perpendicular to both v and B; zero if $v = 0$ or $v \parallel B$; does *no work*.

*The complete force on a charge combines electric and magnetic: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ — the **Lorentz force**. Electric acts on any charge along the field; magnetic acts only on moving charge, across the field.*

Watch out: two traps to dodge

⚠ A charge at rest feels a magnetic force.

It needs *motion* — $v = 0$ gives $F = 0$. Unlike the electric force, which acts on any charge.

⚠ The force is along v or B.

It's perpendicular to *both* (right-hand rule) — and it does no work, so the speed never changes.

Moving charge, cross product, perpendicular to both, zero work — the magnetic force in one breath.

Your turn: the magnetic force

$F = qvB\sin\theta$. *Cover the right column.*

1. $q = 2$ C, $v = 3$, $B = 4$, $\perp$: F? $\Rightarrow$ 24 N

2. Same charge at rest: F? $\Rightarrow$ 0

3. $v \parallel B$: F? $\Rightarrow$ 0

? *Why can the magnetic force change a charge's direction but never its kinetic energy?*

Your turn: magnetic force

Cover the answers.

1. $q = 1$, $v = 5$, $B = 2$, $\perp$: F _____

2. Direction of $\vec{F}$ relative to v, B _____

3. Work done by the magnetic force

_____

4. Full force on a charge (name it)

_____

How does the magnetic force differ from the electric force in what it acts on?

Answers, with the reason

1. $qvB = 1(5)(2)$
$= 10$ N

2. cross product
$\Rightarrow \perp$ both

3. $\vec{F} \perp \vec{v}$
$\Rightarrow 0$

4. electric + magnetic
$\Rightarrow$ Lorentz

Because $\vec{F}$ is always perpendicular to $\vec{v}$, the power $\vec{F} \cdot \vec{v}$ is zero — no energy is added, only the direction turns.

In one breath

$\vec{F} = q\vec{v} \times \vec{B}$, $F = qvB\sin\theta$

Moving charge only; perpendicular to both; zero work; part of the Lorentz force.

A force that's always perpendicular to the velocity and does no work — that's exactly a centripetal force. So a charge crossing a field curls into a circle.

Next up: **B2** Circular motion in a magnetic field

Explore this live | Watch the walk-through

Circular Motion in a Field

The magnetic force as centripetal

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}, \text{ speed-free}$$

Lesson `calcphys.magnetism.B2`

Explore this live

Watch the walk-through

A perpendicular force bends the path into a circle

When a charge moves perpendicular to a uniform field, the magnetic force — always perpendicular to v — acts as a ***centripetal*** *force, curling the path into a circle.*

$$qvB = \frac{mv^2}{r} \quad \Longrightarrow \quad r = \frac{mv}{qB}$$

This is exactly the uniform-circular-motion analysis of Chapter 2 — with the magnetic force playing the role gravity or tension played there. Set magnetic equal to centripetal, and the radius falls out.

The period ignores the speed

A remarkable consequence: the period doesn't depend on speed or radius at all.

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

The v cancels! Faster charges travel bigger circles but take the same time to go around. This is the principle of the ***cyclotron*** *— and it works because the force does no work, so the speed stays constant and the motion is truly uniform.*

Watch every step: double the speed

Two identical charges enter the same field, one moving twice as fast. Compare their radii and periods.

$$r = \frac{mv}{qB} \propto v \Rightarrow \boxed{r \text{ doubles}} \qquad \text{bigger circle}$$

$$T = \frac{2\pi m}{qB} \ (\text{no } v) \Rightarrow \boxed{T \text{ unchanged}} \qquad \text{same time}$$

Bigger circle, same time — the cyclotron principle. The faster charge simply has farther to travel, and its extra speed exactly covers the extra distance.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B7` — the mass spectrometer and velocity selector, where $r = mv/qB$ sorts particles.

The rule, in one line

$$qvB = mv^2/r \Rightarrow r = mv/(qB);\ T = 2\pi m/(qB)$$

Period independent of speed. Uniform circular motion — the field steers, the speed is constant.

A stronger field or slower speed tightens the circle; a heavier or faster particle widens it. Because r depends on m/q, a magnetic field sorts particles by mass-to-charge — the mass spectrometer.

Watch out: two traps to dodge

⚠ The radius is speed-independent.
No — $r \propto v$. It's the *period* that's speed-independent.

⚠ The speed changes in the field.
It's constant — the force does no work. Only the direction turns.

$r \propto v$ (bigger circle), T fixed (same time), speed constant (no work). The cyclotron in three facts.

Your turn: circular motion

$r = mv/qB$, $T = 2\pi m/qB$. Cover the right column.

1. Double B: radius changes by? ⇒ halved
2. Double v: period changes by? ⇒ unchanged
3. Does the speed change in the field? ⇒ no

❓ *Why do faster charges in a cyclotron still complete each loop in the same time?*

Your turn: charges in a field

Cover the answers.

1. $m = 2$, $v = 3$, $q = 1$, $B = 6$: r _____

2. What supplies the centripetal force? _____

3. What is speed-independent, r or T? _____

4. Stronger field: circle tightens or widens? _____

How does $r = mv/qB$ let a mass spectrometer separate ions?

Answers, with the reason

1. $r = mv/qB = 6/6$
$= 1$ m

2. magnetic force
$\Rightarrow qvB$

3. period
$\Rightarrow T$

4. larger B
$\Rightarrow$ tightens

At fixed v and B, $r = mv/qB$ depends on m/q — so ions of different mass curve to different radii and land apart.

In one breath

$r = mv/(qB)$; $T = 2\pi m/(qB)$, **speed-free**
Chapter 2's uniform circular motion;
the field steers, the speed is constant.

A current is just a stream of moving charges — so a whole current-carrying wire feels this same force, summed over every carrier. That force turns motors.

Next up: **B3** The force on a current-carrying wire

Explore this live | Watch the walk-through

Force on a Current

Moving charges, summed into a wire

$$\vec{F} = I\vec{L} \times \vec{B}$$
$$F = BIL\sin\theta$$

Lesson `calcphys.magnetism.B3`

Explore this live

Watch the walk-through

A current is moving charge — so it feels the force

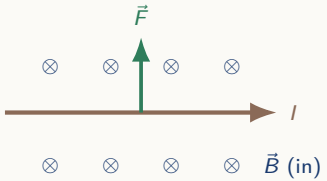
Summing $q\vec{v} \times \vec{B}$ over every carrier in a wire gives the force on the whole wire.

$$\vec{F} = I\vec{L} \times \vec{B}, \qquad F = BIL\sin\theta$$

*$\vec{L}$ points along the wire in the current's direction. The magnitude is largest when the wire is perpendicular to the field and zero when parallel, with the direction perpendicular to both (right-hand rule). This force makes electric **motors** turn.*

The wire jumps in the field

Current to the right, field into the page: the force pushes the wire up (right-hand rule).



The force is on the *wire* — transmitted through the carriers to the metal lattice — which is why a current-carrying wire physically jumps in a field.

Watch every step: perpendicular vs. parallel

A wire of length L carries current I through field B. Find the force when perpendicular, and when parallel.

$$\text{wire} \perp B \;:\; F = BIL\sin 90^\circ = BIL \qquad \text{maximum}$$

$$\text{wire} \parallel B \;:\; F = BIL\sin 0 = 0 \qquad \text{no force}$$

And a wire with no current feels nothing at all — just as a stationary charge does. The force needs both a current and a component of the wire across the field.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B3` — the force on a current-carrying wire, $F = BIL$, without the cross product.

The rule, in one line

$$\vec{F} = I\vec{L} \times \vec{B}, \; F = BIL\sin\theta$$

Max perpendicular, zero parallel; needs a current. The basis of the electric motor.

This is $\vec{F} = q\vec{v} \times \vec{B}$ scaled up to a whole wire: replace "charge times velocity" with "current times length." A loop of such wire feels forces that produce a torque — the next step toward the motor.

Watch out: two traps to dodge

⚠ A current-free wire feels a force.
No current, no force — exactly like a charge at rest. Motion of charge is required.

⚠ Force peaks when the wire is parallel to B.
It's *zero* parallel, *maximum* perpendicular — the $\sin\theta$ again.

Current times length crossed with field; max perpendicular, zero parallel, zero with no current.

Your turn: force on a wire

$F = BIL\sin\theta$. *Cover the right column.*

1. $B = 2$, $I = 3$, $L = 0.5$, $\perp$: F? $\Rightarrow$ 3 N

2. Same wire, no current: F? $\Rightarrow$ 0

3. Wire parallel to B: F? $\Rightarrow$ 0

? *Why does a current-carrying wire physically jump when a magnet is brought near?*

Your turn: current in a field

Cover the answers.

1. $B = 0.4$, $I = 5$, $L = 2$, $\perp$: F _____

2. When is the force maximum? _____

3. What must the wire carry to feel a force? _____

4. What device does this force run? _____

How is $\vec{F} = I\vec{L} \times \vec{B}$ really just $q\vec{v} \times \vec{B}$ added up?

Answers, with the reason

1. $BIL = 0.4(5)(2)$
$= 4$ N

2. $\theta = 90°$
$\Rightarrow$ wire $\perp B$

3. moving charge
$\Rightarrow$ a current

4. rotating loop
$\Rightarrow$ a motor

Each carrier feels $q\vec{v} \times \vec{B}$; multiply by the number of carriers moving at drift speed and it becomes $I\vec{L} \times \vec{B}$ for the whole wire.

In one breath

$$\vec{F} = I\vec{L} \times \vec{B}, \; F = BIL\sin\theta$$

Max perpendicular, zero parallel, zero with no current; runs the motor.

So far fields have acted on charges. Now flip the question: what creates a magnetic field? Moving charges do — and each current element makes a little field by the Biot–Savart law.

Next up: **B4** The magnetic field of a current (Biot–Savart)

Explore this live | Watch the walk-through

The Biot–Savart Law

Currents make circling fields

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$
$$\vec{B} = \int d\vec{B}$$

Lesson `calcphys.magnetism.B4`

Explore this live

Watch the walk-through

What *creates* a magnetic field?

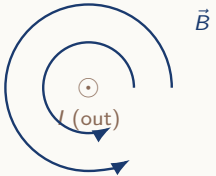
So far fields have acted on charges. Now we ask what makes them — moving charges and currents. Each element $I\,d\vec{l}$ produces a small field:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}, \qquad \vec{B} = \int d\vec{B}$$

The total field is the integral over the whole current — the magnetic cousin of Chapter 13's electric-field integral. Two twists: a cross product ($d\vec{l}\times\hat{r}$), and the constant μ_0 (permeability) in place of k.

The cross product makes it circle

Because of $d\vec{l} \times \hat{r}$, the field circles around the current rather than pointing radially outward.



Magnetic field lines form *closed loops* around currents — they never begin or end. There are no magnetic "charges," unlike electric lines that start on $+$ and end on $-$.

Watch every step: same integral, new geometry

Compare building the electric field of a charge distribution with building the magnetic field of a current.

$$\text{electric (Ch.13)} \;:\; \vec{E} = \int \frac{k\,dq}{r^2}\hat{r} \qquad \text{radial}$$

$$\text{magnetic} \;:\; \vec{B} = \int \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \qquad \text{circling}$$

Same integral idea; the cross product turns "radial" into "circling" — the signature difference between electricity and magnetism. Finding the field of an extended current means setting up and evaluating this integral, with $d\vec{l}\times\hat{r}$ tracking direction.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B4` — the field of a wire and a loop, without the Biot–Savart integral.

The rule, in one line

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}\text{, integrated over the current}$$

The cross product makes $\vec{B}$ circle the current;
field lines are closed loops (no magnetic charges).

This is the fundamental law of how currents make fields — exact but often laborious. When the geometry is symmetric, Ampère's law (next) short-cuts the whole integral, just as Gauss's law short-cut the electric-field integral.

Watch out: two traps to dodge

⚠ Drawing $\vec{B}$ radial like an electric field.
The cross product makes it *circle* the current, not point outward.

⚠ Giving field lines endpoints.
Magnetic lines are *closed loops* — no start, no end. There are no magnetic charges.

Integrate $d\vec{l} \times \hat{r}/r^2$; the field circles the current; the lines close on themselves.

Your turn: Biot–Savart

Field circles the current; total is an integral. Cover the right column.

1. Direction of $\vec{B}$ around a straight current? ⇒ circles it
2. Do magnetic field lines have endpoints? ⇒ no (loops)
3. What replaces k in the magnetic law? ⇒ $\mu_0/4\pi$

? *What is the single mathematical feature that makes magnetic fields circle rather than radiate?*

Your turn: field of a current

Cover the answers.

1. What creates a magnetic field? _____

2. Shape of magnetic field lines _____

3. Which Ch.13 integral is this the cousin of? _____

4. Why are there no line endpoints? _____

How does the Biot–Savart integral differ from the electric-field integral of Chapter 13?

Answers, with the reason

1. moving charge
$\Rightarrow$ current

2. closed loops
$\Rightarrow$ circle the current

3. electric-field $\int$
$\Rightarrow$ $k\,dq/r^2$

4. no magnetic charge
$\Rightarrow$ nowhere to end

The cross product $d\vec{l} \times \hat{r}$ points around the current, so the field wraps it in loops with no beginning or end.

In one breath

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}\text{; } \vec{B} \textbf{ circles the current}$$

The magnetic cousin of Ch.13's field integral; closed-loop lines, no magnetic charges.

Carrying out this integral for the most important case — a long straight wire — gives a clean $1/r$ field. Let's read off the result.

Next up: **B5** The magnetic field of a long straight wire

Explore this live | Watch the walk-through

Field of a Straight Wire

A $1/r$ field, circling the wire

$$B = \frac{\mu_0 I}{2\pi r}$$

parallel currents attract

Lesson `calcphys.magnetism.B5`

Explore this live

Watch the walk-through

The key application of Biot–Savart

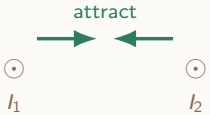
Working the integral for a long straight wire (or, more easily, Ampère's law next) gives a clean result.

$$B = \frac{\mu_0 I}{2\pi r}$$

The field circles the wire in loops, its magnitude falling as $1/r$. The direction follows the right-hand rule — thumb along the current, fingers curl with $\vec{B}$. Note the $1/r$ (not $1/r^2$): the hallmark of a line source.

Parallel currents attract

Each wire sits in the other's field and feels a force. Same-direction currents attract; opposite currents repel.



This mutual force is used to *define the ampere*. Resembles a charged line, whose electric field also fell as $1/r$ — but this one circles rather than points radially.

Watch every step: the $1/r$ falloff

The field is B at distance r from a wire. Find it at $2r$ — and contrast a line source with a point source.

$$B \propto 1/r \Rightarrow B(2r) = B/2 \qquad \text{line source, } 1/r$$

$$\text{(point source)} \quad : \quad \text{would give } B/4 \qquad 1/r^2 \text{ — not this}$$

Halving, not quartering — the $1/r$ falloff is characteristic of a line source, magnetic or electric. A point source would fall as $1/r^2$.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B4` — the field of a wire, $B = \mu_0 I/2\pi r$, without calculus.

The rule, in one line

$B = \mu_0 I/(2\pi r)$, **circling the wire,** $\propto 1/r$

Right-hand rule for direction. Parallel currents attract; antiparallel repel.

Doubling the current doubles the field; doubling the distance halves it. Two parallel wires exert equal and opposite forces through their fields — the effect used to define the ampere as the current giving a set force per metre.

Watch out: two traps to dodge

⚠ Using $1/r^2$ for a wire.
A *line* source falls as $1/r$. Only a point source is $1/r^2$.

⚠ Reversing the parallel-wire rule.
Same-direction currents *attract*; opposite ones repel. (Opposite to charges!)

$B = \mu_0 I/2\pi r$: *circles the wire, falls as* $1/r$*, parallel currents attract.*

Your turn: field of a wire

$B = \mu_0 I/2\pi r$; $B \propto 1/r$. *Cover the right column.*

1. Double the current: field changes by? $\Rightarrow \times 2$
2. Triple the distance: field changes by? $\Rightarrow \times \frac{1}{3}$
3. Two currents same direction: attract or repel? $\Rightarrow$ attract

Why does a wire's field fall off more slowly ($1/r$) than a point charge's ($1/r^2$)?

Your turn: the straight wire

Cover the answers.

1. Field formula for a long wire _____

2. How does B vary with r? _____

3. Antiparallel currents: attract or repel? _____

4. What does this force define? _____

Why do parallel currents attract, opposite to the way like charges behave?

Answers, with the reason

1. line source
$= \mu_0 I/2\pi r$

2. inverse
$\Rightarrow \propto 1/r$

3. opposite currents
$\Rightarrow$ repel

4. mutual force
$\Rightarrow$ the ampere

An infinite line's field spreads over a cylinder (circumference $\propto r$), so it dilutes as $1/r$ — a point source spreads over a sphere ($\propto r^2$), giving $1/r^2$.

In one breath

$B = \mu_0 I/(2\pi r)$, **circling,** $\propto 1/r$

Right-hand rule; parallel currents attract; defines the ampere.

We got this from Biot–Savart — but there's a far faster route. Ampère's law hands back the same field from symmetry alone, exactly as Gauss's law did for electricity.

Next up: **B6** Ampère's law

Explore this live | Watch the walk-through

Ampère's Law

The magnetic sibling of Gauss's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

circulation $\leftrightarrow$ enclosed current

Lesson `calcphys.magnetism.B6`

Explore this live

Watch the walk-through

Symmetry hands back the field

Ampère's law is to magnetism what Gauss's law was to electricity: a deep, exact statement that, with symmetry, gives the field with almost no calculation.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

The line integral of $\vec{B}$ around any closed loop equals μ_0 times the current enclosed. Where Gauss tied electric flux through a surface to enclosed charge, Ampère ties magnetic circulation around a loop to enclosed current.

Two of Maxwell's equations, side by side

The same symmetry strategy extracts a field from each — flux for one, circulation for the other.

Gauss (Ch.13)	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$	surface flux, enclosed charge
Ampère	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$	line circulation, enclosed current

Only the current piercing the loop matters — currents outside contribute zero net circulation, exactly as outside charges gave zero net flux.

Watch every step: the wire field in one line

Use a circular Amperian loop of radius r around a straight wire. By symmetry B is constant along it.

$$\begin{aligned} \oint \vec{B} \cdot d\vec{l} &= B(2\pi r) && B \text{ constant, pulls out} \\ B(2\pi r) &= \mu_0 I && I_{enc} = I \\ B &= \frac{\mu_0 I}{2\pi r} && \text{instantly, no integral} \end{aligned}$$

The same wire field as Biot–Savart, in three lines. That's the power: when symmetry makes B constant on the loop, it comes out of the integral.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B5` — Ampère's law and enclosed current, without the line integral.

The rule, in one line

$$\oint \vec{B} \cdot d\vec{l} \quad = \quad \mu_0 I_{enc}$$

Circulation set by enclosed current alone (outside gives zero). Always true; useful with symmetry.

Like Gauss's law, it is always true but only useful for finding B when symmetry lets you pull it out of the integral. Pick an Amperian loop that follows the field and where B is constant — then read off B.

Watch out: two traps to dodge

⚠ Counting outside currents.
Only current *piercing* the loop counts — outside currents give zero net circulation.

⚠ Confusing it with Gauss's flux.
Ampère integrates $\vec{B}$ *around a loop* (a line), not *through a surface*.

Circulation of $\vec{B}$ around a loop equals μ_0 times the enclosed current — symmetry does the rest.

Your turn: Ampère's law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$. *Cover the right column.*

1. A current outside the loop contributes what? ⇒ 0
2. Ampère is the sibling of which law? ⇒ Gauss
3. Integral: over a loop or a surface? ⇒ a loop

? *Why does Ampère's law give the wire field so much faster than Biot–Savart?*

Your turn: circulation

Cover the answers.

1. What equals $\mu_0 I_{enc}$? _____

2. What current counts in I_{enc}? _____

3. When is Ampère *useful* for finding B? _____

4. Electric analog of this law _____

What must be true of $\vec{B}$ on your loop for you to pull it out of the integral?

Answers, with the reason

1. circulation of $\vec{B}$
$= \oint \vec{B} \cdot d\vec{l}$

2. piercing the loop
$\Rightarrow$ enclosed only

3. symmetric geometry
$\Rightarrow$ B pulls out

4. Gauss's law
$\Rightarrow$ flux

B must be constant in magnitude and parallel to $d\vec{l}$ along the loop — then $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ and B comes straight out.

In one breath

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ — **Gauss's magnetic sibling**
Only enclosed current counts; always true, useful with symmetry.

The perfect symmetric device for Ampère is the solenoid — a long coil that makes a nearly uniform field inside, the magnetic twin of the parallel-plate capacitor.

Next up: **B7** Solenoids & magnetic fields

Explore this live | Watch the walk-through

Solenoids

A uniform field, on demand

$$B = \mu_0 n I$$

n = turns per *length*

Lesson `calcphys.magnetism.B7`

Explore this live

Watch the walk-through

The magnetic workhorse

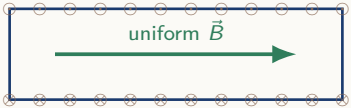
*A **solenoid** — a long coil of many turns — makes a nearly uniform field inside, much as a parallel-plate capacitor made a uniform electric field. Ampère's law gives:*

$$B = \mu_0 n I$$

Here n is the number of turns per unit length. The interior field is uniform and axial; the field outside an ideal long solenoid is essentially zero.

Uniform inside, zero outside

The turns' fields reinforce along the axis inside and cancel outside.



The field depends on turn *density* n, not total turns — pack the same turns into a shorter coil (larger n) and it strengthens. From MRI machines to electromagnets.

Watch every step: density, not total turns

A solenoid has N turns over length L. What field does it make, and how do you strengthen it?

$$n = N/L \qquad \text{turns per length}$$

$$B = \mu_0 n I = \mu_0 (N/L) I \qquad \text{raise } I \text{ or wind tighter}$$

To strengthen it, raise the current or wind more tightly (larger n) — not merely add turns over a longer coil. The uniform interior field is the magnetic analog of the capacitor's uniform field.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B5` — the solenoid field $B = \mu_0 n I$ and Ampère's law, without calculus.

The rule, in one line

$B = \mu_0 n I$ (n = **turns per length**), **uniform & axial inside**
$\sim$zero outside. The magnetic analog of
the parallel-plate capacitor's uniform field.

This is how strong, controllable fields are made. Both the solenoid's uniform interior field and the capacitor's uniform field come from applying the relevant symmetry law (Ampère, Gauss) to a long, symmetric device.

Watch out: two traps to dodge

⚠ Using total turns N instead of n.
The field is $\mu_0 nI$ with $n = N/L$ — turns *per length*, not the raw count.

⚠ Thinking the outside field is strong.
An ideal solenoid's exterior field is nearly *zero*; the field is trapped inside.

$B = \mu_0 nI$: uniform inside, zero outside, set by turn density and current.

Your turn: solenoids

$B = \mu_0 n I$, $n = N/L$. Cover the right column.

1. Double the current: field changes by? $\Rightarrow \times 2$
2. Same turns, half the length: field changes by? $\Rightarrow \times 2$
3. Field outside an ideal solenoid? $\Rightarrow \approx 0$

? *Why does squeezing the same number of turns into a shorter coil strengthen the field?*

Your turn: the solenoid

Cover the answers.

1. Solenoid field formula _____

2. What is n? _____

3. Field outside an ideal solenoid _____

4. Electric analog of the interior field _____

How is the solenoid the magnetic twin of the parallel-plate capacitor?

Answers, with the reason

1. Ampère's law
$= \mu_0 nI$

2. turns per length
$\Rightarrow N/L$

3. ideal
$\Rightarrow \approx 0$

4. capacitor
$\Rightarrow$ uniform field

Both apply a symmetry law to a long, symmetric device: Ampère gives the solenoid a uniform interior field, Gauss gives the capacitor one.

In one breath

$B = \mu_0 n I$ — **uniform axial field inside, zero outside**
Set by turn density $n = N/L$ and current; the magnetic analog of the capacitor.

One structure remains: the current loop. In a field it feels a torque that tries to align it — the principle behind every electric motor.

Next up: **B8** Magnetic dipoles & torque

Explore this live | Watch the walk-through

Magnetic Dipoles & Torque

The heart of the electric motor

$$\vec{\mu} = I\vec{A}; \vec{\tau} = \vec{\mu} \times \vec{B}$$
$$\tau = \mu B \sin\theta$$

Lesson `calcphys.magnetism.B8`

Explore this live Watch the walk-through

A current loop is a magnetic dipole

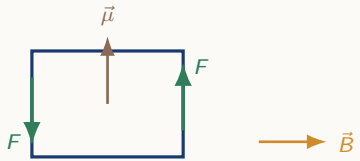
*A loop of current I and area A has a **magnetic dipole moment** $\mu = IA$, a vector perpendicular to the loop. In a field it feels a torque.*

$$\vec{\mu} = I\vec{A}, \qquad \vec{\tau} = \vec{\mu} \times \vec{B}, \qquad \tau = \mu B \sin\theta$$

Here θ is the angle between the moment and the field. The torque tries to align the moment with the field — just as an electric dipole aligns with $\vec{E}$, or a compass needle with Earth's field.

The torque spins the loop

Forces on opposite sides of the loop point opposite ways, producing a couple that rotates it toward alignment.



Torque is *maximum* when $\mu \perp B$ ($\theta = 90^\circ$) and *zero* when aligned ($\theta = 0$, stable). Reverse the current each half-turn and the loop keeps spinning — a motor.

Watch every step: where the torque peaks

A loop of moment μ sits in a field B. Find the torque when perpendicular and when aligned.

$$\theta = 90^\circ \;:\; \tau = \mu B \sin 90^\circ = \mu B \qquad \text{maximum}$$

$$\theta = 0 \;:\; \tau = \mu B \sin 0 = 0 \qquad \text{aligned, stable}$$

The loop, like any dipole, seeks alignment, storing energy $U = -\vec{\mu} \cdot \vec{B}$ (lowest when aligned) when twisted away. More current or area means a stronger dipole ($\mu = IA$) and a bigger torque.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.magnetism.B6` — motors and galvanometers — the torque on a current loop, without calculus.

The rule, in one line

$\vec{\mu} = I\vec{A}$; $\vec{\tau} = \vec{\mu} \times \vec{B}$ ($\tau = \mu B \sin\theta$)

Aligns the moment with B (zero when aligned). A rotating loop is the electric motor.

The moment $\mu = IA$ captures the loop's strength as a magnet — more current or area, stronger dipole. This is also how magnetism arises in matter: countless atomic current loops, each a tiny dipole.

Watch out: two traps to dodge

⚠ Torque is maximum when aligned.
Backwards — torque is *zero* when aligned ($\theta = 0$) and *maximum* when perpendicular.

⚠ Forgetting area in the moment.
$\mu = IA$ — both current *and* area set the dipole strength.

Moment IA; torque $\mu B \sin\theta$; peaks perpendicular, vanishes aligned; spins the motor.

Your turn: dipole torque

$\mu = IA$, $\tau = \mu B \sin\theta$. *Cover the right column.*

1. $I = 2$, $A = 0.5$: moment μ? $\Rightarrow 1\ \text{A m}^2$

2. That loop, $B = 4$, $\theta = 90^\circ$: τ? $\Rightarrow 4\ \text{N m}$

3. Torque when aligned ($\theta = 0$)? $\Rightarrow 0$

? *Why must a DC motor reverse its loop current every half-turn to keep spinning?*

Your turn: magnetic dipoles

Cover the answers.

1. Magnetic moment of a loop _____

2. Torque formula _____

3. Where is the torque zero? _____

4. Energy of a dipole in a field _____

How does a current loop's tendency to align power an electric motor?

Answers, with the reason

1. current $\times$ area
$= IA$

2. cross product
$\Rightarrow \mu B \sin\theta$

3. aligned
$\Rightarrow \theta = 0$

4. lowest when aligned
$\Rightarrow -\vec{\mu} \cdot \vec{B}$

The torque dies as the loop reaches alignment; flipping the current at that instant makes the loop overshoot and keep rotating — continuous spin.

In one breath

$$\vec{\mu} = I\vec{A}; \vec{\tau} = \vec{\mu} \times \vec{B}, \tau = \mu B \sin\theta$$

Aligns with B, zero when aligned; the rotating loop is the motor.

The So-What of Chapter 16: ***magnetism is the cross-product twin of electrostatics*** *— force laws mirror Coulomb, the field integral mirrors Ch.13, Ampère mirrors Gauss. Only one question remains: change a magnetic field in time, and it makes an electric field. That coupling is induction — and it becomes light.*

Chapter 16 complete. Next chapter: **Electromagnetic Induction & EM Waves**

Explore this live | Watch the walk-through

Magnetic Flux

Field through a surface

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$
$$= BA\cos\theta,\ \theta \text{ from the normal}$$

Lesson `calcphys.induction.N1`

Explore this live

Watch the walk-through

How much field passes through

Magnetic flux measures how much field threads a surface — the magnetic twin of Chapter 13's electric flux, the same surface integral.

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \qquad \xrightarrow{\text{uniform}} \qquad \Phi_B = BA\cos\theta$$

Here θ is measured from the surface normal, and the unit is the weber. It is maximum when $\vec{B}$ is perpendicular to the surface ($\theta = 0$) and zero when parallel ($\theta = 90°$).

It's the *changes* that matter

What makes flux the star of this chapter isn't its value — it's how it changes. There are exactly three ways to change $\Phi_B = BA\cos\theta$.

change B	stronger/weaker field	(a moving magnet)
change A	grow/shrink the loop	(a sliding rod)
change θ	rotate the loop	(a generator)

Each of these will induce an EMF — the subject of the whole chapter. A steady flux does nothing; a changing flux drives a current.

Watch every step: flux at an angle

A uniform field $B = 0.4$ T passes through a loop of area $A = 0.5$ m^2 tilted so the normal is 60° from $\vec{B}$. Find the flux.

$$\begin{aligned} \Phi_B &= BA\cos\theta && \theta \text{ from the normal} \\ &= (0.4)(0.5)\cos 60^\circ = (0.2)(0.5) = \boxed{0.1 \text{ Wb}} && \text{tilt cuts it} \end{aligned}$$

Face-on ($\theta = 0$) the flux would be the full 0.2 Wb; the 60° tilt halves it. Measure θ from the normal, not the surface.

Math support Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N1` — magnetic flux $\Phi_B = BA\cos\theta$, without the surface integral.

The rule, in one line

$$\Phi_B \;=\; \int \vec{B}\cdot d\vec{A} \;=\; BA\cos\theta \quad \textbf{(webers)}$$

θ from the normal; max perpendicular, zero parallel. Its time-*changes* drive induction.

This is built exactly like the electric flux of Chapter 13 — a field dotted into an area. But there, Gauss tied the flux to enclosed charge; here it's the flux's rate of change, not its source, that will matter.

Watch out: two traps to dodge

⚠ Measuring θ from the surface.
It's from the *normal*. Face-on to $\vec{B}$ means $\theta = 0$ and *maximum* flux, not zero.

⚠ Thinking flux is fixed.
B, A, or θ can each change it — and every change induces an EMF.

Field dotted into area; angle from the normal; the changes are what count.

Your turn: magnetic flux

$\Phi_B = BA\cos\theta$, θ *from the normal. Cover the right column.*

1. $B = 0.5$, $A = 2$, face-on: Φ_B? $\Rightarrow$ 1 Wb
2. Same loop turned edge-on ($\theta = 90^\circ$): Φ_B? $\Rightarrow$ 0
3. Name a way to change the flux. $\Rightarrow$ B, A, or θ

? *Why does turning a loop edge-on to the field drop its flux to zero?*

Your turn: flux

Cover the answers.

1. $B = 0.2$, $A = 3$, face-on: Φ_B _____

2. From where is θ measured? _____

3. Unit of magnetic flux _____

4. What about flux drives induction? _____

How is magnetic flux built just like the electric flux of Chapter 13?

Answers, with the reason

1. $0.2(3)\cos 0$
$= 0.6$ Wb

2. the normal
$\Rightarrow \perp$ surface

3. weber
$\Rightarrow$ Wb

4. rate of change
$\Rightarrow d\Phi_B/dt$

Both are a field dotted into an area, $\int \vec{B} \cdot d\vec{A}$ vs. $\int \vec{E} \cdot d\vec{A}$ — the same construction for the two fields.

In one breath

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta\text{; } \theta \textbf{ from the normal}$$

Max perpendicular, zero parallel; its *changes* drive everything.

Now the heart of the chapter: a changing flux induces a voltage. It's a derivative — the calculus of rates that opened the course, in its most consequential form.

Next up: **N2** Faraday's law of induction

Explore this live | Watch the walk-through

Faraday's Law

A changing flux makes a voltage

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

a derivative — *changing* flux only

Lesson `calcphys.induction.N2`

Explore this live

Watch the walk-through

The calculus of rates, in its most consequential form

Faraday's law is the heart of the chapter — one of the most consequential equations in physics: a changing magnetic flux induces an EMF.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

This is a derivative — the same calculus of rates that opened the course with $v = dx/dt$, now driving generators, transformers, and the electric grid. The crucial word is changing: a steady flux, however large, induces nothing.

Change B, A, or θ

Since $\Phi_B = BA\cos\theta$, you can induce an EMF three ways — and the last is exactly how a generator works.

$$\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)$$

Change the field B (a transformer), change the area A (a sliding rod), or rotate to change θ — a spinning coil making alternating current, the generator that electrified the world. The size of the EMF depends on how fast the flux changes, not on its value.

Watch every step: rate, not value

One loop has a large flux changing slowly; another a small flux changing quickly. Which has the larger induced EMF?

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \text{depends on the } \textit{rate}$$

$$\Rightarrow \quad \text{small, fast-changing flux wins} \qquad \text{value is irrelevant}$$

The value of the flux doesn't matter — only $d\Phi_B/dt$. A fast-changing small flux beats a slow-changing large one. The course began with $v = dx/dt$ and $I = dQ/dt$; Faraday's is the same calculus, now turning motion into electricity.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N2` — Faraday's law and induced EMF, without the derivative.

The rule, in one line

$\mathcal{E} = -d\Phi_B/dt$ — **only a *changing* flux induces**
A steady flux induces nothing; size $\propto$ rate of change. Change B, A, or θ.

This one law — changing magnetic flux makes electricity — underlies generators, transformers, and induction charging: the technologies that electrified civilization. The minus sign encodes the direction, which is Lenz's law next.

Watch out: two traps to dodge

⚠ A steady flux induces an EMF.
Only a *changing* flux does. A huge but constant flux induces exactly nothing.

⚠ Tying EMF to the flux value.
It's the *rate* $d\Phi_B/dt$ that sets the EMF — how fast, not how much.

Changing flux only; the faster the change, the bigger the EMF; value is irrelevant.

Your turn: Faraday's law

$\mathcal{E} = -d\Phi_B/dt$. *Cover the right column.*

1. Φ_B changes by 0.6 Wb in 0.2 s: $|\mathcal{E}|$? $\Rightarrow$ 3 V

2. Flux large but steady: EMF? $\Rightarrow$ 0

3. How does a generator change the flux? $\Rightarrow$ rotates (θ)

? *Why does a magnet held still inside a coil induce no current, however strong it is?*

Your turn: induced EMF

Cover the answers.

1. Φ_B: $0.8 \to 0.2$ Wb in 0.3 s: $|\mathcal{E}|$ _____

2. What must the flux do to induce? _____

3. EMF depends on value or rate? _____

4. Three ways to change the flux _____

Which earlier course equations share the derivative form of Faraday's law?

Answers, with the reason

1. $|-0.6|/0.3$
$= 2$ V

2. change in time
$\Rightarrow$ vary

3. the rate
$\Rightarrow d\Phi_B/dt$

4. B, A, θ
$\Rightarrow$ any of three

$v = dx/dt$, $I = dQ/dt$, $\mathcal{E} = -d\Phi_B/dt$ — all rates of change, the calculus that runs from Chapter 1 to here.

In one breath

$\mathcal{E} = -d\Phi_B/dt$ — **a changing flux makes a voltage**
Steady flux does nothing; size $\propto$ rate; change B, A, or θ.

That minus sign isn't decoration. It fixes the direction of the induced current — Lenz's law, and it's rooted in energy conservation.

Next up: **N3** Lenz's law

Explore this live Watch the walk-through

Lenz's Law

The induced current opposes the change

oppose the change in flux
the minus sign in Faraday

Lesson `calcphys.induction.N3`

Explore this live Watch the walk-through

The meaning of the minus sign

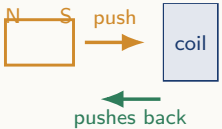
Lenz's law gives the direction of the induced current — the meaning of the minus sign in Faraday's law.

The induced current always opposes the *change* in flux that produced it.

If the flux is increasing, the induced current opposes the increase; if decreasing, it reinforces the flux — always resisting the change. Never the flux itself, only its change.

Rooted in energy conservation

Why must it oppose? Because the alternative would make energy from nothing.



Push a magnet toward a coil and the coil pushes *back* — if the current instead reinforced the change, it would amplify without limit, creating energy from nothing.

Watch every step: a magnet approaches

You push the north pole of a magnet toward a coil. Which way does the induced current flow, and what do you feel?

flux increasing $\Rightarrow$ current opposes it — coil face becomes N

$\Rightarrow$ coil repels the magnet — you must do work

The coil presents a north pole to the incoming north pole, repelling it — so you must push against that resistance. The mechanical work you do is exactly the electrical energy that appears. This is why generators need a force to turn.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N3` — Lenz's law and the direction of the induced current, without calculus.

The rule, in one line

The induced current opposes the *change* in flux

The minus sign in Faraday's law; rooted in energy conservation: push a magnet, it pushes back.

This opposition is why generators need a force to turn — the electricity out is paid for by the mechanical work in, exactly as energy conservation demands. There is no free lunch: every joule of electrical energy is bought with a joule of work against the induced push.

Watch out: two traps to dodge

⚠ The current reinforces the change.
It *opposes* it. Reinforcing would amplify itself endlessly — energy from nothing.

⚠ No resistance when pushing a magnet.
You must do work against the induced push — that work *is* the electrical energy produced.

Oppose the change (not the flux); the pushback is energy conservation made mechanical.

Your turn: Lenz's law

Induced current opposes the change. Cover the right column.

1. Flux increasing: current opposes or reinforces? ⇒ opposes
2. Pull the magnet away: current now? ⇒ reinforces
3. Where does the electrical energy come from? ⇒ your work

? *Why would a self-reinforcing induced current violate conservation of energy?*

Your turn: direction of induction

Cover the answers.

1. What does the induced current oppose? _____

2. What principle underlies Lenz's law? _____

3. Push a magnet at a coil: it _____

4. Why must a generator be turned? _____

How does Lenz's law explain the minus sign in Faraday's law?

Answers, with the reason

1. the change
⇒ in flux

2. energy conservation
⇒ no free lunch

3. pushes back
⇒ repels

4. against the pushback
⇒ work in

The minus sign says the induced EMF drives a current whose own flux fights the change — Lenz's law is that sign, spelled out.

In one breath

Induced current opposes the change in flux

The minus sign; energy conservation; push a magnet, it pushes back.

Now the simplest generator of all: slide a conducting rod through a field. The circuit's area changes — Faraday's law in action — giving a clean $\mathcal{E} = BLv$.

Next up: **N4** Motional EMF

Explore this live | Watch the walk-through

Motional EMF

The simplest generator

$$\mathcal{E} = BLv$$

Faraday, or the qvB force

Lesson `calcphys.induction.N4`

Explore this live

Watch the walk-through

Induction from the moving rod's view

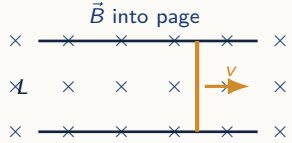
A straight rod of length L moving at speed v through a perpendicular field B develops a voltage across its ends.

$$\mathcal{E} = BLv$$

This is the simplest generator — motion producing voltage. And it isn't a new law: it's Faraday's law where the flux changes because the area changes, seen from the conductor's point of view.

Two views, one answer

The same BLv falls out whether you think in flux or in force.



Faraday: sliding the rod grows the circuit's area, so $d\Phi_B/dt = BLv$. **Force**: each charge feels qvB (Ch.16), pushing charge to one end until BLv builds.

Watch every step: a sliding rod

A rod $L = 0.5$ m slides at $v = 4$ m/s along rails through a field $B = 0.3$ T (perpendicular). Find the EMF.

$$\begin{aligned} \mathcal{E} &= BLv && \text{area grows at rate } Lv \\ &= (0.3)(0.5)(4) = 0.6 \text{ V} && \text{motion makes voltage} \end{aligned}$$

The same $q\vec{v} \times \vec{B}$ that curved charges into circles in Chapter 16 now separates charge along the rod — revealing induction and the magnetic force as one thing. Stop the rod ($v = 0$) and the EMF vanishes.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N4` — motional EMF $\mathcal{E} = BLv$ from a moving conductor, without calculus.

The rule, in one line

$\mathcal{E} = BLv$ — **Faraday with a changing area**

Equivalently the qvB force separating charge along the rod. The simplest generator.

Motional EMF stitches two chapters together: Faraday's changing flux (this chapter) and the $\vec{F} = q\vec{v} \times \vec{B}$ force (Chapter 16) give the identical result. Induction and the magnetic force are not two facts but one.

Watch out: two traps to dodge

⚠ It needs a changing B.
No — here B is steady; the *area* changes as the rod slides, and that changes the flux.

⚠ A rod at rest has an EMF.
Motion is required. $v = 0$ gives $\mathcal{E} = BLv = 0$ — no change, no EMF.

Steady field, moving rod, growing area — $\mathcal{E} = BLv$, and only while it moves.

Your turn: motional EMF

$\mathcal{E} = BLv$. *Cover the right column.*

1. $B = 0.2$, $L = 1$, $v = 5$: $\mathcal{E}$? $\Rightarrow 1$ V

2. Same rod at rest: $\mathcal{E}$? $\Rightarrow 0$

3. Double the speed: EMF changes by? $\Rightarrow \times 2$

? *Which two chapters' ideas both predict the same BLv?*

Your turn: the sliding rod

Cover the answers.

1. $B = 0.4$, $L = 0.5$, $v = 3$: $\mathcal{E}$ _____

2. Which quantity changes the flux here? _____

3. Force-view origin of the EMF _____

4. EMF of a stationary rod _____

How is motional EMF just Faraday's law in disguise?

Answers, with the steps

1. $0.4(0.5)(3)$
$= 0.6$ V

2. sliding rod
$\Rightarrow$ area A

3. qvB on charges
$\Rightarrow q\vec{v} \times \vec{B}$

4. $v = 0$
$\Rightarrow 0$

As the rod moves, the enclosed area grows at rate Lv, so $d\Phi_B/dt = BLv$ — Faraday's law, with the change coming from the area.

In one breath

$\mathcal{E} = BLv$ — **Faraday (changing area) = the qvB force**
The simplest generator; needs motion;
unites induction and the magnetic force.

A coil can change its own flux by changing its own current — and it fights that change. That self-opposition is the inductor, the mirror of the capacitor.

Next up: **N5** Inductance & inductors

Explore this live | Watch the walk-through

Inductance

A coil that fights current changes

$$\mathcal{E} = -L\frac{dI}{dt}$$

the mirror of the capacitor

Lesson `calcphys.induction.N5`

Explore this live

Watch the walk-through

Opposing changes in current

An inductor — usually a coil — opposes changes in current, as a capacitor opposed changes in voltage. When its current changes, its own flux changes, inducing a back-EMF that fights the change.

$$\mathcal{E} = -L\frac{dI}{dt}$$

The self-inductance L relates them, measured in henries. Note the derivative again — the EMF depends on how fast the current is changing, not on its value.

The current-analog of the capacitor

Inductor and capacitor are exact mirror images — one stores energy in a magnetic field, the other in an electric field.

Capacitor	resists V changes	$I = C\,dV/dt$
Inductor	resists I changes	$\mathcal{E} = -L\,dI/dt$

An inductor lets a steady current pass freely (no changing flux, no back-EMF) but fights any rapid change — so the current in an inductor cannot jump instantaneously.

Watch every step: back-EMF from a changing current

A coil has $L = 0.2$ H. The current through it changes at 50 A/s. Find the back-EMF.

$$\begin{aligned} |\mathcal{E}| &= L\frac{dI}{dt} && \text{depends on the } \textit{rate} \\ &= (0.2)(50) = \boxed{10\ \text{V}} && \text{fights the change} \end{aligned}$$

A fast-changing current makes a large back-EMF — the principle of ignition coils and switching supplies, where interrupting a current spikes a high voltage. A steady current ($dI/dt = 0$) makes no back-EMF at all.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N5` — inductance and how a coil resists current changes, without the derivative.

The rule, in one line

$$\mathcal{E} = -L\,dI/dt \quad \textbf{(henries)}$$

Opposes current *changes*, not steady current; the current cannot jump. The current-analog of the capacitor.

The parallel with the capacitor is exact: a capacitor stores energy in an electric field and resists voltage changes; an inductor stores energy in a magnetic field and resists current changes. Two fields, two components, one idea.

Watch out: two traps to dodge

⚠ Tying the EMF to the current value.
It responds to dI/dt — the *rate* of change, not the current itself.

⚠ An inductor blocks steady current.
It passes steady current freely; it only resists *sudden change*, and the current can't jump.

Back-EMF $\propto dI/dt$; steady current flows free; the current changes smoothly, never in a jump.

Your turn: inductance

$|\mathcal{E}| = L\,dI/dt$. *Cover the right column.*

1. $L = 0.1$ H, $dI/dt = 20$: back-EMF? ⇒ 2 V
2. Steady current: back-EMF? ⇒ 0
3. Inductor is the analog of which component? ⇒ capacitor

? *Why can't the current through an inductor change instantaneously?*

Your turn: inductors

Cover the answers.

1. $L = 0.5$ H, $dI/dt = 8$: back-EMF _____

2. Unit of inductance _____

3. Does an inductor block steady current? _____

4. Capacitor $V \leftrightarrow$ inductor _____

Why does interrupting an inductor's current produce a voltage spike?

Answers, with the reason

1. 0.5(8)
$= 4$ V

2. henry
$\Rightarrow$ H

3. no changing flux
$\Rightarrow$ no

4. voltage $\leftrightarrow$ current
$\Rightarrow V \leftrightarrow I$

Cutting the current means a huge dI/dt, so $\mathcal{E} = -L\,dI/dt$ spikes — the same effect that fires a spark plug.

In one breath

$\mathcal{E} = -L\,dI/dt$ **(henries) — resists current changes**

Steady current passes free; current
can't jump; the mirror of the capacitor.

Building that current from zero takes work against the back-EMF — and that work is stored in the magnetic field as $\frac{1}{2}LI^2$, the exact mirror of the capacitor's energy.

Next up: **N6** Energy stored in an inductor

Explore this live Watch the walk-through

Energy in an Inductor

Stored in the magnetic field

$$U = \tfrac{1}{2}LI^2$$

mirror of $\tfrac{1}{2}CV^2$

Lesson `calcphys.induction.N6`

Explore this live | Watch the walk-through

Like a capacitor, but magnetic

An inductor stores energy — but in a magnetic field. A coil carrying current I holds:

$$U = \tfrac{1}{2} L I^2$$

This is the exact magnetic mirror of the capacitor's $\frac{1}{2} C V^2$ (Chapter 14). The half-factor arises for the same reason: you build the current from zero against the back-EMF, integrating the work as it rises.

One formula, roles swapped

Swap $C \leftrightarrow L$ and $V \leftrightarrow I$, and one energy formula becomes the other.

Capacitor	$\frac{1}{2}CV^2$	energy in the *electric* field, $\frac{1}{2}\epsilon_0 E^2$
Inductor	$\frac{1}{2}LI^2$	energy in the *magnetic* field, $B^2/2\mu_0$

The energy lives in the field — with density $u = B^2/2\mu_0$, the magnetic counterpart of the electric $\frac{1}{2}\epsilon_0 E^2$. This is exactly why fields can carry energy, the idea that lets electromagnetic waves transport energy through empty space.

Watch every step: energy in a coil

A coil with $L = 0.4$ H carries $I = 3$ A. How much energy does it store?

$$\begin{aligned} U &= \tfrac{1}{2}LI^2 && \text{mirror of } \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2}(0.4)(3)^2 = \tfrac{1}{2}(0.4)(9) = \boxed{1.8\ \text{J}} && \text{in the magnetic field} \end{aligned}$$

The half-factor comes from integrating the work done against the back-EMF as the current climbs from 0 to I — exactly as the capacitor's half came from charging against the rising voltage.

🎓 **Math support** Shaky on this step? See **Physics (Algebra)** lesson `physics.induction.N5` — inductors storing magnetic energy, without the field-density integral.

The rule, in one line

$U = \frac{1}{2}LI^2$ — **mirror of** $\frac{1}{2}CV^2$ **(**$C \leftrightarrow L$**,** $V \leftrightarrow I$**)**

Half-factor from the rising current; energy in the magnetic field, density $B^2/2\mu_0$.

Two fields, two energies, one formula — the clearest sign that electricity and magnetism are two aspects of one subject. Chapter 14 stored $\frac{1}{2}CV^2$ in an electric field; here $\frac{1}{2}LI^2$ in a magnetic one.

Watch out: two traps to dodge

⚠ Dropping the half.
$U = \frac{1}{2}LI^2$ — the current builds from zero, so the work integrates to a half, just like the capacitor.

⚠ Energy in an electric field.
The inductor's energy lives in the *magnetic* field, density $B^2/2\mu_0$.

Half LI^2, magnetic field, exact mirror of the capacitor — one formula, two subjects united.

Your turn: inductor energy

$U = \frac{1}{2}LI^2$. *Cover the right column.*

1. $L = 2$ H, $I = 2$ A: U? ⇒ 4 J

2. Double the current: energy changes by? ⇒ $\times 4$

3. Where is the energy stored? ⇒ magnetic field

❓ *Why does the inductor's energy formula carry the same one-half as the capacitor's?*

Your turn: stored magnetic energy

Cover the answers.

1. $L = 0.5$ H, $I = 4$ A: U _____

2. Capacitor energy formula _____

3. Magnetic field energy density _____

4. $C \leftrightarrow L$, $V \leftrightarrow$ _____

Why does energy living in fields make electromagnetic waves possible?

Answers, with the steps

1. $\frac{1}{2}(0.5)(16)$
$= 4$ J

2. electric field
$\Rightarrow \frac{1}{2}CV^2$

3. magnetic
$\Rightarrow B^2/2\mu_0$

4. current
$\Rightarrow I$

Because the energy resides in the field itself, a field travelling through empty space carries its energy with it — the seed of the light wave to come.

In one breath

$U = \frac{1}{2}LI^2$ — **magnetic mirror of** $\frac{1}{2}CV^2$

Half from the rising current; energy in the field, density $B^2/2\mu_0$.

Faraday tied a changing $\vec{B}$ to an $\vec{E}$. Maxwell now adds the missing half — a changing $\vec{E}$ makes a $\vec{B}$ — and his four equations snap together into one whole.

Next up: **N7** Maxwell's equations & the displacement current

Explore this live Watch the walk-through