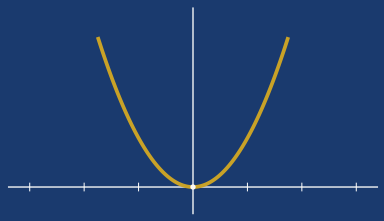
QUANTUM TUTORING SERIES

EVERY WRONG ANSWER HAS A NAME

THE DIAGNOSTIC PSAT / NMSQT

Math · Reading · Writing

Student Edition



Algebra · Advanced Math · Problem-Solving & Data

Geometry & Trigonometry · Reading · Writing & Language

"Pure mathematics is, in its way, the poetry of logical ideas."

— Albert Einstein

RONALD MILLER, PhD

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Every Wrong Answer Has a Name: The Diagnostic PSAT / NMSQT

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Published by Quantum Tutoring LLC

Quantum Tutoring Series • `QuantumTutoring.ai`

ISBN 979-8-9976071-1-1 (print)

ISBN 979-8-9976071-2-8 (digital)

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First edition, 2026.

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Full-Length Practice Test 403

Part I

Approach

How to use this manual

The digital PSAT/NMSQT has two sections. The **Reading and Writing** section is **54 questions** in two adaptive modules of 27, 32 minutes each; the **Math** section is **44 questions** in two adaptive modules of 22, 35 minutes each. In Math, about **three questions in four are multiple choice** and **one in four is student-produced response** (a grid-in with no choices); a Desmos graphing calculator and a formula reference sheet are available on *every* Math question. In both sections the questions run easy to hard, and how you do on the first module sets the difficulty of the second. The PSAT/NMSQT is also the qualifying test for the **National Merit Scholarship Program**, so a strong junior-year performance can carry real weight. This manual covers **both sections — Math and Reading and Writing** — the same way.

This manual is built on one idea: **on this test, a wrong answer is not random — it is a specific, nameable mistake.** For every skill you will find:

- **The skill** — the rule or method, stated plainly.
- **The Approach** — *Read, Plan, Solve, Check* (RPSC), the same four-step habit used across the Quantum Tutoring Series. Most points are lost not to missing knowledge but to careless slips — a sign error, a misread of what is asked, the wrong grid-in form; the *Check* step is where you catch them.
- **Common traps** — the handful of named wrong-answer patterns for that skill. Learn to recognize the trap and you eliminate the distractor.
- **Practice** — items graded easy to hard, in Math and in Reading and Writing. Work every item before you check yourself; the full **Answer Keys** are gathered at the back of the manual.

Do not just note whether you were right. For each miss, name the trap you fell for. That is the whole method.

How to Approach a Problem

You have about a minute and a half per question. You do not need cleverness; you need a **habit** that keeps you from beating yourself. Every skill in this manual runs on the same four steps — *Read, Plan, Solve, Check* — and most lost points are caught at the last one.

Read — what is given, and what is asked

Name the quantities and, above all, **what the question actually wants**. The most common way to miss an easy question is to solve for x when it asked for $2x$, or to find a price before tax when it asked for the total. Underline the asked-for quantity before you compute anything.

Plan — choose the method or formula

Decide the route before you calculate: substitute, factor, use the formula sheet, set up a proportion, or let Desmos graph it. A short plan beats a long scramble. The reference sheet is provided — your job is to *select* the right formula, not to recall it (except surface area and trigonometry, which are *not* on the sheet; memorize those).

Solve — execute carefully

Do the algebra one step at a time, watching signs, units, and grouping. Carry units through so the answer's units make sense. On a grid-in there are no choices to catch a slip, so slow down by a beat.

Check — the step that saves the points

Ask three things: Does the answer **answer the question** that was asked? Is it **reasonable** in size and sign? Is it in the **right form** — and for a grid-in, does it fit the entry rules (a fraction or decimal, no mixed numbers, enough decimal places)?

Worked example: Read · Plan · Solve · Check

A jacket is discounted 20% to a sale price of $48. What was the original price?

Read. Asked: the *original* price, not the discount. **Plan.** Sale price is 80% of the original, so original $= 48 \div 0.80$.

Solve. $48 \div 0.80 = 60$. **Check.** A $60 jacket, 20% off, drops $12 to $48 — consistent, and $60 is larger than the sale price, as it must be. **Answer: $60.**

The trap here is answering **$57.60** — taking 20% *off the $48* instead of recognizing $48 as the already-discounted price. Naming that trap is what the *Check* step is for.

Coming next: the adaptive tutor

This manual shows you the procedure. An interactive tutor (in development) will serve these skills adaptively, name the exact trap behind each wrong answer, and follow your progress skill by skill — turning *show* into *do*.

Part II

Mathematics

DOMAIN 1

Algebra

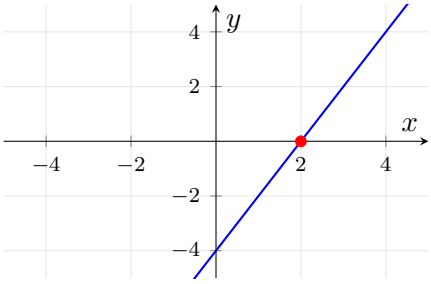
The grammar of the whole section

"As long as algebra is taught in school, there will be prayer in school."
— Cokie Roberts

About 35% of the section — tied for the largest domain. Linear equations, inequalities, systems, and graphs; equivalent expressions; and absolute value.

Solve linear equations in one variable

The skill. Isolate the variable using inverse operations on both sides, distributing and combining like terms first. A linear equation has exactly one solution unless it reduces to a contradiction (no solution) or an identity (all real numbers).



A linear equation has exactly one solution: $2x - 4 = 0$ crosses the x-axis once, at $x = 2$.

Worked example

Solve $4(x - 1) = 2x + 10$**.**

Read. Given the equation $4(x - 1) = 2x + 10$; find the value of x.

Plan. Distribute the 4, collect the variable terms on one side and constants on the other, then divide by the coefficient.

Solve.

$$\begin{aligned} 4(x-1) &= 2x+10 && \text{start with the given equation} \\ 4x-4 &= 2x+10 && \text{distribute the 4 across } (x-1) \\ 2x-4 &= 10 && \text{subtract } 2x \text{ from both sides} \\ 2x &= 14 && \text{add 4 to both sides} \\ x &= 7 && \text{divide both sides by 2} \end{aligned}$$

Check. Left: $4(7-1)=24$. Right: $2(7)+10=24$. Both sides equal, so $x=7$ is correct.
Answer: $x=7$.

Approach Read · Plan · Solve · Check

Read. Clear parentheses and fractions, then get all variable terms on one side and constants on the other.

Plan. Apply the inverse operation to both sides at each step (subtract to undo addition, divide to undo multiplication).

Solve. Track signs through every transposition; perform each operation on both sides.

Check. If the variable cancels: $0 =$ nonzero means no solution; $0 = 0$ means all reals. Otherwise substitute the answer back to check.

Common traps — every wrong answer is one of these

transposition sign error.
Moves a term across the equals sign without flipping its sign (x + 5 = 12 -> x = 12 + 5), or applies the same operation instead of the inverse.

distribution error.
Fails to distribute to every term (3(x+2) -> 3x+2), or mis-signs when distributing a negative.

both-sides imbalance.
Operates on only one side, or adds variable terms that should be subtracted when collecting them.

coefficient/division error.
From ax = b gives b - a or a/b instead of b/a, or forgets to divide by the coefficient.

special-case blindness.
Treats a contradiction (2x+1 = 2x+3) as solvable, or an identity (2x+2 = 2(x+1)) as having a single solution instead of infinitely many.

Practice

1. *Solve 3x + 5 = 20.*

What is x?

(A) 45
(B) 25/3
(C) 5
(D) 15

2. *Solve 5x - 3 = 2x + 9.*

What is x?

(A) 12
(B) 4
(C) 12/7
(D) 2

3. *Solve 2(x + 3) = 16.*

What is x?

(A) 10
(B) 11
(C) 5
(D) 6.5

4. *Solve 10 - 2(x - 1) = 4.*

What is x?

(A) 8
(B) -8
(C) 2
(D) 4

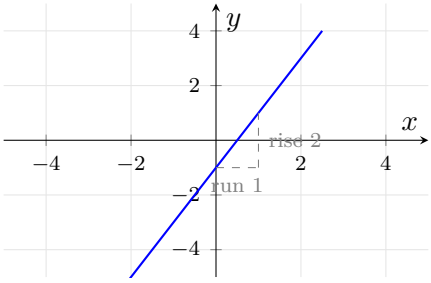
5. *The equation 3x + 6 = 3(x + 2).*

How many solutions does it have?

(A) No solution
(B) One solution, x = 2
(C) One solution, x = 0
(D) Infinitely many (all real numbers)

Slope as ratio (rise/run, geometric)

The skill. The slope of a line is rise over run, dy/dx = (y2 - y1)/(x2 - x1), for any two distinct points on it. It measures steepness and direction: positive rises left-to-right, negative falls, zero is horizontal, and a vertical line has undefined slope.



Slope $= \frac{rise}{run}$*: this line rises 2 for every 1 to the right, so slope* $= 2$*.*

Worked example

A line passes through $(-1, 2)$ **and** $(4, -8)$**. What is its slope?**

Read. Two points on the line are given, $(-1, 2)$ and $(4, -8)$, and we want the slope. Slope measures how steeply the line rises or falls, and any two distinct points on a line determine it, so this is enough information.

Plan. Use the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$. Fix $(-1, 2)$ as point 1 and $(4, -8)$ as point 2, and keep that same order in both the numerator and the denominator so the signs stay consistent.

Solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{slope is rise over run}$$

$$m = \frac{-8 - 2}{4 - (-1)} \qquad \text{substitute both points}$$

$$m = \frac{-10}{5} \qquad \text{subtract in numerator and denominator}$$

$$m = -2 \qquad \text{divide}$$

Check. The line falls from left to right, so a negative slope is expected. The magnitude 2 means the line drops 2 units for every 1 unit to the right, which matches the steepness shown.

Answer: -2.

Approach Read · Plan · Solve · Check

Read. Identify two points on the line as ordered pairs (x1,y1), (x2,y2); fix which is 'point 1'.

Plan. slope = (y2 - y1)/(x2 - x1) – change in y over change in x, the same point first in numerator and denominator.

Solve. Subtract in matching order; reduce.

Check. Sign matches the visual direction; horizontal -> 0, vertical -> undefined; any two points on the line give the same value.

Common traps — every wrong answer is one of these

run/rise inversion (dx/dy).
Computes run over rise. Geometric root of the contextual inverted-rate error.

sign/direction.
Right magnitude, wrong sign; treats a falling line as positive. Absorbs inconsistent-point-order subtraction, which flips the sign and is answer-indistinguishable from a direction misread (process-diagnosable only, via the compute step).

horizontal/vertical swap.
Says a horizontal line is undefined and a vertical line is zero (the two special cases swapped).

single-point ratio / coordinate-as-slope.
Uses y/x of one point, or reads a single coordinate as the slope, instead of a difference.

pair-dependence belief.
Thinks the slope changes depending on which two points are chosen.

Practice

1. *A line passes through (1, 3) and (3, 7).*

What is the slope of the line?

(A) -2
(B) 3
(C) 1/2
(D) 2

2. *A line passes through (-2, 5) and (3, -5).*

What is the slope of the line?

(A) 2
(B) -1/2
(C) -2
(D) -2.5

3. *A line passes through (-3, 4) and (5, 4).*

What is the slope of the line?

(A) 8
(B) undefined
(C) 4
(D) 0

4. *A line passes through (2, -1) and (2, 6).*

What is the slope of the line?

(A) 7
(B) 0
(C) undefined
(D) 2

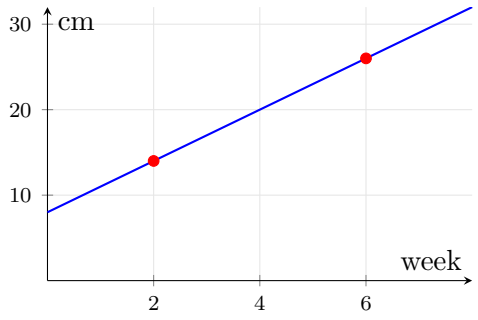
5. *Points (0, 1), (2, 4), and (4, 7) all lie on the same line.*

What is the slope of the line?

(A) 7/4
(B) 3/2, and any pair of the three points gives the same value
(C) It depends on which two points you choose
(D) 2/3

Slope as rate of change (contextual)

The skill. A rate of change is the constant amount y changes per one-unit increase in x. Compute it as $\Delta y/\Delta x$ from any two points (or read it as the coefficient of x), and state it with units: y-units per x-unit.



Constant growth: from $(2, 14)$ *to* $(6, 26)$ *the height rises* 3 *cm per week — the slope.*

Worked example

A seedling grows at a constant rate. At week 2 it is 14 cm tall; at week 6 it is 26 cm tall. What is the rate of change of height with respect to time?

Read. Two height readings are given: $(t, H) = (2, 14)$ and $(6, 26)$, with t in weeks and H in cm. Because the seedling grows at a constant rate, the graph is a line and the rate of change is its slope.

Plan. Compute the rate as $\dfrac{\Delta H}{\Delta t}$, the change in height divided by the change in time. Carry the units of cm per week through, since a rate of change must be stated with its units.

Solve.

$$\begin{aligned} \text{rate} &= \frac{\Delta H}{\Delta t} && \text{change in height over change in time} \\ &= \frac{26 - 14}{6 - 2} && \text{substitute the two readings} \\ &= \frac{12}{4} && \text{subtract in top and bottom} \\ &= 3 && \text{divide; units are cm per week} \end{aligned}$$

Check. Height increases over time, so a positive rate is expected. The units come out as cm per week (not week per cm), which is the correct "how much taller per week" interpretation.

Answer: 3 cm per week.

Approach Read · Plan · Solve · Check

Read. Name x and y and their units; treat 'per / each / every' as the rate cue.

Plan. Pick two clean points (table, words, or graph); rate = Δy/Δx carrying units.

Solve. Reduce to per-one-unit.

Check. Sign matches direction; units are y-per-x not x-per-y; magnitude is plausible.

Common traps — every wrong answer is one of these

inverted rate (Δx/Δy).
Computes run/rise; 'min per dollar' for 'dollars per min'. Spikes when the question names the quantity of interest first.

non-constancy belief.
Thinks rate is local (only consecutive rows) or changes; or the reverse failure: over-applies 'any two points' to non-linear data.

total-for-rate.
Uses a single (x,y) as the rate (y/x) instead of Δy/Δx; breaks on nonzero intercept. Parent of L6:M2.

sign/direction.
Decreasing quantity reported with a positive rate, or vice versa.

units inverted/dropped.
Bare number, or right number with the 'per' swapped.

step-size error.
On a table, treats row spacing as $\Delta x = 1$ instead of the actual step.

point-as-intercept.
Reads a single data point's y-value as the fixed/initial value (the intercept).

Practice

1. *A candle burns at a constant rate. t (min): 0, 10, 20, 30 -> H (cm): 20, 17, 14, 11.*

What is the rate of change of height with respect to time?

(A) -3 cm per minute
(B) -0.3 minutes per cm
(C) +0.3 cm per minute
(D) -0.3 cm per minute

2. *Same candle table.*

Which two times must you use to find the rate?

(A) Any two rows give the same rate
(B) Only the first and last rows
(C) The starting value 20 is the rate
(D) Only consecutive rows (0 and 10)

3. *A gym charges a sign-up fee plus a monthly rate. After 2 months a member has paid $190; after 5 months, $340.*

What is the monthly rate?

(A) $95 per month
(B) $68 per month
(C) $50 per month
(D) $50 (no time unit)

4. *Same gym scenario.*

What is the sign-up fee?

(A) $0
(B) $90
(C) $50
(D) $190

5. *A table that looks linear but whose successive differences are not constant.*

What is the rate of change of this relationship?

(A) A single rate computed from the first two points

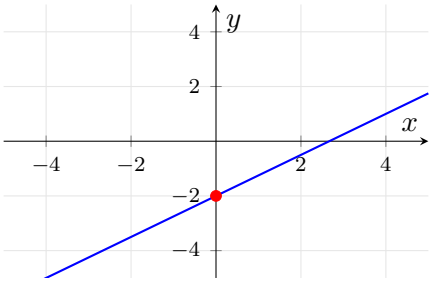
(B) A single rate computed from the last two points

(C) The relationship is not linear, so it has no single rate of change

(D) A single rate computed from the first and last points

Forms of a line (slope-intercept, point-slope, standard)

The skill. A line can be written as y = mx + b (slope-intercept), y - y1 = m(x - x1) (point-slope), or Ax + By = C (standard). Read slope and intercepts from each: in slope-intercept m and b are direct; in standard form the slope is -A/B and y-intercept C/B; convert by isolating y.



$3x - 4y = 8$ becomes $y = \frac{3}{4}x - 2$: slope $\frac{3}{4}$, y-intercept -2.

Worked example

What is the slope of the line $3x - 4y = 8$?

Read. The line is given in standard form $3x - 4y = 8$, and we want its slope. In standard form the slope cannot be read off directly, so the equation must first be rearranged.

Plan. Isolate y to put the equation into slope-intercept form $y = mx + b$. Once y is alone, the coefficient of x is the slope m.

Solve.

$$\begin{aligned} 3x - 4y &= 8 && \text{start with the given equation} \\ -4y &= -3x + 8 && \text{subtract } 3x \text{ from both sides} \\ y &= \frac{-3x + 8}{-4} && \text{divide both sides by } -4 \\ y &= \frac{3}{4}x - 2 && \text{simplify; } m \text{ is the coefficient of } x \end{aligned}$$

Check. The shortcut slope $= -A/B = -3/(-4) = \frac{3}{4}$ gives the same value, confirming the rearrangement was done correctly.

Answer: $\frac{3}{4}$.

Approach Read · Plan · Solve · Check

Read. Identify which form the equation is in before reading anything off it.

Plan. Slope-intercept: m is the x-coefficient, b the constant. Standard: isolate y first (slope = -A/B). Point-slope: the point is (x1, y1) with the signs as written.

Solve. Rearrange to the needed form; track signs through every move.

Check. Does the slope sign match the direction implied? Is the intercept on the correct axis (x-intercept has y = 0, y-intercept has x = 0)?

Common traps — every wrong answer is one of these

intercept-as-slope swap.
Within y = mx + b, reads b as the slope (or m as the intercept). Algebraic root of the contextual role-swap.

coefficient-as-slope ignoring form.
From Ax + By = C, takes the x-coefficient as the slope without first solving for y.

standard-form rearrangement error.
Solves for y but mis-signs or inverts: reports slope as A/B, B/A, or +A/B instead of -A/B.

point-slope sign error.
In y - y1 = m(x - x1), mis-signs the substituted point or swaps its coordinates.

intercept confusion.
Swaps x- and y-intercepts (sets the wrong variable to zero), or reads the constant C directly as an intercept.

Practice

1. *The line y = 3x - 5.*

What is the y-intercept?

(A) -5
(B) 5/3
(C) 3
(D) (-5, 0)

2. *The line 2x + 3y = 12.*

What is the slope?

(A) -3/2
(B) -2/3

(C) 2

(D) 2/3

3. *The line 2x + 3y = 12.*

What is the x-intercept?

(A) 3

(B) 6

(C) 12

(D) 4

4. *A line has slope 2 and passes through (-3, 5).*

Which is its point-slope equation?

(A) y - 3 = 2(x + 5)

(B) y - 5 = 2(x - 3)

(C) y + 5 = 2(x - 3)

(D) y - 5 = 2(x + 3)

5. *The line 4x - 2y = 6.*

Which equation describes the same line?

(A) y = 2x - 3

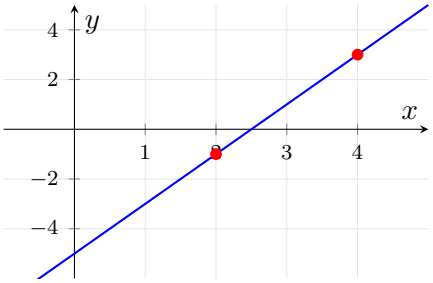
(B) y = -2x - 3

(C) y = 2x + 6

(D) y = -2x + 3

Write a line from conditions (two points / point+slope / table / parallel-perpendicular)

The skill. To write a line: get the slope (given, or from two points as dy/dx, or from a parallel/perpendicular condition), then pin the intercept by substituting one known point and solving for b. Parallel lines share a slope; perpendicular slopes are negative reciprocals.



Two known points pin one line: through $(2,-1)$ and $(4,3)$ gives $y = 2x - 5$.

Worked example

Write the equation of the line through $(2,-1)$ and $(6,7)$.

Read. Two points on the line are given, $(2,-1)$ and $(6,7)$, and we want the equation in slope-intercept form $y = mx + b$. To write that equation we need both the slope m and the y-intercept b.

Plan. First find the slope from $\dfrac{\Delta y}{\Delta x}$. Then substitute one known point into $y = mx + b$ and solve for b, which pins the line to that point.

Solve.

$m = \dfrac{7-(-1)}{6-2}$	slope from the two points
$m = \dfrac{8}{4} = 2$	simplify the slope
$-1 = 2(2) + b$	substitute $(2,-1)$ into $y = mx + b$
$-1 = 4 + b$	multiply
$b = -5$	subtract 4 from both sides
$y = 2x - 5$	write with $m = 2$ and $b = -5$

Check. Test the other point $(6,7)$: at $x = 6$, $y = 2(6) - 5 = 7$, which matches. Both given points lie on the line, so the equation is correct.

Answer: $y = 2x - 5$.

Approach Read · Plan · Solve · Check

Read. Identify what is given: a slope, two points, a table, or a parallel/perpendicular reference line.

Plan. Step 1 get m (compute dy/dx from two points if needed; flip-and-negate for perpendicular). Step 2 substitute one point into y = mx + b and solve for b.

Solve. Carry signs through the substitution; distribute fully if expanding point-slope.

Check. Does the line actually pass through the given point(s)? Check by plugging one back in.

Common traps — every wrong answer is one of these

y-value-as-intercept.
After finding the slope, uses a given point's coordinate as b without solving (e.g. y = 3x + 7 for slope 3 through (2,7), instead of y = 3x + 1).

expansion/solve sign error.
Distributes or signs incorrectly when expanding point-slope to slope-intercept, or when solving

for b.

parallel/perpendicular slope error.
Uses the same slope for a perpendicular line, takes the reciprocal without the sign flip, or negates without reciprocating.

Practice

1. *A line has slope 3 and passes through (2, 7).*

Which is its equation?

(A) y = 3x + 13
(B) y = 3x + 7
(C) y = 3x + 1
(D) y = 3x + 2

2. *A line passes through (1, 4) and (3, 10).*

Which is its equation?

(A) y = 3x + 4
(B) y = (1/3)x + 4
(C) y = 3x - 1
(D) y = 3x + 1

3. *The point-slope equation y - 5 = 3(x - 2).*

Write it in slope-intercept form.

(A) y = 3x - 1
(B) y = 3x - 11
(C) y = 3x + 5
(D) y = 3x + 3

4. *A line passes through (0, 2) and is perpendicular to y = 2x + 1.*

Which is its equation?

(A) y = -1/2 x + 2
(B) y = -2x + 2
(C) y = 2x + 2
(D) y = 1/2 x + 2

5. *A line passes through (-1, 3) and (2, -3).*

Which is its equation?

(A) y = -2x - 3

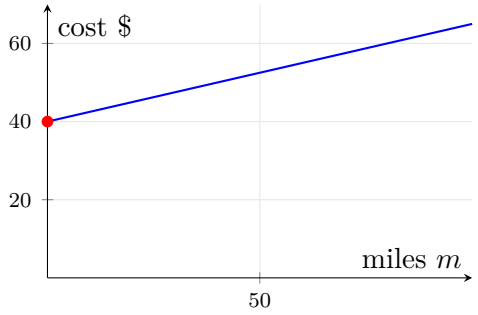
(B) y = -2x + 3

(C) y = -2x + 1

(D) y = 2x + 1

Interpret slope & intercept of a linear model in context

The skill. In y = mx + b modeling a real quantity, m is the per-unit rate (units of y per unit of x) and b is the value of y when x = 0. Match each to its real-world referent and units.



$C = 0.25m + 40$: the intercept $40 is the fixed fee; the slope $0.25 is the cost per mile.

Worked example

A rental car's cost is $C = 0.25m + 40$ dollars, where m is miles driven. What does the 0.25 represent?

Read. The model is $C = 0.25m + 40$, where C is cost in dollars and m is miles driven. The question asks what the coefficient 0.25 means in this real-world setting, so we must match it to a part of the linear-model template.

Plan. Compare the model with $y = mx + b$. The coefficient of the variable is the per-unit rate, carrying units of y per unit of x — here dollars per mile — while the constant term is the fixed amount.

Solve.

$C = 0.25m + 40$	compare with $y = mx + b$
coefficient of $m = 0.25$	this coefficient is the slope
slope = \$0.25 per mile	cost added for each extra mile

Check. The other number, 40, is the fixed value at $m = 0$ (the base fee), while 0.25 is the per-mile rate. The two roles are distinct and not swapped, so the interpretation holds.

Answer: \$0.25 per mile.

Approach Read · Plan · Solve · Check

Read. Identify what x and y physically are, and their units.

Plan. m carries units y/x; b carries units of y at x = 0.

Solve. Read the coefficients off the equation.

Check. Check sign; b is a fixed/starting quantity, m a per-each quantity.

Common traps — every wrong answer is one of these

role swap.
Reads b as the rate and m as the fixed amount.

rate-as-total.
Treats m as the value at one unit, dropping b.

coefficient/variable conflation.
Says the coefficient 'is' the variable (45 'represents the hours').

sign/direction.
Negative slope read as an increase; 'is' vs 'decreases by'.

units drop.
Right number, wrong dimension ('dollars' not 'dollars per hour').

Practice

1. *A technician's total charge C (dollars) is C = 45h + 75, where h = hours.*

What does 45 represent?

(A) The hourly labor rate
(B) The charge for a one-hour repair
(C) The number of hours
(D) The fixed service fee

2. *Same technician scenario.*

What does 75 represent?

(A) The charge when no labor time is needed
(B) The total for the first hour
(C) The hourly rate
(D) The total charge for the longest possible job

3. *A pool drains so its depth D (inches) after t minutes is D = 60 - 1.5t.*

What does -1.5 represent?

(A) The depth rises by 1.5 inches per minute

(B) The depth drops by 1.5 inches per minute

(C) The pool starts at 1.5 inches

(D) The depth is 1.5 inches

4. *Same pool scenario.*

Which gives the correct interpretation of the slope, with units?

(A) 1.5 minutes per inch

(B) 1.5 inches

(C) The depth is -1.5 inches at the start

(D) The depth decreases by 1.5 inches per minute

5. *Two phone plans: $P = 30 + 12n$ and $Q = 18n$, where n = gigabytes.*

Which interpretation correctly compares the plans' fixed costs and per-GB rates?

(A) P has a \$30 fixed cost and \$12/GB; Q has no fixed cost and \$18/GB; they cross at n = 5

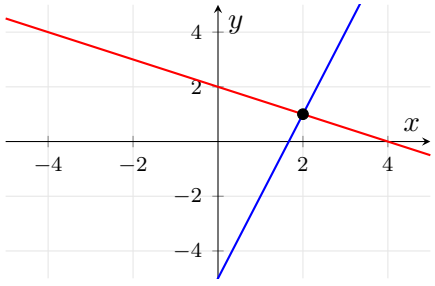
(B) The 30 and 18 are both per-GB rates

(C) P costs \$12 per GB regardless of n, so P is the flat plan

(D) Q is always cheaper because $18 > 12$

Solve linear systems (substitution / elimination)

The skill. A 2x2 linear system is solved by substitution (isolate one variable, put it into the OTHER equation) or elimination (scale equations so one variable's coefficients are opposite, then add). Solve the resulting one-variable equation, then back-substitute for the second variable. The system has one solution unless the lines are parallel (none) or identical (infinitely many).



The system's solution is where the two lines cross: $(2, 1)$.

Worked example

Solve the system $3x - y = 5$ and $x + 2y = 4$.

Read. Two equations in x and y are given, $3x - y = 5$ and $x + 2y = 4$. The solution is the single pair (x, y) that makes both equations true at once.

Plan. The first equation solves for y with one step ($y = 3x - 5$), so substitution is cleanest: replace y in the second equation, solve the resulting one-variable equation, then back-substitute for y.

Solve.

$x + 2y = 4$	second equation
$x + 2(3x - 5) = 4$	substitute $y = 3x - 5$
$x + 6x - 10 = 4$	distribute the 2
$7x - 10 = 4$	combine like terms
$7x = 14$	add 10 to both sides
$x = 2$	divide both sides by 7
$y = 3(2) - 5 = 1$	back-substitute for y

Check. Substitute $(2, 1)$ into both originals: second gives $2 + 2(1) = 4$, and first gives $3(2) - 1 = 5$. Both hold, so the pair is correct.

Answer: $(2, 1)$.

Approach Read · Plan · Solve · Check

Read. Pick a method: substitution if a variable is already isolated, elimination if coefficients align easily.

Plan. Substitution: solve one equation for a variable, insert into the other. Elimination: scale whole equations to make one variable cancel, then add.

Solve. Reduce to one variable and solve; back-substitute into an original equation for the other.

Check. If the variables all cancel: 0 = nonzero means no solution (parallel); 0 = 0 means infinitely many (same line). Otherwise check the pair (x, y) in both equations.

Common traps — every wrong answer is one of these

substitution scope error.
Substitutes a variable's expression back into the same equation it came from, producing $0 = 0$ and a false 'infinitely many'.

elimination sign/scaling error.
Adds equations when they should be subtracted, or scales one side or one term of an equation without scaling the whole thing.

incomplete / back-substitution error.
Solves for one variable and stops, mis-signs the back-substitution, or reports the answer with x and y swapped.

solution-count misread.
Reaches parallel or identical lines but reports a single numeric solution, or confuses no-solution with infinitely-many.

Practice

1. *Solve the system: $y = 2x + 1$ and $3x + y = 11$.*

What is the solution (x, y)?

(A) x = 2 (no y found)
(B) (2, 5)
(C) (10, 21)
(D) Infinitely many solutions

2. *Solve the system: $2x + 3y = 12$ and $2x - y = 4$.*

What is the solution (x, y)?

(A) (1, 2)
(B) (2, 3)
(C) (4, 4)
(D) (3, 2)

3. *Solve the system: $3x + 2y = 16$ and $x + y = 6$.*

What is the solution (x, y)?

(A) (4, 2)
(B) (2, 4)
(C) (10, -4)
(D) (4, 6)

4. *Solve the system: $y = 3x - 2$ and $y = x + 4$.*

What is the solution (x, y)?

(A) (3, 7)
(B) (1, 5)
(C) (6, 10)

(D) (3, 11)

5. *The system:* $y = 2x + 3$ *and* $2y = 4x + 6$.

How many solutions does it have?

(A) Exactly one solution

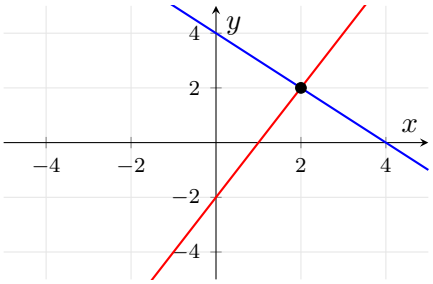
(B) Infinitely many (the two equations describe the same line)

(C) (0, 3)

(D) No solution

Systems in context & number of solutions

The skill. Translate a situation into a system (one equation per constraint), solve it, and interpret the result in context with units. Determine the number of solutions from structure: different slopes give one solution, same slope with different intercepts gives none (parallel), same slope and intercept gives infinitely many (one line).



Each constraint is a line; different slopes cross once, giving one solution $(2, 2)$.

Worked example

A theater sells adult tickets for \$8 and child tickets for \$5. One show sold 30 **tickets for \$195 total. How many adult tickets were sold?**

Read. Let a be the number of adult tickets and c the number of child tickets. The problem gives two facts: a count constraint (total tickets) and a money constraint (total revenue), which is exactly enough for a two-variable system.

Plan. Translate the two facts into $a + c = 30$ and $8a + 5c = 195$. Solve the first for c, substitute into the second, and report the adult count a that the question actually asks for.

Solve.

$$\begin{aligned} a + c &= 30 && \text{count constraint} \\ c &= 30 - a && \text{solve the count equation for } c \\ 8a + 5(30 - a) &= 195 && \text{substitute into the money equation} \\ 8a + 150 - 5a &= 195 && \text{distribute the 5} \\ 3a + 150 &= 195 && \text{combine like terms} \\ 3a &= 45 && \text{subtract 150 from both sides} \\ a &= 15 && \text{divide both sides by 3} \end{aligned}$$

Check. If $a = 15$ then $c = 30 - 15 = 15$. Revenue is $8(15) + 5(15) = 120 + 75 = 195$ and the count is $15 + 15 = 30$, so both constraints are satisfied.

Answer: 15 adult tickets.

Approach Read · Plan · Solve · Check

Read. Name each unknown with units; write one equation per stated constraint.

Plan. Solve by substitution or elimination; or, for a count question, compare slopes and intercepts without solving.

Solve. Carry the solve through, then map the numeric answer back to the quantity the question actually asks for.

Check. Does the answer name the right quantity with the right units? Does a 'no solution' or 'infinitely many' result make sense in the scenario?

Common traps — every wrong answer is one of these

translation/setup error.
Builds the wrong system from the words: swaps the count and total equations, mis-assigns a rate, or drops a constraint.

solution-count from structure.
Judges the number of solutions by eyeballing the equations: assumes one solution without comparing slopes, or calls proportional equations 'one line' when the intercepts differ.

contextual answer-interpretation.
Solves correctly but reports the wrong piece: gives the other variable, the total, or the intersection coordinate instead of the asked quantity, or drops units.

Practice

1. *A shop sells small drinks for \$3 and large for \$5. One day it sold 80 drinks for \$310 total. Let s = small drinks, l = large drinks.*

Which system represents this situation?

(A) 3s + 5l = 80; s + l = 310

(B) s + l = 80; 3s + 5l = 310

(C) s + l = 310; 3s + 5l = 80

(D) 5s + 3l = 310; s + l = 80

2. *From the same shop: $s + l = 80$ and $3s + 5l = 310$ (so $s = 45$, $l = 35$).*

How many large drinks were sold?

(A) 45

(B) 35

(C) 80

(D) 70

3. *The system $2x + 4y = 8$ and $x + 2y = 5$.*

How many solutions does it have?

(A) One solution at (1, 2)

(B) Infinitely many solutions

(C) No solution (the lines are parallel)

(D) Exactly one solution

4. *Plan A charges \$40 plus \$10 per month; Plan B charges \$25 plus \$15 per month.*

After how many months do the two plans cost the same?

(A) 13 months

(B) 3 months

(C) -3 months

(D) \$70

5. *Plan X costs $30 + 20m$ dollars; Plan Y costs $20m + 30$ dollars, where m is months.*

For how many values of m do the plans cost the same?

(A) All values of m – the plans always cost the same

(B) Only m = 0

(C) No value of m

(D) Exactly one value of m

Linear inequalities & systems of inequalities

The skill. Solve a linear inequality like an equation, but reverse the inequality whenever you multiply or divide both sides by a negative. Graph it with a boundary line (solid if inclusive, dashed if strict) and shade the half-plane that satisfies it (test a point). A system of inequalities is solved by the overlap (intersection) of the regions.



The solution $x > -3$: an open dot at -3 (strict), shaded to the right.

Worked example

Solve $-3x + 4 < 13$.

Read. This is a one-variable linear inequality, $-3x + 4 < 13$, and we want the full solution set — every value of x that makes it true, not a single number.

Plan. Isolate x using the same inverse operations as for an equation. The one extra rule: reverse the inequality sign whenever both sides are multiplied or divided by a negative number.

Solve.

$-3x + 4 < 13$	start with the given inequality
$-3x < 9$	subtract 4 from both sides
$x > -3$	divide by -3 and flip the sign

Check. Pick a value in the solution set, say $x = 0$ (which is > -3): $-3(0) + 4 = 4 < 13$ is true. A test point confirms the direction of the inequality is right.

Answer: $x > -3$.

Approach Read · Plan · Solve · Check

Read. Decide: one-variable solution set, a single half-plane, or the overlap of several.

Plan. Isolate the variable; flip the sign on any negative multiply/divide. For graphs, draw the boundary (solid = inclusive, dashed = strict) and test a point to choose the side.

Solve. Carry the solve through with the flip rule; for systems, shade each constraint and keep only the common region.

Check. Does a test point in the shaded region actually satisfy every inequality? Are included/excluded endpoints drawn correctly?

Common traps — every wrong answer is one of these

sign-flip omission.
Fails to reverse the inequality when multiplying or dividing both sides by a negative (-2x > 6 -> x > -3 instead of x < -3).

boundary type error.
Confuses strict and inclusive: open vs closed endpoint, dashed vs solid boundary, or includes an endpoint that should be excluded.

wrong-side shading.
Shades the wrong half-plane: assumes 'greater than = above' without testing a point, or tests a point incorrectly.

system region error.
Takes the union (or complement) of the shaded regions instead of their intersection, or keeps only one constraint.

Practice

1. *Solve -2x + 3 >= 11.*

What is the solution set?

(A) x >= -4
(B) x <= -4
(C) x <= -7
(D) x <= 4

2. *Solve and graph x - 2 <= 3 on a number line.*

Which graph is correct?

(A) Closed dot at 5, shaded to the left
(B) Open dot at 5, shaded to the left
(C) Closed dot at 5, shaded to the right
(D) Open dot at 5, shaded to the right

3. *The inequality y > 2x - 1 in the xy-plane.*

Which region is its graph?

(A) Above the line, boundary included (solid)
(B) Below the line, boundary excluded (dashed)
(C) Above the line, boundary excluded (dashed)
(D) Below the line, boundary included (solid)

4. *The system y <= x + 2 and y >= -x.*

Which region is the solution?

(A) The overlap (intersection) of the two shaded regions

(B) The region outside both

(C) Only where y <= x + 2

(D) The union of the two shaded regions

5. *The compound inequality -2 < x + 1 <= 3.*

For how many integer values of x does it hold?

(A) 5

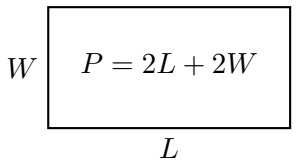
(B) 4

(C) 6

(D) Infinitely many

Literal equations & rearranging formulas

The skill. To solve a literal equation for one variable, treat every other letter as a constant and undo operations in reverse order, the same as a numeric equation. Divide EVERY term by a coefficient, not just one. If the target variable appears in two terms, collect those terms and factor it out.



Rearranging a formula: to solve $P = 2L + 2W$ for L, treat W as a constant and undo operations.

Worked example

The simple-interest formula is $I = Prt$. Solve for r.

Read. The formula $I = Prt$ is to be rearranged to isolate r. Every other letter — I, P, and t — is treated as a constant, exactly as we would treat numbers.

Plan. On the right side r is multiplied by both P and t. To undo that multiplication, divide both sides by the whole product Pt, which is the inverse operation.

Solve.

$$I = Prt \qquad \text{start with the formula}$$

$$\frac{I}{Pt} = \frac{Prt}{Pt} \qquad \text{divide both sides by } Pt$$

$$\frac{I}{Pt} = r \qquad \text{cancel } Pt \text{ on the right}$$

$$r = \frac{I}{Pt} \qquad \text{write with } r \text{ on the left}$$

Check. Multiply the result back in: $P \cdot \dfrac{I}{Pt} \cdot t = I$, which recovers the original formula, so the rearrangement is valid.

Answer: $r = \dfrac{I}{Pt}$.

Approach Read · Plan · Solve · Check

Read. Identify which variable to isolate; treat the others as constants.

Plan. Undo operations in reverse order (add/subtract first, then multiply/divide). If the target appears twice, gather its terms and factor.

Solve. Apply each inverse operation to the WHOLE side; divide every term by the coefficient.

Check. Did every term get divided? Is the target variable isolated and absent from the other side?

Common traps — every wrong answer is one of these

inverse/order error.
Undoes operations in the wrong order or applies the wrong inverse (multiplies instead of dividing, or moves a term with the wrong sign).

incomplete division.
When dividing by a coefficient, divides only one term instead of every term on that side.

fails to factor a repeated variable.
When the target variable appears in two terms, fails to collect and factor it out, leaving the variable on both sides.

Practice

1. *The perimeter of a rectangle is P = 2L + 2W.*

Solve for L.

(A) L = P/2 - 2W
(B) L = 2(P - 2W)
(C) L = P - 2W - 2
(D) L = (P - 2W)/2

2. *y = x/3 + 4.*

Solve for x.

(A) x = (y - 4)/3
(B) x = y/3 - 4
(C) x = 3(y - 4)
(D) x = 3y - 4

3. *ax = bx + c.*

Solve for x.

(A) x = c/(ab)

(B) x = c/(a - b)

(C) x = (c + bx)/a

(D) x = c/(a + b)

4. *The temperature conversion formula is F = (9/5)C + 32.*

Solve for C.

(A) C = (5/9)F - 32

(B) C = (5/9)F + 32

(C) C = (9/5)(F - 32)

(D) C = (5/9)(F - 32)

5. *The area of a trapezoid is A = (1/2)(b1 + b2)h.*

Solve for h.

(A) h = A/(2(b1 + b2))

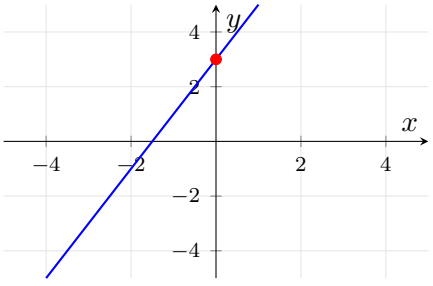
(B) h = 2A - (b1 + b2)

(C) h = 2A/b1 + b2

(D) h = 2A/(b1 + b2)

Build & interpret linear models in context

The skill. To model a real-world situation, the constant per-unit rate of change is the slope and the one-time or starting value is the y-intercept: y = (rate)x + (initial). A decrease makes the rate negative. 'At most' means <=, 'at least' means >=; use an inequality when the context sets a limit rather than an exact value.



A model y = (rate)x + (initial): the intercept is the starting value, the slope is the per-unit rate.

Worked example

A messaging plan charges a $25 monthly base fee plus $0.05 per text. Write an equation for the total cost y of sending x texts in a month.

Read. The plan has a per-text rate of $0.05 (a per-unit rate) and a fixed $25 base fee (a one-time amount). The total cost is an exact value, so the model is an equation, not an inequality.

Plan. Build the model from the template $y = (\text{rate})x + (\text{initial})$: the per-unit rate becomes the coefficient of x, and the fixed amount becomes the constant term.

Solve.

$y = (\text{rate})x + (\text{initial})$	linear-model template
$y = 0.05x + 25$	rate $0.05, base $25

Check. At $x = 0$ (no texts) the cost is $25, the base fee alone; each additional text adds $0.05. Both match the description, so the model is right.

Answer: $y = 0.05x + 25$.

Approach Read · Plan · Solve · Check

Read. Identify the variable, the per-unit rate, the fixed/starting amount, and whether the limit is exact or a bound.

Plan. Place the rate as the coefficient of the variable and the fixed amount as the constant. Make the rate negative for a decrease. Choose =, <=, or >= to match the context.

Solve. Assemble y = rate*x + initial (or the matching inequality).

Check. Is the per-unit amount the slope and the one-time amount the intercept (not swapped)? Right sign? Right inequality direction?

Common traps — every wrong answer is one of these

rate/initial role swap.
Uses the starting value as the rate (coefficient) and the per-unit rate as the constant, swapping slope and intercept.

rate sign/operation error.
Models a decrease as an increase (wrong sign), or combines the rate and the fixed amount into one term.

inequality direction/type error.
Chooses the wrong inequality direction (<= vs >=), or uses an equation where the context requires an inequality.

Practice

1. *A gym charges a $30 one-time sign-up fee plus $20 per month.*

Which equation gives the total cost y for x months?

(A) y = 20x - 30

(B) y = 30x + 20

(C) y = 50x

(D) y = 20x + 30

2. *A tank holds 200 liters and drains at 8 liters per minute.*

Which equation gives the volume y after x minutes?

(A) y = 200 - 8x

(B) y = 200 + 8x

(C) y = 8x - 200

(D) y = 8x + 200

3. *Maria has $50 and each ticket costs $4.*

Which inequality shows she can buy x tickets without exceeding her money?

(A) 4x <= 50

(B) 4x = 50

(C) 50x <= 4

(D) 4x >= 50

4. *A plumber charges a $75 visit fee plus $60 per hour, and a job costs $315.*

Which equation finds the number of hours x?

(A) 135x = 315

(B) 75x + 60 = 315

(C) 60x - 75 = 315

(D) 60x + 75 = 315

5. *A printing service charges $0.10 per page plus a $5 setup fee, and a school's budget is at most $50.*

Which inequality gives the number of pages x they can print?

(A) 5x + 0.10 <= 50

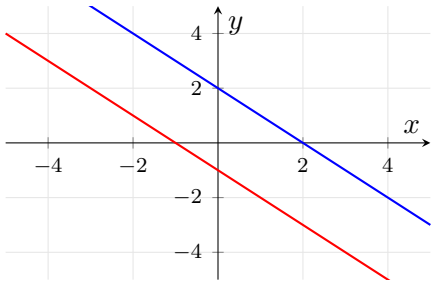
(B) 0.10x + 5 >= 50

(C) 0.10x + 5 = 50

(D) 0.10x + 5 <= 50

Parametric linear equations & systems (solve for the constant controlling solution count)

The skill. To find the constant that gives a target number of solutions: a one-variable equation ax = c has one solution if a is nonzero, no solution if a = 0 and c is nonzero, and infinitely many if a = 0 and c = 0. A two-variable system has exactly one solution when the slopes differ; NO solution when the lines are parallel (coefficients proportional but constants not: a1/a2 = b1/b2 not equal to c1/c2); and infinitely many when the equations are equivalent / scalar multiples (a1/a2 = b1/b2 = c1/c2). Set up the matching condition and solve for the unknown constant.



Equal slopes but different intercepts: the lines are parallel and never meet — no solution.

Worked example

For what value of k does the system $kx + 2y = 5$ and $6x + 4y = 9$ have no solution?

Read. We need the value of k that makes the system have no solution. No solution means the two lines are parallel: their x- and y-coefficients are proportional, but the constants are not in that same ratio (otherwise the lines would coincide).

Plan. Set the coefficient ratios equal, $\frac{k}{6} = \frac{2}{4}$, and solve for k. Then confirm the constant ratio is different, which rules out infinitely many solutions.

Solve.

$$\frac{k}{6} = \frac{2}{4} \qquad \text{match the coefficient ratios}$$
$$\frac{k}{6} = \frac{1}{2} \qquad \text{reduce } \frac{2}{4}$$
$$2k = 6 \qquad \text{cross-multiply}$$
$$k = 3 \qquad \text{divide both sides by 2}$$

Check. With $k = 3$ the constant ratio is $\frac{5}{9} \neq \frac{1}{2}$, so the lines are parallel but not identical. That is exactly the no-solution case, as required.

Answer: $k = 3$.

Approach Read · Plan · Solve · Check

Read. Identify the target count (one / none / infinite) and which coefficient or constant is the unknown.

Plan. For ax = c, decide what makes a = 0 matter. For a system, write the slope-equality condition for parallel (no solution) or the full-proportionality condition for equivalent lines (infinitely many), then solve for the parameter.

Solve. Solve the proportionality equation for the constant; check whether the constants also match (that distinguishes none from infinite).

Check. Right condition for the target count? No solution = equal slopes but NOT identical; infinite = fully identical. ax = c fails only when a = 0.

Common traps — every wrong answer is one of these

count-to-condition mapping error.
Applies the wrong structural condition for the target count: requires the equations to be identical for a 'no solution' answer, or sets only equal-slopes for an 'infinitely many' answer, confusing the parallel (none) and identical (infinite) cases.

degenerate-coefficient error.
For ax = c, misses that the equation has no solution only when the coefficient a = 0; picks a nonzero value (which still gives a solution) or thinks a nonzero coefficient can produce no solution.

proportionality setup/sign error.
Sets up the slope/coefficient-matching condition incorrectly: matches the wrong terms, cross-multiplies wrong, makes a sign error, or fails to scale the constant when scaling the equation.

Practice

1. *In the equation $ax = 5$, a is a constant.*

For what value of a does the equation have no solution?

(A) 0

(B) 10

(C) 5

(D) 1

2. *The system $kx - 3y = 4$ and $4x - 5y = 7$ has no solution, where k is a constant.*

What is the value of k?

(A) 16/7

(B) -16/7

(C) -12/5

(D) 12/5

3. *The system 3s - 2t = a and -15s + bt = -7 has infinitely many solutions, where a and b are constants.*

What is the value of a?

(A) -7/5

(B) -35

(C) 7

(D) 7/5

4. *The system 2x + 3y = 6 and 4x + 6y = k has a constant k.*

For what value of k does the system have infinitely many solutions?

(A) 3

(B) 12

(C) Any value except 12

(D) 6

5. *In the equation 4x + c = 4x + 7, c is a constant.*

For what value of c does the equation have infinitely many solutions?

(A) 0

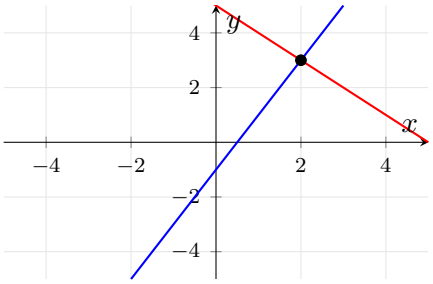
(B) 7

(C) Any value of c

(D) No value of c

Solve a linear system graphically (intersection point)

The skill. The solution of a two-variable linear system is the single point (x, y) where the two lines cross. Solve by graphing both lines and reading the intersection, or match a system to its graph by comparing slopes and intercepts. Parallel lines (same slope, different intercept) never cross, so the system has no solution; identical lines overlap everywhere, so it has infinitely many.



$y = 2x - 1$ and $y = -x + 5$ cross at $(2, 3)$ — the system's solution.

Worked example

The lines $y = 2x - 1$ and $y = -x + 5$ are graphed. At what point do they intersect?

Read. Both lines are already solved for y. Their intersection is the single point (x, y) that lies on both lines at once — the one place where the two y-values agree.

Plan. Since both equations give y, set the two expressions for y equal to each other. Solve that one-variable equation for x, then substitute x back into either line to get y.

Solve.

$2x - 1 = -x + 5$	set the two expressions for y equal
$3x - 1 = 5$	add x to both sides
$3x = 6$	add 1 to both sides
$x = 2$	divide both sides by 3
$y = 2(2) - 1 = 3$	substitute back for y

Check. Test the point in the other line: $-x + 5 = -2 + 5 = 3$, matching $y = 3$. So $(2, 3)$ lies on both lines and is the intersection.

Answer: $(2, 3)$.

Approach Read · Plan · Solve · Check

Read. The solution is where the two lines meet — a point that satisfies BOTH equations.

Plan. Put each equation in slope-intercept form; graph or compare slopes/intercepts; read the crossing point.

Solve. Locate the intersection (x, y); verify it lies on both lines by substitution.

Check. Same slope + different intercept = no solution; identical lines = infinitely many; did you read x then y?

Common traps — every wrong answer is one of these

coordinate order flip.
Reads the intersection as (y, x) instead of (x, y), swapping the coordinates.

parallel-as-solution.
Sees two parallel lines and still names a point on one of them as the solution, missing that parallel lines never intersect.

single-line read.
Uses one line's intercept or a point on just one line as the system's solution, ignoring the second equation.

slope/intercept mismatch.
Matches the system to the wrong graph by confusing which line has which slope or intercept.

Practice

1. *Two lines cross at the point 3 right and 2 up from the origin.*

What is the solution (x, y)?

(A) (3, 2)
(B) (2, 3)
(C) (0, 2)
(D) (3, 0)

2. *A system graphs as two distinct parallel lines.*

How many solutions does the system have?

(A) Infinitely many
(B) Exactly two
(C) No solution
(D) One solution at their y-intercepts

3. *Lines $y = x + 1$ and $y = -x + 5$ are graphed.*

At what point do they intersect?

(A) (3, 2)
(B) (0, 1)
(C) (0, 5)
(D) (2, 3)

4. *The system $y = 2x - 3$ and $y = 2x + 4$ is graphed.*

What is the solution?

(A) (0, -3)

(B) No solution (parallel lines)

(C) Infinitely many

(D) (0, 4)

5. *A system's two equations reduce to the SAME line.*

How many solutions?

(A) No solution

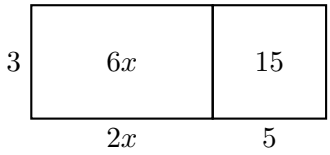
(B) Two

(C) Exactly one

(D) Infinitely many solutions

Equivalent linear expressions (distribute and combine like terms)

The skill. To simplify a linear expression, distribute every coefficient across its parentheses — multiply it by EACH term inside — then combine like terms (add the coefficients of matching variables; add the constants separately). Two expressions are equivalent when they simplify to the same form. A minus sign in front of parentheses multiplies every inside term by -1.



Distributing $3(2x+5)$ *as areas:* $3 \cdot 2x + 3 \cdot 5 = 6x + 15$ *— multiply each inside term.*

Worked example

Simplify $3(2x+5) - 2(x-4)$**.**

Read. The goal is to rewrite $3(2x+5) - 2(x-4)$ in simplest form by clearing both sets of parentheses and combining what is left.

Plan. Distribute each factor across every term inside its parentheses. Watch the second factor especially: the -2 must multiply both terms of $(x-4)$, including the sign. Then combine like terms.

Solve.

$$\begin{aligned} & 3(2x+5)-2(x-4) && \text{start with the expression} \\ &= 6x+15-2(x-4) && \text{distribute the 3} \\ &= 6x+15-2x+8 && \text{distribute the } -2 \text{ to both terms} \\ &= 6x-2x+15+8 && \text{group like terms} \\ &= 4x+23 && \text{combine like terms} \end{aligned}$$

Check. Substitute a test value, $x = 1$: the original is $3(7) - 2(-3) = 21 + 6 = 27$, and the simplified form is $4(1) + 23 = 27$. They agree, so the two expressions are equivalent.

Answer: $4x + 23$.

Approach Read · Plan · Solve · Check

Read. Rewrite the expression in simplest form; two expressions match only if they simplify identically.

Plan. Distribute across every set of parentheses, then group like terms.

Solve. Multiply each factor through; add variable coefficients together and constants together.

Check. Did you distribute to EVERY term, keep the sign on a negative factor, and avoid adding x-terms to constants?

Common traps — every wrong answer is one of these

partial distribution.
Multiplies the factor by only the first term inside the parentheses, e.g. 3(2x - 4) = 6x - 4.

negative-sign drop.
Distributing a negative, changes the sign of only the first term, e.g. -(2x - 5) = -2x - 5.

combines unlike terms.
Adds a variable term to a constant as if like terms, e.g. 6x + 9 = 15x.

coefficient arithmetic slip.
Combines like terms but adds their coefficients wrong, e.g. 5x - 2x = 7x.

Practice

1. Which is equivalent to 3(2x - 4) + 5?

(A) 6x - 12

(B) 6x + 1

(C) 6x - 7

(D) 6x - 4 + 5

2. Which is equivalent to -(2x - 5) + 3x?

(A) 5x + 5

(B) x + 5

(C) x - 5

(D) -5x + 5

3. Simplify: 4x + 7 - 2(x + 3).

(A) 2x + 1

(B) 2x + 13

(C) 6x + 1

(D) 2x + 4

4. Which is equivalent to 5(x + 2) - 3x?

(A) 12x

(B) 2x + 2

(C) 2x + 10

(D) 8x + 10

5. Simplify: 2(3x - 1) - (x - 4).

(A) 5x - 6

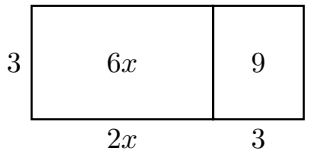
(B) 5x - 2

(C) 7x + 2

(D) 5x + 2

Equivalent linear expressions by factoring a common factor

The skill. A linear expression can be rewritten by factoring out a factor shared by every term: pull out the greatest common factor k so that ax + b = k(px + q), where k divides both a and b. Check by distributing back — the factored form must multiply out to the original. Factoring out a negative flips the sign of each remaining term.



Factoring $6x + 9 = 3(2x + 3)$: the common side 3 is pulled out front, leaving $2x + 3$ inside.

Worked example

Factor $8x - 20$ completely.

Read. To factor $8x - 20$ completely means to rewrite it as (common factor)(what is left) without changing its value, pulling out the largest factor shared by both terms.

Plan. Find the greatest common factor of the coefficients 8 and 20. Write that factor outside the parentheses, and divide each term by it to get what stays inside.

Solve.

$$\begin{aligned} 8x - 20 &= 4 \cdot 2x - 4 \cdot 5 && \text{GCF of 8 and 20 is 4} \\ &= 4(2x - 5) && \text{factor the 4 out front} \end{aligned}$$

Check. Distribute back: $4(2x) - 4(5) = 8x - 20$, the original expression. Because the 4 divided both terms evenly, the factoring is complete.

Answer: $4(2x - 5)$.

Approach Read · Plan · Solve · Check

Read. Rewrite the expression as (common factor)(what's left) without changing its value.

Plan. Find the greatest factor dividing EVERY term; write it outside, the quotient of each term inside.

Solve. Divide each term by the common factor; place results inside the parentheses.

Check. Distribute back to confirm you recover the original; did the factor divide every term, signs included?

Common traps — every wrong answer is one of these

incomplete factoring.
Factors from only one term, leaving the other unchanged, e.g. 6x + 9 = 3(2x) + 9.

factor doesn't divide all terms.
Pulls out a number that does not divide every term evenly, e.g. 6x + 8 = 3(2x + 8/3).

sign error factoring a negative.
Factors out a negative but does not flip the sign of every remaining term, e.g. -6x + 9 = -3(2x + 3).

quotient arithmetic slip.
Divides one term by the common factor incorrectly, e.g. 12x + 8 = 4(3x + 4).

Practice

1. Which is equivalent to 6x + 9?

(A) 3(2x + 3)

(B) 3(2x) + 9

(C) 2(3x + 4.5)

(D) 3(2x + 6)

2. Factor completely: 12x + 8.

(A) 4(3x + 4)

(B) 2(6x + 8)

(C) 4(3x + 8)

(D) 4(3x + 2)

3. Which is equivalent to -6x + 9?

(A) -3(2x - 3)

(B) -3(2x + 9)

(C) -3(2x + 3)

(D) 3(2x - 3)

4. Which is equivalent to 10x - 15?

(A) 5(2x - 3)

(B) 5(2x - 15)

(C) 5(2x - 5)

(D) 5(2x + 3)

5. Factor: 9x + 6.

(A) 3(3x + 6)

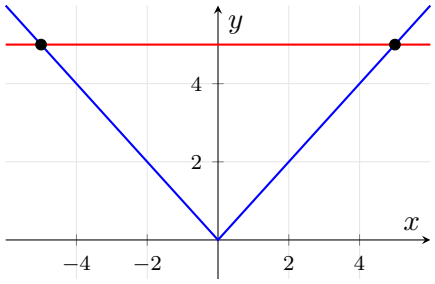
(B) 2(4.5x + 3)

(C) 3(3x + 2)

(D) 3(3x + 3)

Absolute-value equations & functions

The skill. Absolute value is distance from zero, so it is never negative. To solve |expression| = k with k > 0, split into TWO cases: expression = k and expression = -k, then solve each as a linear equation. If k < 0 there is NO solution; if k = 0 there is one. The graph of y = |x - h| + k is a V opening upward with vertex at (h, k). For inequalities, |x| < k means -k < x < k (between), while |x| > k means x < -k or x > k (outside).



$|x| = 5$ has two solutions where the V meets the line $y = 5$: $x = 5$ and $x = -5$.

Worked example

Solve $|x + 2| = 6$.

Read. This is an absolute-value equation, $|x + 2| = 6$, with a positive number on the right. Because absolute value is distance from zero, the inside can be either $+6$ or -6, so we expect two solutions.

Plan. Split into two cases that drop the absolute-value bars: $x + 2 = 6$ and $x + 2 = -6$. Solve each as an ordinary linear equation and keep both answers.

Solve.

$x + 2 = 6$	Case 1: inside equals $+6$
$x = 4$	subtract 2 from both sides
$x + 2 = -6$	Case 2: inside equals -6
$x = -8$	subtract 2 from both sides

Check. Test both: $|4 + 2| = |6| = 6$ and $|-8 + 2| = |-6| = 6$. Each value gives 6, so both cases are valid solutions.

Answer: $x = 4$ or $x = -8$.

Approach Read · Plan · Solve · Check

Read. Is this an equation, an inequality, or a graph question? Check the sign of the right-hand side before solving.

Plan. Equation with positive RHS: write both cases (= k and = -k). Negative RHS: no solution. Graph: read vertex (h, k) from |x - h| + k. Inequality: 'less than' is between, 'greater than' is outside.

Solve. Solve each case as a linear equation; for graphs/inequalities apply the vertex or between/outside rule.

Check. Did I keep BOTH cases? Did I check that the RHS is non-negative? Did I read the vertex sign correctly (x - h gives +h)? Between vs outside matches < vs >?

Common traps — every wrong answer is one of these

dropped-negative-case error.
Solves |expression| = k using only expression = k, omitting the expression = -k case, and reports a single solution instead of two.

negative-RHS blindness.
Treats |expression| = (negative number) as solvable, setting up cases or solving one branch, instead of recognizing that an absolute value cannot be negative so there is no solution.

vertex/graph sign error.
Misreads the vertex of y = |x - h| + k, e.g. taking the vertex x-coordinate as -h instead of h, or the sign of k, or thinking the V opens downward.

inequality and/or error.
Uses the 'outside' (or) form for |x| < k or the 'between' (and) form for |x| > k, mixing up which compound inequality matches less-than versus greater-than.

Practice

1. *Solve |x| = 5.*

What are all solutions?

(A) x = 0
(B) x = 5
(C) x = -5
(D) x = 5 or x = -5

2. *Solve |x - 3| = 7.*

What are all solutions?

(A) x = 10 or x = 4
(B) x = 7 or x = -7
(C) x = 10
(D) x = 10 or x = -4

3. *Solve |2x + 1| = -3.*

What are all solutions?

(A) No solution
(B) x = -3
(C) x = -2 or x = 1
(D) x = -2

4. *The graph of y = |x - 4| + 2.*

Where is its vertex?

(A) (4, 2)

(B) (-4, 2)

(C) (-4, -2)

(D) (4, -2)

5. *Solve the inequality |x| < 4.*

Which describes all solutions?

(A) x > 4

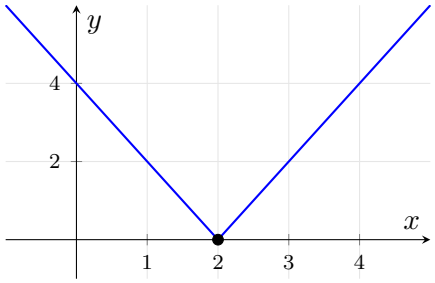
(B) x < -4 or x > 4

(C) x < 4

(D) -4 < x < 4

Absolute-value equations with a coefficient

The skill. For |ax + b| = k (k > 0), split into ax + b = k and ax + b = -k, then solve each linear equation. Both cases must be kept, and the coefficient a must be divided out.



$y = |2x - 4|$ is a V with vertex at $x = 2$, where the inside $2x - 4$ equals zero.

Worked example

Solve $|2x - 4| = 10$.

Read. The equation is $|2x - 4| = 10$ with a positive right side, so there will be two cases. The inside now carries a coefficient of 2, which must be divided out when solving each case.

Plan. Drop the bars into two linear equations, $2x - 4 = 10$ and $2x - 4 = -10$. Solve each for x by undoing the -4 and then the coefficient 2.

Solve.

$$\begin{aligned} 2x - 4 &= 10 && \text{Case 1: inside equals } +10 \\ 2x &= 14 && \text{add 4 to both sides} \\ x &= 7 && \text{divide both sides by 2} \\ 2x - 4 &= -10 && \text{Case 2: inside equals } -10 \\ 2x &= -6 && \text{add 4 to both sides} \\ x &= -3 && \text{divide both sides by 2} \end{aligned}$$

Check. Test both: $|2(7) - 4| = |10| = 10$ and $|2(-3) - 4| = |-10| = 10$. Both check out, so the coefficient was handled correctly.

Answer: $x = 7$ or $x = -3$.

Approach Read · Plan · Solve · Check

Read. Positive right side -> two cases.

Plan. ax + b = k and ax + b = -k.

Solve. Solve each for x.

Check. Did I keep BOTH cases and divide by a?

Common traps — every wrong answer is one of these

dropped the negative case.
Solves only ax + b = k.

sign-flipped both.
Reports the negatives of both solutions.

ignored the coefficient.
Solves as if a = 1.

Practice

1. *Solve |2x + 2| = 6.*

Which is correct?

(A) x = 2 or x = -4
(B) x = 2
(C) x = -2 or x = 4
(D) x = 6

2. *Solve |3x + 6| = 6.*

Which is correct?

(A) x = 0

(B) x = 0 or x = 4

(C) x = 6

(D) x = 0 or x = -4

3. *Solve |3x + 3| = 6.*

Which is correct?

(A) x = 1 or x = -3

(B) x = 6

(C) x = -1 or x = 3

(D) x = 1

4. *Solve |3x + 6| = 9.*

Which is correct?

(A) x = 1 or x = -5

(B) x = -1 or x = 5

(C) x = 1

(D) x = 9

5. *Solve |3x - 6| = 9.*

Which is correct?

(A) x = 5

(B) x = 5 or x = -1

(C) x = 15 or x = -3

(D) x = -5 or x = 1

Absolute-value inequalities: less-than (between)

The skill. |x - h| < k means the distance from h is less than k, so x is BETWEEN: h - k < x < h + k. 'Less than' gives a single compound (and) inequality, not an 'or'.



'Less than' is BETWEEN: $|x-4|<3$ means $1<x<7$, shaded between two open ends.

Worked example

Solve $|x - 4| < 3$.

Read. The inequality $|x - 4| < 3$ says the distance from x to 4 is less than 3. "Less than" packs x between two bounds, so the answer is a single compound (and) inequality, not two separate pieces.

Plan. Rewrite the absolute value as the compound form $-3 < x - 4 < 3$. Then add 4 to all three parts at once to isolate x in the middle.

Solve.

$\lvert x - 4\rvert < 3$	start with the given inequality
$-3 < x - 4 < 3$	less-than means between (and)
$1 < x < 7$	add 4 to all three parts

Check. Test a middle value, $x = 5$: $|5 - 4| = 1 < 3$ is true, and 5 does lie between 1 and 7. The interval is correct.

Answer: $1 < x < 7$.

Approach Read · Plan · Solve · Check

Read. 'Less than' -> between (and).

Plan. -k < x - h < k, then add h.

Solve. h - k < x < h + k.

Check. Is it between (not outside)? Did I add h to all parts?

Common traps — every wrong answer is one of these

used outside form.
Writes x < h-k or x > h+k.

one-sided.
Keeps only one bound.

ignored the shift.
Forgets to add h.

Practice

1. *Solve |x - 3| < 4.*

Which is correct?

(A) x < 7

(B) x < -1 or x > 7

(C) -1 < x < 7

(D) -4 < x < 4

2. *Solve |x - 5| < 4.*

Which is correct?

(A) x < 1 or x > 9

(B) -4 < x < 4

(C) x < 9

(D) 1 < x < 9

3. *Solve |x - 2| < 2.*

Which is correct?

(A) x < 4

(B) -2 < x < 2

(C) x < 0 or x > 4

(D) 0 < x < 4

4. *Solve |x - 2| < 4.*

Which is correct?

(A) -4 < x < 4

(B) -2 < x < 6

(C) x < 6

(D) x < -2 or x > 6

5. *Solve |x - 2| < 5.*

Which is correct?

(A) -3 < x < 7

(B) x < 7

(C) -5 < x < 5

(D) x < -3 or x > 7

Absolute-value inequalities: greater-than (outside)

The skill. |x - h| > k means the distance from h exceeds k, so x is OUTSIDE: x < h - k OR x > h + k. 'Greater than' splits into two separate pieces joined by 'or'.



'Greater than' is OUTSIDE: $|x-3|>2$ means $x<1$ or $x>5$, two shaded pieces.

Worked example

Solve $|x-2|>5$.

Read. The inequality $|x-2|>5$ says the distance from x to 2 is greater than 5. "Greater than" pushes x outside both bounds, so the answer splits into two separate pieces joined by "or".

Plan. Break the absolute value into two inequalities, $x-2<-5$ or $x-2>5$. Solve each piece by adding 2; keep both pieces in the final answer.

Solve.

$\lvert x-2\rvert>5$	start with the given inequality
$x-2<-5 \quad \text{or} \quad x-2>5$	greater-than means outside (or)
$x<-3 \quad \text{or} \quad x>7$	add 2 to each piece

Check. Test an outside value $x=8$: $|8-2|=6>5$ is true. Test a between value $x=0$: $|-2|=2$, which is not >5, so it is correctly excluded.

Answer: $x<-3$ or $x>7$.

Approach Read · Plan · Solve · Check

Read. 'Greater than' -> outside (or).

Plan. x - h < -k or x - h > k; add h.

Solve. x < h-k or x > h+k.

Check. Two pieces with 'or'? Added h to each?

Common traps — every wrong answer is one of these

used between form.
Writes h-k < x < h+k.

one piece only.
Keeps just one inequality.

ignored the shift.
Forgets to add h.

Practice

1. *Solve |x - 3| > 2.*

Which is correct?

(A) $x < 1$ or $x > 5$
(B) $x < -2$ or $x > 2$
(C) $1 < x < 5$
(D) $x > 5$

2. *Solve $|x - 3| > 4$.*

Which is correct?

(A) $x > 7$
(B) $-1 < x < 7$
(C) $x < -4$ or $x > 4$
(D) $x < -1$ or $x > 7$

3. *Solve $|x - 2| > 2$.*

Which is correct?

(A) $x > 4$
(B) $0 < x < 4$
(C) $x < 0$ or $x > 4$
(D) $x < -2$ or $x > 2$

4. *Solve $|x - 5| > 4$.*

Which is correct?

(A) $x > 9$
(B) $x < 1$ or $x > 9$
(C) $1 < x < 9$
(D) $x < -4$ or $x > 4$

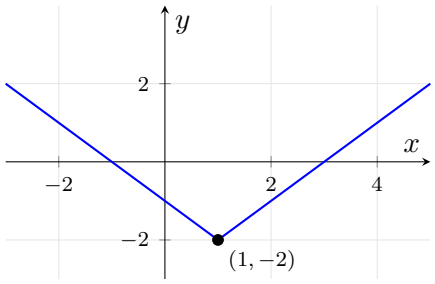
5. *Solve $|x - 1| > 3$.*

Which is correct?

(A) $-2 < x < 4$
(B) $x > 4$
(C) $x < -2$ or $x > 4$
(D) $x < -3$ or $x > 3$

Graphs of absolute-value functions (vertex form)

The skill. $y = |x + h| - k$ is a V with vertex where the inside is zero: $x + h = 0$ gives $x = -h$, and $y = -k$, so the vertex is $(-h, -k)$. The x-shift is opposite the inside sign; the outside constant is the y-value directly.



$y = |x - 1| - 2$: the vertex sits where the inside is zero, at $(1, -2)$.

Worked example

What is the vertex of $y = |x + 6| - 1$?

Read. The function $y = |x + 6| - 1$ is in vertex form, and we want the vertex — the tip of the V. The graph turns exactly where the absolute-value expression bottoms out.

Plan. An absolute value is smallest (zero) when its inside is zero, so set $x + 6 = 0$ to find the vertex x. The outside constant, -1, is then the vertex y-value.

Solve.

$x + 6 = 0$	vertex is where the inside is zero
$x = -6$	subtract 6 from both sides
$y = -1$	outside constant is the y-value
vertex $= (-6, -1)$	combine the coordinates

Check. At $x = -6$: $y = |0| - 1 = -1$. Any other x makes $|x + 6|$ positive, giving a larger y, so $(-6, -1)$ is the lowest point — the vertex.

Answer: $(-6, -1)$.

Approach Read · Plan · Solve · Check

Read. Vertex is where the inside = 0.

Plan. x + h = 0 -> x = -h; read y = -k.

Solve. (-h, -k).

Check. Did the x-shift flip sign? Is y the outside constant?

Common traps — every wrong answer is one of these

x-sign error.
Uses +h instead of -h for the vertex x.

y-sign error.
Uses +k instead of -k for the vertex y.

both signs wrong.
Reports (h, k).

Practice

1. *What is the vertex of $y = |x + 4| - 2$?*

Which is correct?

(A) (4, -2)
(B) (4, 2)
(C) (-4, -2)
(D) (-4, 2)

2. *What is the vertex of $y = |x + 2| - 5$?*

Which is correct?

(A) (-2, -5)
(B) (2, 5)
(C) (2, -5)
(D) (-2, 5)

3. *What is the vertex of $y = |x + 3| - 2$?*

Which is correct?

(A) (3, 2)
(B) (-3, -2)
(C) (3, -2)
(D) (-3, 2)

4. *What is the vertex of $y = |x + 3| - 5$?*

Which is correct?

(A) (3, -5)
(B) (-3, 5)
(C) (-3, -5)
(D) (3, 5)

5. *Find the vertex of $y = |x - 5| + 3$.*

Which is correct?

(A) (-5, -3)

(B) (5, 3)

(C) (5, -3)

(D) (-5, 3)

DOMAIN 2

Advanced Math

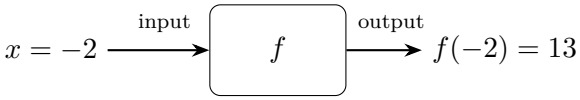
Functions, and the structure beneath them

"The book of nature is written in the language of mathematics."
— Galileo Galilei

About 35% of the section. Quadratics, polynomials, exponentials, radicals, rational expressions, nonlinear systems, function transforms, and complex numbers.

Evaluate & interpret function notation f(x)

The skill. f(x) names a function: f(a) is the output when the input is a, found by substituting a for x. f(a) = b means the point (a, b) is on the graph. f(x + h) means substitute the whole expression x + h for x – it is not f(x) + h, and f(a) is not f times a.



A function as an input-output machine: feed in $x = -2$ and the rule $f(x) = x^2 - 4x + 1$ returns the output 13.

Worked example

If $f(x) = x^2 - 4x + 1$, find $f(-2)$.

Read. We are given the rule $f(x) = x^2 - 4x + 1$ and asked for $f(-2)$ – the single output the function returns when the input is -2.

Plan. Substitute -2 for every x, always wrapped in parentheses so the sign is squared correctly, then simplify one operation at a time.

Solve.

$f(-2) = (-2)^2 - 4(-2) + 1$	substitute -2 for every x
$f(-2) = 4 + 8 + 1$	square -2 and multiply $-4(-2) = +8$
$f(-2) = 13$	add the three terms

Check. The easiest slip is the sign of the middle term: $-4(-2) = +8$, not -8. With $+8$ the sum is $4 + 8 + 1 = 13$, confirming the result.

Answer: 13.

Approach Read · Plan · Solve · Check

Read. Identify the input (what goes in for x) and what is being asked (an output value, or an input given an output).

Plan. To evaluate f(a): replace every x with a, in parentheses. To read f(a) = b: a is the input, b is the output.

Solve. Substitute carefully, keeping signs and grouping; simplify.

Check. Did you substitute for x everywhere? Is the answer an output (not the input you plugged in)?

Common traps — every wrong answer is one of these

notation-as-multiplication.
Reads f(a) as f times a, or distributes the input over the whole expression (f(4) as 4 times (3x+2)) instead of substituting.

input/output swap.
Confuses input and output: returns the input value, reads the wrong table row/column, or solves for x when the output was asked (and vice versa).

transformation-vs-evaluation.
Computes f(x + h) as f(x) + h (or f(x) + h as f(x + h)), conflating substituting an expression with adding to the output.

Practice

1. *f(x) = 3x + 2.*

What does f(4) represent?

(A) f multiplied by 4

(B) The value of the function when x = 4

(C) 4 times the whole expression, 4(3x + 2)

(D) The value of x for which f(x) = 4

2. *f(x) = 2x - 5.*

Find f(-3).

(A) 1

(B) -16

(C) -11

(D) -1

3. *A table of values: f(1) = 5, f(2) = 8, f(3) = 11.*

Find f(2).

(A) 8

(B) 11

(C) 2

(D) 5

4. *f(x) = 2x + 1.*

Find f(x + 3).

(A) 2x + 7

(B) 2x + 4

(C) 6x + 1

(D) 2x + 6

5. *C(n) gives the cost in dollars of buying n items. C(10) = 45.*

What does this mean?

(A) 10 items cost $45

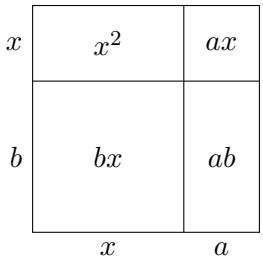
(B) 45 items cost $10

(C) C times 10 equals 45

(D) Each item costs $45

Expand binomials / FOIL

The skill. Multiply two binomials by distributing every term across every term: $(x+a)(x+b) = x^2 + (a+b)x + ab$. The middle coefficient is the SUM a+b; the constant is the PRODUCT ab. Special cases: $(x+a)^2 = x^2 + 2ax + a^2$ (note the doubling) and $(x+a)(x-a) = x^2 - a^2$ (the middle cancels).



Area model of $(x+a)(x+b)$: the four pieces sum to $x^2+(a+b)x+ab$ – the middle term collects both cross-products.

Worked example

Expand $(x+7)(x-2)$.

Read. We must expand the product of two binomials, $(x+7)$ and $(x-2)$. Keep the signs attached: the constants are $+7$ and -2.

Plan. Use FOIL – multiply First, Outer, Inner, Last – then combine the two like middle terms. As a check, the middle coefficient should be the sum $7+(-2)$ and the constant the product $7(-2)$.

Solve.

$(x+7)(x-2) = x\cdot x + x(-2) + 7\cdot x + 7(-2)$	FOIL: each term of the first times each of the second
$(x+7)(x-2) = x^2 - 2x + 7x - 14$	carry out the four products with signs
$(x+7)(x-2) = x^2 + 5x - 14$	combine the like middle terms $-2x+7x$

Check. The middle coefficient is the sum $7+(-2)=5$ and the constant is the product $7(-2)=-14$; the answer has three terms, as expected for a general product.

Answer: $x^2+5x-14$.

Approach Read · Plan · Solve · Check

Read. Identify the two binomials and the sign of each term.

Plan. Distribute first*first, outer, inner, last; combine the outer+inner into the middle term. For a square, remember the middle is 2ax, not ax.

Solve. Multiply with signs; collect like terms.

Check. Three terms for a general product (unless the middle cancels); is the middle the SUM and the constant the PRODUCT? Did you double the middle on a perfect square?

Common traps — every wrong answer is one of these

missing cross terms (incomplete FOIL).
Drops the outer+inner middle term: (x+a)(x+b) -> x^2 + ab, or $(x+a)^2$ -> x^2 + a^2. An incomplete distribution.

sign error in products.
Mis-signs one of the product terms, especially with a negative binomial.

perfect-square doubling error.
Expands $(x+a)^2$ as $x^2 + ax + a^2$, writing a single ax and forgetting to double the middle term.

sum/product confusion.
Uses the product for the middle coefficient and/or the sum for the constant, swapping which operation belongs where.

Practice

1. *Expand $(x + 3)(x + 2)$.*

Which is the result?

(A) $x^2 + 5x + 6$
(B) $x^2 + 6x + 6$
(C) $x^2 + 5x + 5$
(D) $x^2 + 6$

2. *Expand $(x - 4)(x + 2)$.*

Which is the result?

(A) $x^2 - 8$
(B) $x^2 - 2x - 8$
(C) $x^2 - 2x + 8$
(D) $x^2 + 2x - 8$

3. *Expand $(x + 5)^2$.*

Which is the result?

(A) $x^2 + 25$
(B) $x^2 + 5x + 25$
(C) $x^2 + 10x + 10$
(D) $x^2 + 10x + 25$

4. *Expand $(2x + 3)(x - 1)$.*

Which is the result?

(A) $2x^2 + x - 3$
(B) $2x^2 - x - 3$

(C) $2x^2 + x + 3$

(D) $2x^2 - 2x$

5. *Expand (x + 6)(x - 6).*

Which is the result?

(A) $x^2 - 6$

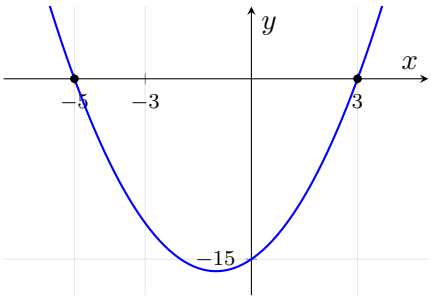
(B) $x^2 + 36$

(C) $x^2 - 36$

(D) $x^2 - 12x - 36$

Factor quadratics

The skill. To factor $x^2 + bx + c$, find two numbers whose SUM is b and PRODUCT is c; the factors are (x + p)(x + q). For a leading coefficient a != 1, account for it (factor by grouping or trial). Pull out a common factor first, and recognize special forms: difference of squares $x^2 - a^2 = (x+a)(x-a)$, perfect-square trinomial $x^2 + 2ax + a^2 = (x+a)^2$. Check by expanding.



$y = x^2 + 2x - 15 = (x + 5)(x - 3)$: the factors reveal the roots $x = -5$ and $x = 3$ where the curve meets the x-axis.

Worked example

Factor $x^2 + 2x - 15$.

Read. We are factoring $x^2 + 2x - 15$. There is no common factor to pull out, so we look for two numbers whose sum is the middle coefficient 2 and whose product is the constant -15.

Plan. List the factor pairs of -15 and pick the one that adds to 2. Because the product is negative, the two numbers have opposite signs; $+5$ and -3 give product -15 and sum $+2$.

Solve.

$x^2 + 2x - 15 = x^2 + (5 - 3)x + (5)(-3)$	split the middle using 5 and -3
$x^2 + 2x - 15 = (x + 5)(x - 3)$	write the factors from those two numbers

Check. Expanding recovers the original: $(x + 5)(x - 3) = x^2 - 3x + 5x - 15 = x^2 + 2x - 15$. The factorisation is complete.

Answer: $(x + 5)(x - 3)$.

Approach Read · Plan · Solve · Check

Read. Factor out any GCF first; note the signs of b and c.

Plan. Find a number pair with the right sum (b) and product (c). For a != 1, use grouping. Watch for difference of squares and perfect squares.

Solve. Write the factors; place signs so the product gives c and the sum gives b.

Check. Expand your factors (Q2) – do you recover the original? Is it factored completely?

Common traps — every wrong answer is one of these

sum/product factor-pair error.
Picks a number pair with the right product but the wrong sum (or vice versa), not checking both conditions.

sign error in factors.
Right magnitudes, wrong signs: e.g. (x-2)(x-3) where the middle term should be positive, or sign misplacement with a negative constant.

leading-coefficient/common-factor mishandling.
Factors $ax^2 + bx + c$ as if a = 1, misplaces the inner/outer factors, or drops a common factor.

special-form error.
Mis-factors a difference of squares as a perfect square, or stops before factoring completely.

Practice

1. *Factor $x^2 + 7x + 12$.*

Which is correct?

(A) (x - 3)(x - 4)
(B) (x + 2)(x + 6)
(C) (x + 1)(x + 12)
(D) (x + 3)(x + 4)

2. *Factor $x^2 - x - 6$.*

Which is correct?

(A) (x + 1)(x - 6)
(B) (x + 2)(x - 3)
(C) (x + 6)(x - 1)
(D) (x - 2)(x + 3)

3. *Factor $2x^2 + 7x + 3$.*

Which is correct?

(A) (2x + 3)(x + 1)
(B) (x + 1)(x + 3)
(C) (2x - 1)(x - 3)
(D) (2x + 1)(x + 3)

4. *Factor $x^2 - 16$.*

Which is correct?

(A) $(x + 4)^2$
(B) (x - 16)(x + 1)
(C) $(x - 4)^2$
(D) (x + 4)(x - 4)

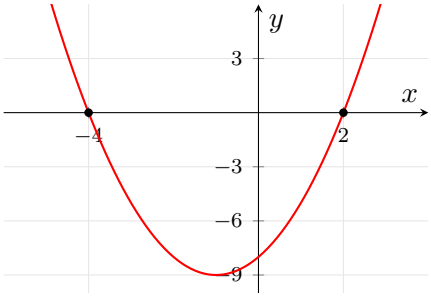
5. *Factor completely: $x^3 - 4x$.*

Which is correct?

(A) (x + 2)(x - 2) [dropped the common factor x]
(B) $x(x^2 - 4)$ [left unfactored]
(C) x(x + 2)(x - 2)
(D) $x(x - 2)^2$

Solve quadratics by factoring & the zero-product property

The skill. To solve a quadratic by factoring: first move everything to one side so it equals 0, factor, then apply the zero-product property – set EACH factor equal to zero and solve the resulting linear equations. The zero-product property works only when the product equals zero. A repeated factor gives one (double) root.



Zero-product property: $y = (x + 4)(x - 2)$ equals 0 only where a factor is 0, i.e. at the x-intercepts $x = -4$ and $x = 2$.

Worked example

Solve $x^2 + 2x - 8 = 0$**.**

Read. The equation $x^2 + 2x - 8 = 0$ is already set equal to zero, which is exactly what the zero-product property needs. We solve for x.

Plan. Factor the quadratic, then apply the zero-product property: if a product is zero, at least one factor is zero, so set each factor equal to 0 and solve the two linear equations.

Solve.

$x^2 + 2x - 8 = 0$	start from the given equation
$(x + 4)(x - 2) = 0$	factor: 4 and -2 sum to 2, multiply to -8
$x + 4 = 0$ or $x - 2 = 0$	zero-product property
$x = -4$ or $x = 2$	solve each linear factor

Check. Substitute both roots: $(-4)^2+2(-4)-8 = 16-8-8 = 0$ and $2^2+2(2)-8 = 4+4-8 = 0$. Both check.

Answer: $x = -4$ or $x = 2$.

Approach Read · Plan · Solve · Check

Read. Is the equation set equal to zero? If not, move all terms to one side first.

Plan. Factor the quadratic, then set each factor = 0 and solve each linear equation.

Solve. Solve each factor equation for x; keep every distinct root.

Check. Did you set it to zero before factoring? Did you keep all roots (including x = 0)? Substitute each root back.

Common traps — every wrong answer is one of these

not-set-to-zero.
Applies the zero-product property when the equation is not equal to zero – e.g. sets each factor equal to the constant on the right side.

dropped or miscounted root.
Reports only one solution when there are two, or mishandles a repeated (double) root.

divide-by-variable.
Divides both sides by the variable ($x^2 = 4x$ -> $x = 4$), silently discarding the $x = 0$ root.

Practice

1. *Solve x^2 - 5x + 6 = 0.*

What are the solutions?

(A) x = -2 or x = -3

(B) x = 1 or x = 6

(C) x = 2 or x = 3

(D) x = 2 only

2. *Solve (x - 2)(x - 3) = 2.*

What are the solutions?

(A) x = 2 or x = 3

(B) x = 4 or x = 5

(C) x = -1 or x = -4

(D) x = 1 or x = 4

3. *Solve $x^2 = 4x$.*

What are the solutions?

(A) x = 4 only

(B) x = 0 only

(C) x = 0 or x = -4

(D) x = 0 or x = 4

4. *Solve $4x^2 - 12x + 9 = 0$.*

What are the solutions?

(A) x = 3/2 or x = -3/2

(B) No real solution

(C) x = 3/2 (a double root)

(D) x = 2/3

5. *The graph $y = x^2 - 7x + 10$ crosses the x-axis where $x^2 - 7x + 10 = 0$.*

What are the x-intercepts?

(A) x = 2 and x = 5

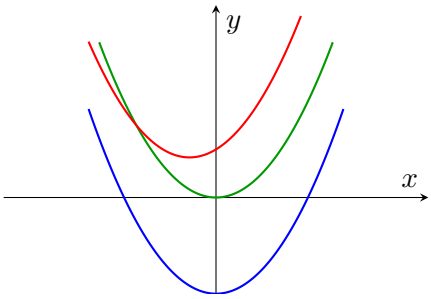
(B) x = 7 and x = 10

(C) x = -2 and x = -5

(D) x = 2 only

Quadratic formula & discriminant (number of real solutions)

The skill. Any quadratic ax^2 + bx + c = 0 is solved by x = (-b +/- $\sqrt{b^2 - 4ac}$) / (2a). The discriminant b^2 - 4ac counts the real solutions: positive gives two, zero gives one (a double root), negative gives none. The whole numerator is divided by 2a, and both the + and - branches are kept.



The discriminant $b^2 - 4ac$ counts real roots: two if positive (crosses twice), one if zero (just touches), none if negative (stays off the axis).

Worked example

Solve $x^2 - 6x + 8 = 0$ with the quadratic formula.

Read. The equation $x^2 - 6x + 8 = 0$ is in standard form, so we read off the coefficients $a = 1$, $b = -6$, and $c = 8$, keeping the sign of b.

Plan. First compute the discriminant $b^2 - 4ac$ to learn how many real roots there are, then substitute into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, dividing the whole numerator by $2a$ and keeping both branches.

Solve.

$b^2 - 4ac = (-6)^2 - 4(1)(8)$	substitute $a = 1,\ b = -6,\ c = 8$
$b^2 - 4ac = 36 - 32$	evaluate the square and the product
$b^2 - 4ac = 4$	a positive discriminant means two real roots
$x = \dfrac{-(-6) \pm \sqrt{4}}{2(1)}$	substitute into the quadratic formula
$x = \dfrac{6 \pm 2}{2}$	simplify $-(-6) = 6$ and $\sqrt{4} = 2$
$x = 4$ or $x = 2$	take the $+$ and $-$ branches

Check. A positive discriminant matches the two roots found. As a further check, $4+2 = 6 = -b$ and $4 \cdot 2 = 8 = c$, agreeing with the sum and product of the roots.

Answer: $x = 4$ or $x = 2$.

Approach Read · Plan · Solve · Check

Read. Identify a, b, c with their signs (the equation must equal zero first).

Plan. Compute the discriminant b^2 - 4ac; its sign gives the number of real solutions. If you need the roots, substitute into the full formula.

Solve. Carry -b (positive when b is negative); divide the entire numerator by 2a; keep both branches.

Check. Does the count match the discriminant's sign? Did you divide everything by 2a and keep both +/- roots?

Common traps — every wrong answer is one of these

discriminant solution-count misread.
Misreads the discriminant: claims real solutions when b^2 - 4ac < 0, says two when it is 0, or otherwise mismaps the sign to the count.

formula substitution sign error.
Mis-substitutes signs into the formula: uses -b as negative when b is already negative, or mishandles the sign of 4ac when c is negative.

formula structure error.
Mis-structures the formula: drops the +/- (keeps one branch), or divides only part of the numerator by 2a.

Practice

1. *Solve $x^2 + 3x - 4 = 0$ with the quadratic formula.*

What are the solutions?

(A) x = 2 or x = -8

(B) x = 1 or x = -4

(C) x = -1 or x = 4

(D) x = 1 only

2. *The equation $x^2 + x - 6 = 0$ (discriminant 25).*

How many real solutions does it have?

(A) One real solution

(B) Two real solutions

(C) Infinitely many solutions

(D) No real solutions

3. *The equation $x^2 + x + 5 = 0$ (discriminant -19).*

How many real solutions does it have?

(A) Two real solutions

(B) Two solutions, x = (-1 +/- $\sqrt{-19}$)/2, treated as real

(C) No real solutions

(D) One real solution

4. *Solve x^2 - 5x + 6 = 0 with the quadratic formula (a=1, b=-5, c=6).*

What are the solutions?

(A) x = 6 or x = 4

(B) x = -3 or x = -2

(C) x = 3 or x = 2

(D) x = 6 or x = -1

5. *x^2 + kx + 9 = 0 is to have exactly one real solution.*

For what value(s) of k?

(A) No such k exists

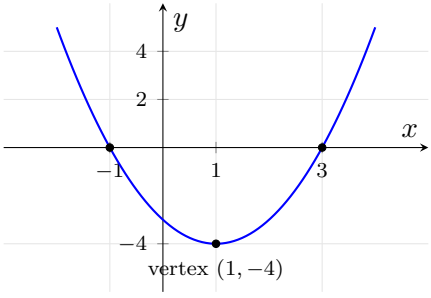
(B) k = 6 or k = -6

(C) k = 6 only

(D) k = 3 or k = -3

Forms of a quadratic & the parabola

The skill. A quadratic has three useful forms. Standard $y = ax^2 + bx + c$: y-intercept c, axis x = -b/(2a). Vertex $y = a(x - h)^2 + k$: vertex (h, k), axis x = h. Factored y = a(x - r1)(x - r2): x-intercepts r1, r2. The sign of a sets direction: $a > 0$ opens up (vertex is a minimum), $a < 0$ opens down (maximum). The number of x-intercepts equals the discriminant count.



$y = (x - 3)(x + 1)$: factored form shows roots -1 and 3; the vertex lies on the axis of symmetry midway between them at $(1, -4)$.

Worked example

Find the vertex of $y = (x - 3)(x + 1)$.

Read. The quadratic $y = (x - 3)(x + 1)$ is in factored form, so its roots are read directly: $x = 3$ and $x = -1$. We want the vertex.

Plan. A parabola is symmetric, so its axis of symmetry lies halfway between the two roots. Average the roots to get the vertex's x, then substitute that value back into the equation to get the vertex's y.

Solve.

$x = \dfrac{3 + (-1)}{2}$	the axis is the midpoint of the roots
$x = 1$	simplify the average
$y = (1 - 3)(1 + 1)$	substitute $x = 1$ into the equation
$y = (-2)(2)$	evaluate each factor
$y = -4$	multiply to get the vertex height

Check. The vertex $(1, -4)$ sits midway between the roots, as it must. Since $a > 0$ the parabola opens upward, so this vertex is a minimum – consistent with the negative y.

Answer: $(1, -4)$.

Approach Read · Plan · Solve · Check

Read. Identify which form the quadratic is in and which feature is asked (vertex, axis, intercepts, direction).

Plan. Vertex form reads the vertex directly (mind the sign of h). Standard: axis x = -b/(2a), then substitute for k. Factored: x-intercepts are the roots. Sign of a gives direction and min/max.

Solve. Extract the feature, tracking signs; convert forms if needed.

Check. Does (h, k) use h with the opposite sign of what's in (x - h)? Does the direction match the sign of a? Does the x-intercept count match the discriminant?

Common traps — every wrong answer is one of these

vertex-form h sign.
Reads the vertex of $y = a(x - h)^2 + k$ as (-h, k) instead of (h, k), flipping the sign of h.

axis from standard form.
Computes the axis as b/(2a) with the wrong sign, or fails to divide by 2a.

direction/extremum confusion.
Reverses opens-up/minimum ($a > 0$) and opens-down/maximum ($a < 0$), or assumes the vertex is always a maximum.

feature-from-form confusion.
Reads the wrong feature off a form: the constant as a root, a coefficient as a coordinate, or the y-intercept as an x-intercept.

Practice

1. *$y = 2(x - 3)^2 + 4$.*

What is the vertex?

(A) (-3, 4)
(B) (2, 4)
(C) (4, 3)
(D) (3, 4)

2. *$y = x^2 - 6x + 5$.*

What is the axis of symmetry?

(A) x = 5
(B) x = 3
(C) x = 6
(D) x = -3

3. *$y = -2(x - 1)^2 + 8$.*

What is the maximum or minimum value?

(A) Minimum value of -8
(B) Maximum value of 8
(C) Minimum value of 8
(D) Maximum value of 1

4. *$y = (x + 2)(x - 5)$.*

What are the x-intercepts?

(A) x = -2 and x = 5
(B) x = -2 only
(C) x = 2 and x = -5
(D) (0, -10), the y-intercept

5. *The parabola $y = x^2 - 4x + 7$ (discriminant -12).*

How many times does it cross the x-axis?

(A) At x = 7

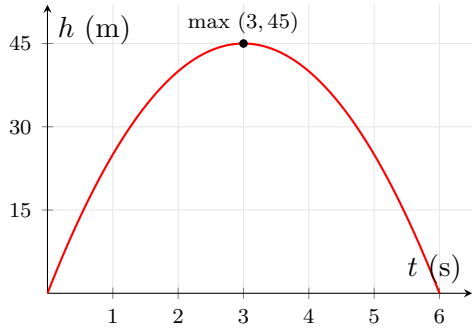
(B) It does not cross the x-axis (0 times)

(C) Once

(D) Twice

Interpret a quadratic model in context

The skill. In a quadratic model, the vertex is the maximum or minimum: its y-coordinate is the extreme VALUE (how much), its x-coordinate is WHERE/WHEN it occurs. The zeros are real-world events (e.g. when height returns to 0); discard context-invalid roots. The y-intercept is the initial value. Always answer the quantity asked, with units.



Height model $h = -5t^2 + 30t$: the downward parabola's vertex $(3, 45)$ gives the maximum height (45 m) and the time it occurs ($t = 3$ s).

Worked example

A rocket's height in meters after t seconds is $h = -5t^2 + 30t$, with vertex $(3, 45)$. What is the maximum height, and when does it occur?

Read. The height model $h = -5t^2 + 30t$ opens downward (the leading coefficient is negative), so its vertex is the highest point. We are told the vertex is $(3, 45)$ and must report both the maximum height and the time it occurs.

Plan. Read the maximum height from the vertex's y-coordinate and the time it occurs from the vertex's x-coordinate, attaching the correct units. To confirm, substitute $t = 3$ back into the model.

Solve.

$t = 3$	the vertex x-coordinate: when the maximum occurs (seconds)
$h(3) = -5(3)^2 + 30(3)$	substitute $t = 3$ into the height model
$h(3) = -45 + 90$	evaluate each term
$h(3) = 45$	the vertex height: the maximum, in meters

Check. Because $a < 0$ the vertex is indeed a maximum, not a minimum. The computed $h(3) = 45$ matches the given vertex y-value, so the maximum height is 45 m at $t = 3$ s.

Answer: 45 m at $t = 3$ s.

Approach Read · Plan · Solve · Check

Read. Identify what each variable and feature means physically, with units.

Plan. Max/min value -> vertex y; when it occurs -> vertex x; an event like landing -> a relevant zero; starting amount -> y-intercept.

Solve. Read the feature, attach the correct units, and discard roots that make no physical sense.

Check. Did you answer the quantity asked (value vs time)? Right units? Did you keep the physically meaningful root?

Common traps — every wrong answer is one of these

extremum interpretation.
Confuses when the extremum occurs (vertex x) with how much it is (vertex y), or mistakes the initial value for the maximum.

zero-in-context meaning.
Misinterprets the roots: reports the launch (t = 0) as the landing, the vertex time as a zero, or keeps a physically invalid root.

answer-interpretation/units.
Solves correctly but reports the wrong quantity or drops/uses wrong units.

Practice

1. *A ball's height (feet) after t seconds is $h = -16t^2 + 64t$, with vertex (2, 64).*

What is the maximum height?

(A) 2 feet

(B) 64 seconds

(C) 64 feet

(D) 2 seconds

2. *Same ball: $h = -16t^2 + 64t$, vertex (2, 64), with a zero at $t = 4$.*

At what time does the ball reach its maximum height?

(A) 2 seconds

(B) 2 feet

(C) 64 seconds

(D) 4 seconds

3. *Same ball: $h = -16t^2 + 64t$, with zeros at $t = 0$ and $t = 4$.*

When does the ball hit the ground?

(A) 4 seconds

(B) At 0 and 4 seconds

(C) 2 seconds

(D) 0 seconds

4. *A company's profit (dollars) is $P = -2x^2 + 40x - 50$, with vertex (10, 150).*

What is the maximum profit?

(A) \$50

(B) \$10

(C) \$150

(D) 150 units

5. *$h = -5t^2 + 20t + 25$ models the height (meters) of an object after t seconds. The vertex is (2, 45), a zero is at $t = 5$, and the initial height is 25 m.*

Which statement is correct?

(A) Maximum height 45 m at t = 2 s; it lands at t = 5 s

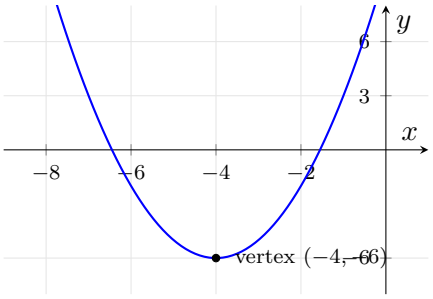
(B) Maximum height 2 m at t = 45 s

(C) The maximum height is 25 m

(D) It lands at t = 2 s

Completing the square

The skill. To complete the square on $x^2 + bx$, take HALF of b, square it, and add $(b/2)^2$, since $x^2 + bx + (b/2)^2 = (x + b/2)^2$. This converts a quadratic from standard form to vertex form $a(x - h)^2 + k$, exposing the vertex (h, k). When the leading coefficient a is not 1, factor a out of the x^2 and x terms first. To keep an equation balanced, whatever you add to complete the square must also be added to the other side (or subtracted back). Solving by completing the square ends with a plus-or-minus square root, giving two solutions.



Completing the square rewrites $x^2 + 8x + 10$ as $(x + 4)^2 - 6$, exposing the vertex $(-4, -6)$ directly.

Worked example

Rewrite $x^2 + 8x + 10$ in vertex form.

Read. We are rewriting $x^2 + 8x + 10$ in vertex form. The leading coefficient is 1, so we can complete the square directly on the $x^2 + 8x$ part.

Plan. Take half of the middle coefficient and square it: $(b/2)^2 = (8/2)^2 = 16$. Add 16 to build the perfect-square trinomial, and subtract it back in the same line so the expression's value is unchanged.

Solve.

$x^2 + 8x + 10 = (x^2 + 8x + 16) - 16 + 10$	add and subtract $(8/2)^2 = 16$
$x^2 + 8x + 10 = (x + 4)^2 - 16 + 10$	fold the perfect-square trinomial into $(x + 4)^2$
$x^2 + 8x + 10 = (x + 4)^2 - 6$	combine the constants $-16 + 10$

Check. Expanding returns the original: $(x + 4)^2 - 6 = x^2 + 8x + 16 - 6 = x^2 + 8x + 10$. The vertex $(-4, -6)$ can now be read off directly.

Answer: $(x + 4)^2 - 6$.

Approach Read · Plan · Solve · Check

Read. Am I rewriting an expression into vertex form, finding a vertex, or solving an equation by completing the square? Is the leading coefficient 1?

Plan. If a != 1, factor a from the x-terms first. Take $(b/2)^2$. Add it to form the perfect square $(x + b/2)^2$, compensating to stay balanced. Read the vertex (h, k); to solve, take the square root of both sides with plus-or-minus.

Solve. Form the perfect-square trinomial, fold it into $(x + b/2)^2$, and finish the conversion or the solve.

Check. Did I HALVE b before squaring? Did I keep the expression/equation balanced? Is the sign of h read as the opposite of the sign in the parenthesis? Did I handle a != 1?

Common traps — every wrong answer is one of these

half-then-square error.
Mishandles the completing constant: adds b^2 (squaring without halving), adds b/2 (halving without squaring), or adds b itself, instead of $(b/2)^2$.

balance error.
Adds $(b/2)^2$ to complete the square without compensating (not subtracting it back in an expression, or not adding it to both sides of an equation), producing a wrong constant or k.

vertex-sign/form error.
Reads the vertex with the wrong sign of h, e.g. taking $(x + 3)^2 + k$ to have vertex $x = 3$ instead of -3, or otherwise mis-signs the completed form.

leading-coefficient error.
When a != 1, fails to factor a out of the x^2 and x terms before completing the square, or mishandles a while doing so.

Practice

1. *You are completing the square for $x^2 + 6x$.*

Which constant should be added to form a perfect square?

(A) 9
(B) 3
(C) 36
(D) 6

2. *Rewrite $x^2 + 6x + 5$ in vertex form.*

Which is correct?

(A) $(x + 3)^2 - 4$
(B) $(x + 3)^2 + 4$
(C) $(x - 3)^2 - 4$
(D) $(x + 3)^2 + 5$

3. *Consider the expression $(x - 2)^2 + 7$.*

What is the vertex of its graph?

(A) (-2, 7)
(B) (2, -7)
(C) (2, 7)
(D) (-2, -7)

4. *You want to complete the square for $2x^2 + 8x + 3$.*

What is the correct first step?

(A) Divide every term by 3

(B) Add $(8/2)^2 = 16$ directly to the expression

(C) Add 16 to get $2x^2 + 8x + 16 + 3$

(D) Factor 2 from the x-terms: $2(x^2 + 4x) + 3$

5. *Solve $x^2 + 4x - 1 = 0$ by completing the square.*

What are the solutions?

(A) $x = -4 \pm \sqrt{5}$

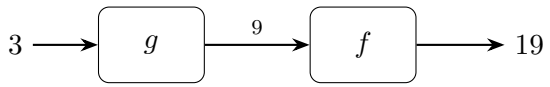
(B) $x = -2 \pm \sqrt{3}$

(C) $x = -2 \pm \sqrt{5}$

(D) $x = 2 \pm \sqrt{5}$

Composite functions & evaluating compositions

The skill. A composite f(g(x)) means: evaluate the INNER function g first, then feed that result into the OUTER function f. Work inside-out. Composition is substitution, not multiplication, so f(g(x)) is not f(x)*g(x). Order matters: f(g(x)) is generally NOT equal to g(f(x)). To evaluate from a table or graph, read the inner value first (g at the given input), then look up the outer function at that value.



$f(g(3))$ works inside-out: the inner machine gives $g(3) = 9$, then the outer gives $f(9) = 19$.

Worked example

Let $f(x) = 2x + 1$ and $g(x) = x^2$. Find $f(g(3))$.

Read. We want $f(g(3))$, a composition. The inner function is g and the outer is f, so we work inside-out: evaluate g at 3 first, then feed that result into f.

Plan. Compute the inner value $g(3)$, then substitute that number as the input of f. Composition is substitution, not multiplication, so we never form $f(x) \cdot g(x)$.

Solve.

$g(3) = 3^2$	evaluate the inner function at 3
$g(3) = 9$	this output becomes the input of f
$f(9) = 2(9) + 1$	substitute 9 into the outer function
$f(9) = 19$	simplify to the final value

Check. The inner function was done first, and each step substituted rather than multiplied. Feeding $g(3) = 9$ into f gives $f(9) = 19$, so $f(g(3)) = 19$.

Answer: 19.

Approach Read · Plan · Solve · Check

Read. Which is the inner function and which is the outer? Is this an algebraic composition, a numeric evaluation, or a table/graph lookup?

Plan. Evaluate the inner function at the input, then apply the outer function to that output. For algebraic forms, substitute the inner expression into the outer. Never multiply the two functions.

Solve. Carry out inner-then-outer; keep the order as written in f(g(x)).

Check. Did I do the inner function FIRST? Did I substitute rather than multiply? Did I respect that f(g(x)) and g(f(x)) can differ? For tables, did I read the correct function at each step?

Common traps — every wrong answer is one of these

order/inside-out error.
Applies the outer function first or works left-to-right, effectively computing g(f(x)) when f(g(x)) is asked, or stops after the inner step.

composition-as-multiplication error.
Treats f(g(x)) as the product f(x)*g(x) instead of substituting g(x) into f.

table/graph lookup error.
Reads the wrong function or the wrong value from a table or graph when evaluating a composite, e.g. looking up f at the original input instead of at the inner output.

commutativity error.
Assumes composition is commutative, taking f(g(x)) = g(f(x)) to be always true.

Practice

1. *Let $f(x) = x^2$ and $g(x) = x + 1$.*

What is f(g(3))?

(A) 9
(B) 4
(C) 16
(D) 10

2. *Let f(x) = x + 2 and g(x) = 3x.*

What is f(g(x))?

(A) (x + 2)(3x)
(B) $3x^2 + 6x$
(C) 3x + 2
(D) 3x + 6

3. *A table gives these values: g(1) = 4, g(4) = 5, f(1) = 2, f(4) = 7.*

What is f(g(1))?

(A) 5
(B) 2
(C) 7
(D) 4

4. *Let f(x) = x^2 and g(x) = x + 1.*

Is f(g(x)) equal to g(f(x))?

(A) Yes; both equal $x^2 + 1$
(B) No; f(g(x)) = $x^2 + 2x + 1$ while g(f(x)) = $x^2 + 1$
(C) Yes; both equal $x^2 + 2x + 1$
(D) Yes; composition is always commutative

5. *Let f(x) = 2x - 1 and g(x) = x^2.*

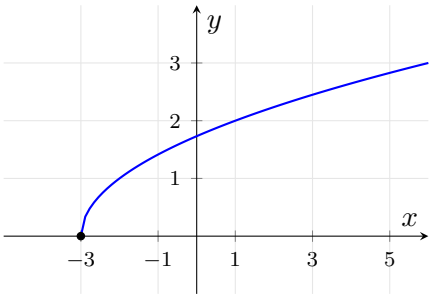
What is g(f(2))?

(A) 9
(B) 12
(C) 3
(D) 7

Domain & range of functions

The skill. The domain is the set of allowed inputs (x-values); the range is the set of possible outputs (y-values). A domain is all real numbers unless something restricts it: a denominator cannot be

zero (rational functions) and the quantity under an even root cannot be negative (radicals). The range often comes from a feature: a parabola y = a(x - h)^2 + k has range y >= k when it opens up and y <= k when it opens down, because the vertex is the extreme output. From a graph, the domain is the horizontal extent and the range is the vertical extent.



$f(x) = \sqrt{x+3}$ is defined only where $x + 3 \geq 0$; the graph begins at $x = -3$, so the domain is $x \geq -3$.

Worked example

Find the domain of $f(x) = \sqrt{x+3}$.

Read. We need the domain of $f(x) = \sqrt{x+3}$ – the set of inputs the rule allows. Because this is an even (square) root, the quantity under the radical cannot be negative.

Plan. Set the radicand greater than or equal to zero, $x + 3 \geq 0$, and solve the inequality for x; that inequality is the domain.

Solve.

$x + 3 \geq 0$	the radicand of an even root cannot be negative
$x \geq -3$	subtract 3 from both sides

Check. At the boundary $x = -3$, $\sqrt{0} = 0$ is defined, so -3 belongs to the domain. Just outside it, $x = -4$ gives $\sqrt{-1}$, which is undefined – confirming $x \geq -3$.

Answer: $x \geq -3$.

Approach Read · Plan · Solve · Check

Read. Is the question about inputs (domain) or outputs (range)? Is the function rational, radical, a parabola, or given as a graph?

Plan. Domain: start with all reals, then exclude denominator zeros and negative even-radicands. Range: read it from the vertex/extreme for a parabola, or the vertical extent of a graph.

Solve. Apply the relevant restriction or feature; state the answer in the correct variable.

Check. Did I report x-values for domain and y-values for range (not swapped)? Did I check the right source of restriction? For a parabola, did I bound the range at the vertex with the correct direction?

Common traps — every wrong answer is one of these

domain/range swap.
Confuses inputs and outputs: gives x-values for the range or y-values for the domain, or reads the wrong axis from a graph.

restriction-source error.
Identifies the domain restriction from the wrong place: uses numerator zeros instead of denominator zeros, forgets the radicand-nonnegative rule, or misses a restriction entirely.

range-from-vertex error.
For a parabola gives the range as all real numbers (ignoring the vertex bound) or uses the wrong direction (<= vs >=) or the wrong coordinate (h instead of k).

graph-reading extent error.
Misreads the domain or range extent from a graph: wrong endpoints, confusing open/closed, or reading the incorrect interval.

Practice

1. *Consider f(x) = 1/(x - 5).*

What is the domain?

(A) All real numbers except x = 5
(B) All real numbers
(C) All real numbers except x = 0
(D) All real numbers except x = -5

2. *A function accepts every real number as an input, and its outputs are always at least 2.*

What is its range?

(A) x >= 2
(B) y <= 2
(C) All real numbers
(D) y >= 2

3. *Consider $f(x) = (x - 3)^2 + 4$.*

What is the range?

(A) y >= 3
(B) y >= 4
(C) All real numbers
(D) y <= 4

4. *Consider* $f(x) = \sqrt{x-2}$.

What is the domain?

(A) x >= -2

(B) All real numbers

(C) x <= 2

(D) x >= 2

5. *A continuous curve is graphed; its points span x from -1 to 5 and y from 0 to 8.*

What are its domain and range?

(A) Domain -1 <= x <= 8; range 0 <= y <= 5

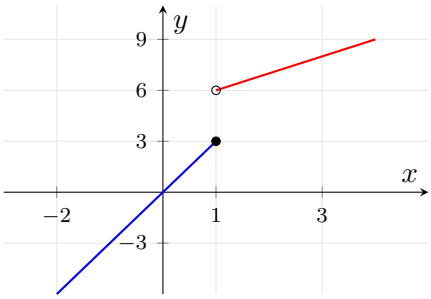
(B) Domain 0 <= x <= 8; range -1 <= y <= 5

(C) Domain -1 <= x <= 5; range 0 <= y <= 8

(D) Domain -1 <= y <= 5; range 0 <= x <= 8

Piecewise-defined functions

The skill. A piecewise function gives different rules over different intervals of the input. To evaluate it at a value, first find which interval the input falls in, then apply ONLY that interval's rule. At a boundary value, the inequalities decide which piece owns it (x >= 3 includes 3, x < 3 does not). Do not plug the input into every rule or combine the pieces; exactly one rule applies to each input.



A piecewise rule: $f(x) = 3x$ *on* $x \le 1$ *(closed dot) and* $f(x) = x + 5$ *on* $x > 1$ *(open dot); each input uses exactly one piece.*

Worked example

Let $f(x) = 3x$ **for** $x \le 1$ **and** $f(x) = x + 5$ **for** $x > 1$**. Find** $f(4)$**.**

Read. This is a piecewise function with two rules. To find $f(4)$ we must first decide which interval the input 4 falls into, then use only that piece.

Plan. Compare the input to the boundary 1. Since $4 > 1$, the input belongs to the second piece, so we apply only the rule $x + 5$ and ignore the first.

Solve.

$f(4) = 4 + 5$	use the second rule $x + 5$, since $4 > 1$
$f(4) = 9$	simplify

Check. The input 4 is not ≤ 1, so the first rule $3x$ does not apply. Only one piece was used, as a piecewise evaluation requires.

Answer: 9.

Approach Read · Plan · Solve · Check

Read. What is the input, and which interval contains it? Is the input at a boundary where the inequality matters?

Plan. Locate the input on the number line of intervals, pick the single matching rule, and evaluate. At a boundary, read the inequality (<= vs <) to see which piece owns the endpoint.

Solve. Apply only the selected rule.

Check. Did I pick the one correct branch? At a boundary, did the inequality decide ownership? Did I avoid evaluating all pieces or combining them?

Common traps — every wrong answer is one of these

wrong-branch selection.
Applies the rule for the wrong interval to the given input.

boundary/endpoint error.
At a boundary value, assigns the endpoint to the wrong piece (misreading <= vs <), or treats it as undefined or as belonging to both pieces.

evaluate-all-pieces error.
Plugs the input into every rule and reports multiple values, or adds/combines the pieces, instead of selecting one.

interval-misread error.
Misreads an interval condition (e.g. x > 0 vs x < 0), selecting the branch by a flipped inequality.

Practice

1. *Let f(x) = 2x + 1 for x < 3, and f(x) = x^2 for x >= 3.*

What is f(1)?

(A) 3

(B) Both 3 and 1

(C) 1

(D) Undefined

2. *Let f(x) = 2x + 1 for x < 3, and f(x) = x^2 for x >= 3.*

What is f(4)?

(A) 16
(B) 8
(C) 9
(D) Both 9 and 16

3. *Let f(x) = 2x + 1 for x < 3, and f(x) = x^2 for x >= 3.*

What is f(3)?

(A) Undefined
(B) 9
(C) Both 7 and 9
(D) 7

4. *Let g(x) = 5 for x > 0, g(x) = -5 for x < 0, and g(x) = 0 for x = 0.*

What is g(-2)?

(A) 0
(B) Both 5 and -5
(C) 5
(D) -5

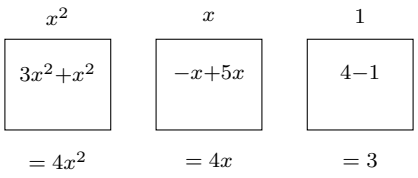
5. *Let f(x) = 2x + 1 for x < 3, and f(x) = x^2 for x >= 3.*

What is f(2) + f(5)?

(A) 29
(B) 45
(C) 30
(D) 16

Polynomial arithmetic & structure

The skill. A polynomial is a sum of terms a*x^n; its degree is the highest exponent and its leading coefficient is the coefficient of that term. Add or subtract by combining only LIKE terms (same exponent); when subtracting a polynomial, distribute the minus to every term. Multiply by distributing each term times each term, adding exponents (product rule) and combining like terms; the degree of a product is the sum of the degrees.



Adding polynomials: only like terms (the same power of x) combine, column by column, giving $4x^2 + 4x + 3$.

Worked example

Simplify $(3x^2 - x + 4) + (x^2 + 5x - 1)$**.**

Read. We are adding two polynomials, $(3x^2 - x + 4)$ and $(x^2 + 5x - 1)$. Only like terms – terms with the same power of x – may be combined.

Plan. Group the terms by degree, then add the coefficients within each group: the x^2 terms together, the x terms together, and the constants together.

Solve.

$(3x^2 - x + 4) + (x^2 + 5x - 1) = (3x^2 + x^2) + (-x + 5x) + (4 - 1)$ group like terms by power of x

$(3x^2 - x + 4) + (x^2 + 5x - 1) = 4x^2 + 4x + 3$ add the coefficients in each group

Check. Only like terms were combined, and the degree stays 2 (no exponents were added). The three groups give $4x^2$, $4x$, and 3.

Answer: $4x^2 + 4x + 3$.

Approach Read · Plan · Solve · Check

Read. Identify the operation and the degree of each term; note which terms are like (same exponent).

Plan. Add/subtract: combine like terms only, distributing any subtraction across all terms. Multiply: every term times every term, adding exponents, then combine.

Solve. Carry out the operation, tracking signs and exponents.

Check. Only like terms combined? Minus distributed to all terms? Exponents ADDED (not multiplied) when multiplying? Degree of product = sum of degrees?

Common traps — every wrong answer is one of these

combining unlike terms.
Combines terms of different degrees (e.g. $3x^2 + 5x = 8x$), or adds exponents when adding like terms ($3x^2 + 2x^2 = 5x^4$).

subtraction sign distribution.
When subtracting a polynomial, negates only the first term instead of distributing the minus to every term.

multiplication/exponent error.
Multiplies exponents instead of adding when multiplying terms ($x^2 * x^3 = x^6$), adds coefficients instead of multiplying, or fails to multiply every pair of terms.

degree/leading-coefficient error.
Misidentifies the degree (e.g. counts the number of terms, or takes the first term's exponent rather than the highest) or the leading coefficient.

Practice

1. *Simplify $3x^2 + 5x + 2x^2 - x$.*

What is the result?

(A) 9x
(B) $5x^3 + 4x$
(C) $5x^2 + 4x$
(D) $5x^2 + 6x$

2. *Simplify $(4x^2 + 3x - 2) - (x^2 - 2x + 5)$.*

What is the result?

(A) $3x^2 + x + 3$
(B) $5x^2 + x - 7$
(C) $3x^2 + 5x + 3$
(D) $3x^2 + 5x - 7$

3. *Multiply $2x^2 * 3x^3$.*

What is the result?

(A) 6x
(B) $6x^6$
(C) $5x^5$
(D) $6x^5$

4. *Consider the polynomial $7x^3 - 2x^5 + 4x - 1$.*

What is its degree?

(A) 4
(B) 5
(C) 3

(D) 7

5. *Expand (x + 2)(x² - 3x + 1).*

What is the result?

(A) $x^3 - x^2 - 5x + 2$

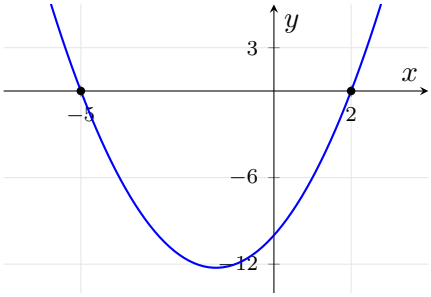
(B) $x^3 - x^2 + 7x + 2$

(C) $x^3 - 3x^2 + x + 2$

(D) $x^3 + x^2 - 5x + 2$

Zeros, factors & the factor theorem

The skill. The factor theorem: (x - a) is a factor of a polynomial P(x) if and only if a is a zero, meaning P(a) = 0. Zeros are the x-intercepts of the graph. From a factor (x - a), the zero is +a (set the factor to 0); from a zero a, the factor is (x - a). A polynomial of degree n has AT MOST n real zeros; a repeated factor counts as one distinct zero (with multiplicity).



$P(x) = (x+5)(x-2)$: each factor gives a zero of opposite sign – $x = -5$ and $x = 2$ – where the graph crosses the x-axis.

Worked example

Find the zeros of $P(x) = (x+5)(x-2)$.

Read. $P(x) = (x+5)(x-2)$ is already factored. Its zeros are the inputs that make $P(x) = 0$, and a product is zero exactly when one of its factors is zero.

Plan. Set each factor equal to zero and solve. Each zero comes out with the opposite sign of the constant inside its factor.

Solve.

$x + 5 = 0$	set the first factor equal to zero
$x = -5$	subtract 5: opposite sign of the $+5$
$x - 2 = 0$	set the second factor equal to zero
$x = 2$	add 2: opposite sign of the -2

Check. Each root makes a factor vanish: $P(-5) = 0 \cdot (-7) = 0$ and $P(2) = 7 \cdot 0 = 0$. Both

are genuine zeros.

Answer: $x = -5$ and $x = 2$.

Approach Read · Plan · Solve · Check

Read. Decide whether you are going from factors to zeros, zeros to factors, or using P(a) = 0.

Plan. Set each factor equal to zero to get a zero (opposite sign of the number in the factor); for a known zero a, the factor is (x - a). Bound the count of zeros by the degree.

Solve. Solve each factor = 0, or assemble factors from zeros; count distinct zeros.

Check. Zero has the OPPOSITE sign of the factor's constant? P(a) = 0 means (x - a) is a factor (not that a is a factor)? At most (degree) zeros?

Common traps — every wrong answer is one of these

factor/zero sign error.
Reads the zero of (x - a) as -a (or writes the factor of zero a as (x + a)), flipping the sign between factor and zero.

factor-theorem meaning error.
Misinterprets P(a) = 0: thinks a itself is a factor, confuses the value with the factor, or misreads what a zero means.

number-of-zeros error.
Believes a degree-n polynomial can have more than n real zeros, or counts a repeated (multiplicity) root as several distinct zeros.

Practice

1. *P(x) = (x - 4)(x + 2).*

What are the zeros?

(A) x = 4 and x = -2

(B) x = 4 and x = 2

(C) x = -4 and x = -2

(D) x = -4 and x = 2

2. *3 is a zero of the polynomial P(x).*

Which must be a factor of P(x)?

(A) 3

(B) (x + 3)

(C) (x - 3)

(D) 3x

3. *For a polynomial P(x), it is known that P(2) = 0.*

What does this tell you?

(A) (x - 2) is a factor of P(x)

(B) P(x) has no real zeros

(C) (x + 2) is a factor of P(x)

(D) 2 is a factor of P(x)

4. *Consider a polynomial of degree 4.*

What is the maximum number of real zeros it can have?

(A) 8

(B) Infinitely many

(C) 3

(D) 4

5. $P(x) = (x - 1)^2 (x + 3)$.

What are the distinct real zeros?

(A) x = -1 and x = 3

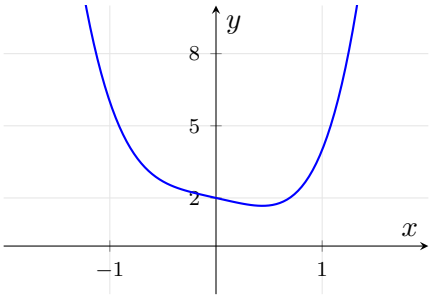
(B) x = 1, x = 1, and x = -3

(C) x = 1 and x = 3

(D) x = 1 and x = -3

Polynomial graphs: end behavior, zeros, multiplicity

The skill. A polynomial's end behavior is set by two things together: the parity of its degree (odd degrees have opposite-direction tails, even degrees have same-direction tails) and the sign of its leading coefficient. The real zeros are the x-intercepts, the x-values where the polynomial equals zero, read from the factors. At a zero with even multiplicity the graph TOUCHES the x-axis and turns back; at odd multiplicity it CROSSES. A degree-n polynomial has at most n real zeros, possibly fewer once repeated or complex roots are counted.



$f(x) = 3x^4 - x + 2$: an even degree with a positive leading coefficient makes both tails rise as $x \to \pm\infty$.

Worked example

Describe the end behavior of $f(x) = 3x^4 - x + 2$.

Read. End behavior is set by the leading term alone. For $f(x) = 3x^4 - x + 2$ that term is $3x^4$: the degree is 4 (even) and the leading coefficient is 3 (positive).

Plan. Combine the two facts. An even degree makes the two tails point the same direction; a positive leading coefficient sends that shared direction upward. So both ends should rise.

Solve.

$f(x) \to +\infty$	right tail: as the input grows large and positive
$f(x) \to +\infty$	left tail: even degree gives a matching direction

Check. The even degree forces same-direction tails and the positive lead makes them upward, so the graph rises on both the left and the right – consistent with both limits going to $+\infty$.

Answer: rises on both the left and the right.

Approach Read · Plan · Solve · Check

Read. Is the question about the tails (end behavior), the x-intercepts (zeros), or behavior at a repeated zero (multiplicity)?

Plan. End behavior: combine degree parity and leading-coefficient sign. Zeros: set each factor to zero and read x-values. Multiplicity: even -> touch, odd -> cross. Count: at most degree-many real zeros.

Solve. Apply the relevant rule; for zeros solve each factor; for tails check parity and sign.

Check. Did I use BOTH parity and sign for the tails? Are the zeros x-values (not y)? Did I check multiplicity for touch vs cross? Is the zero count at most the degree?

Common traps — every wrong answer is one of these

end-behavior error.
Gets the tails wrong by using only the leading-coefficient sign or only the degree, instead of combining degree parity and sign; e.g. assuming a positive leading coefficient means both ends rise.

zeros-as-x-intercepts error.
Reads zeros with flipped signs straight off the factor constants, calls them y-values, or otherwise misidentifies the x-intercepts.

multiplicity error.
Treats every zero as a simple crossing, missing that an even-multiplicity factor makes the graph touch and turn back rather than cross.

degree-vs-zero-count error.
Assumes a degree-n polynomial must have exactly n distinct real zeros (or must cross the axis n times), ignoring repeated and complex roots.

Practice

1. *Consider $f(x) = 2x^3 - 5x + 1$.*

How does the graph behave at its far left and far right?

(A) Rises on both ends
(B) Falls on both ends
(C) Rises to the left, falls to the right
(D) Falls to the left, rises to the right

2. *Let $f(x) = (x - 2)(x + 3)(x - 1)$.*

What are the real zeros?

(A) y = 2, y = -3, and y = 1
(B) x = -2, x = 3, and x = -1
(C) x = 2, x = -3, and x = 1
(D) x = 6

3. *Let $f(x) = (x - 1)^2 (x + 2)$.*

What does the graph do at x = 1?

(A) Touches the x-axis and turns back without crossing
(B) Never reaches the x-axis
(C) Has a vertical asymptote
(D) Crosses the x-axis

4. *Consider $f(x) = -x^4 + 3x^2$.*

What is the end behavior?

(A) Falls on both the left and the right

(B) Rises on both ends

(C) Rises to the left, falls to the right

(D) Falls to the left, rises to the right

5. *A polynomial has degree 4.*

Which statement must be true?

(A) Its graph must cross the x-axis 4 times

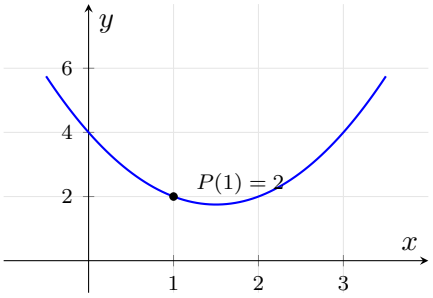
(B) It has at most 4 real zeros

(C) It has exactly 4 distinct real zeros

(D) All 4 of its zeros are real

Polynomial division & the remainder theorem

The skill. Dividing a polynomial P(x) by (x - a) gives P(x) = (x - a)Q(x) + r, where the remainder r is a constant. The remainder theorem says r = P(a), so you can find the remainder by evaluating P at a rather than dividing. The factor theorem is the special case: (x - a) is a factor exactly when P(a) = 0, i.e. the remainder is zero. When doing long or synthetic division, include a zero placeholder for every missing degree, and remember the remainder's degree is less than the divisor's.



Remainder theorem: dividing $P(x) = x^2 - 3x + 4$ by $(x - 1)$ leaves remainder $P(1) = 2$ – just the height of the graph at $x = 1$.

Worked example

Find the remainder when $P(x) = x^2 - 3x + 4$ is divided by $(x - 1)$.

Read. We want the remainder when $P(x) = x^2 - 3x + 4$ is divided by $(x - 1)$. The divisor $(x - 1)$ has root $a = 1$, which is exactly what the remainder theorem uses.

Plan. The remainder theorem says the remainder of division by $(x - a)$ equals $P(a)$. So instead of long division we simply evaluate $P(1)$.

Solve.

$P(1) = 1^2 - 3(1) + 4$	substitute $x = 1$ (the root of the divisor)
$P(1) = 1 - 3 + 4$	evaluate each term
$P(1) = 2$	the remainder

Check. The remainder is 2, which is not 0, so $(x - 1)$ is not a factor of $P(x)$ – the division does not come out even.

Answer: 2.

Approach Read · Plan · Solve · Check

Read. Is the task to find a remainder, test a factor, carry out the division, or solve for an unknown using a factor condition?

Plan. Remainder of division by (x - a): evaluate P(a). Factor test: check whether P(a) = 0. Long/synthetic division: use placeholders for missing terms. Unknown constant: set P(a) = 0 when (x - a) is a factor.

Solve. Evaluate or divide accordingly; keep track of the sign of a from the divisor (x - a).

Check. Did I use a (not -a) from (x - a)? Is a zero remainder the same as a factor? Did I keep placeholders for missing degrees? Did I interpret a nonzero remainder correctly?

Common traps — every wrong answer is one of these

remainder-theorem sign error.
Evaluates P(-a) instead of P(a) when dividing by (x - a), or otherwise uses the wrong value for the divisor's root.

remainder-vs-factor direction error.
Confuses the root with the remainder, or the direction of the factor theorem: e.g. taking P(a) = 0 to mean a is the remainder, or flipping the sign of the factor.

division mechanics error.
Drops terms by omitting zero placeholders for missing degrees, or makes a subtraction-sign slip during long/synthetic division, producing a wrong quotient.

nonzero-remainder interpretation error.
Ignores a nonzero remainder (claims exact division or a factor when r is not 0), or misreads the remainder as a root or quotient.

Practice

1. *Let $P(x) = x^2 + 2x + 5$.*

What is the remainder when P(x) is divided by (x - 2)?

(A) 5 (the constant term)

(B) -13

(C) 5

(D) 13

2. *For a polynomial P, it is known that P(3) = 0.*

Which statement is true?

(A) (x - 3) is a factor of P

(B) 3 is the remainder when P is divided by (x - 3)

(C) (x + 3) is a factor of P

(D) P has no real zeros

3. *Divide x^3 - 8 by (x - 2).*

What is the quotient?

(A) x^2 - 2x + 4

(B) x^2 - 8

(C) x^2 + 2x + 4

(D) x^2 + 4

4. *Dividing $P(x) = x^2 + 1$ by (x - 1) leaves a remainder of 2.*

Which statement is true?

(A) (x - 1) is a factor of P

(B) (x - 1) is not a factor of P, since the remainder is not 0

(C) The remainder means x = 2 is a zero of P

(D) P(x) = (x - 1)(x + 1) exactly

5. *Let $P(x) = 2x^3 - x + k$, and suppose (x - 1) is a factor of P.*

What is the value of k?

(A) k = 1

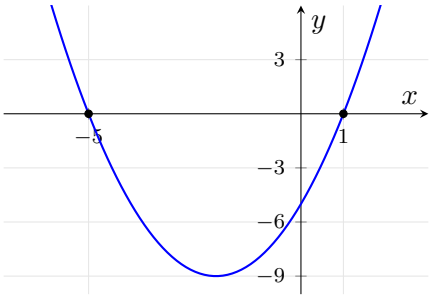
(B) k = 0

(C) k = -3

(D) k = -1

Synthetic division & the quotient

The skill. If x = r is a root of a quadratic, then (x - r) is a factor and the quotient is the other factor. For x^2 + bx - (rs) = (x - r)(x + s), dividing by (x - r) leaves the quotient x + s.



Dividing $x^2 + 4x - 5$ by the factor $(x - 1)$ (root $x = 1$) leaves the quotient $x + 5$, whose root $x = -5$ is the other intercept.

Worked example

Divide $x^2 + 4x - 5$ by $(x - 1)$, given that $x = 1$ is a root. What is the quotient?

Read. We divide $x^2 + 4x - 5$ by $(x - 1)$, and we are told $x = 1$ is a root. By the factor theorem, that makes $(x - 1)$ a factor, so the division comes out even with no remainder.

Plan. Since $(x - 1)$ is one factor, factor the quadratic completely; the other factor is the quotient. Look for two numbers that multiply to -5 and add to 4: they are -1 and 5.

Solve.

$$x^2 + 4x - 5 = (x - 1)(x + 5) \qquad \text{factor: } -1 \text{ and } 5 \text{ multiply to } -5, \text{ add to } 4$$

$$\text{quotient} = x + 5 \qquad \text{the factor other than the divisor } (x - 1)$$

Check. Multiplying back confirms it: $(x - 1)(x + 5) = x^2 + 5x - x - 5 = x^2 + 4x - 5$, the original polynomial.

Answer: $x + 5$.

Approach Read · Plan · Solve · Check

Read. x = r is a root, so (x - r) is a factor.

Plan. Factor to find the OTHER factor (the quotient).

Solve. (x - r)(x + s) -> quotient x + s.

Check. Does (x - r)(quotient) rebuild the polynomial? Right sign on s?

Common traps — every wrong answer is one of these

used the divisor.
Reports x + r (the divisor's number) as the quotient.

sign error.
Reports x - s instead of x + s.

used the coefficient.
Uses the middle coefficient b in the quotient.

Practice

1. *Divide (x^2 + 1x - 6) by (x - 2), given that x = 2 is a root. What is the quotient?*

Which is correct?

(A) x + 2
(B) x + 3
(C) x + 1
(D) x - 3

2. *Divide (x^2 + 2x - 15) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 5
(B) x + 5
(C) x + 3
(D) x + 2

3. *Divide (x^2 + 1x - 12) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 4
(B) x + 4
(C) x + 1
(D) x + 3

4. *Divide (x^2 + 3x - 10) by (x - 2), given that x = 2 is a root. What is the quotient?*

Which is correct?

(A) x - 5
(B) x + 2
(C) x + 5
(D) x + 3

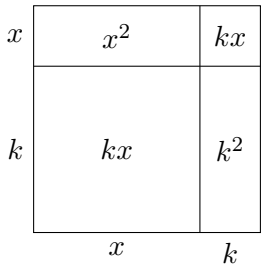
5. *Divide (x^2 + 2x - 15) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 5
(B) x + 5
(C) x - 3
(D) x + 2

Binomial expansion (perfect squares)

The skill. $(x + k)^2 = x^2 + 2kx + k^2$. Squaring a binomial produces a middle term $2kx$ (from the two cross products) — it is not just $x^2 + k^2$. The constant is k^2, not k.



$(x + k)^2$ as a square: the two off-diagonal kx tiles combine into the middle term $2kx$, so $(x + k)^2 = x^2 + 2kx + k^2$.

Worked example

Expand $(x + 6)^2$.

Read. We are squaring the binomial $(x + 6)$, so this fits the perfect-square pattern with $k = 6$.

Plan. Apply $(x + k)^2 = x^2 + 2kx + k^2$. The middle term is twice the cross product, $2kx$, and the constant is k^2 – both are easy to forget.

Solve.

$(x + 6)^2 = x^2 + 2(6)x + 6^2$	apply $(x + k)^2 = x^2 + 2kx + k^2$ with $k = 6$
$(x + 6)^2 = x^2 + 12x + 36$	compute $2(6) = 12$ and $6^2 = 36$

Check. The middle term is $2 \cdot 6 = 12$ (not 6) and the constant is $6^2 = 36$ (not 6), so both easy slips are avoided.

Answer: $x^2 + 12x + 36$.

Approach Read · Plan · Solve · Check

Read. A binomial squared: $(x + k)(x + k)$.

Plan. FOIL; the two middle terms combine to $2kx$.

Solve. $x^2 + 2kx + k^2$.

Check. Did I include the $2kx$ middle term? Is the constant k^2?

Common traps — every wrong answer is one of these

dropped the middle term.
Writes $x^2 + k^2$, omitting $2kx$.

did not double.
Uses kx instead of $2kx$ for the middle term.

did not square the constant.
Leaves k instead of k^2 as the constant.

Practice

1. *Expand $(x + 2)^2$.*

Which is correct?

(A) $x^2 + 4x + 2$
(B) $x^2 + 4$
(C) $x^2 + 2x + 4$
(D) $x^2 + 4x + 4$

2. *Expand $(x + 5)^2$.*

Which is correct?

(A) $x^2 + 25$
(B) $x^2 + 10x + 5$
(C) $x^2 + 5x + 25$
(D) $x^2 + 10x + 25$

3. *Expand $(x + 3)^2$.*

Which is correct?

(A) $x^2 + 3x + 9$
(B) $x^2 + 6x + 3$
(C) $x^2 + 9$
(D) $x^2 + 6x + 9$

4. *Expand $(x + 4)^2$.*

Which is correct?

(A) $x^2 + 8x + 4$
(B) $x^2 + 4x + 16$
(C) $x^2 + 16$
(D) $x^2 + 8x + 16$

5. *Expand $(x + 5)^2$.*

Which is correct?

(A) $x^2 + 5x + 25$

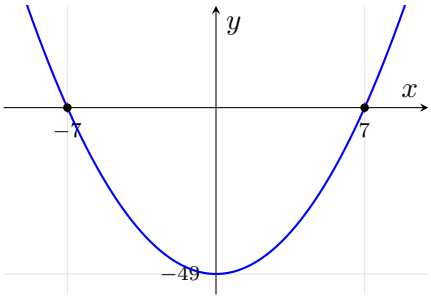
(B) $x^2 + 25$

(C) $x^2 + 10x + 25$

(D) $x^2 + 10x + 5$

Special products & factoring patterns

The skill. $(x + k)(x - k) = x^2 - k^2$ (difference of squares): the two middle terms cancel, leaving no x-term and a MINUS sign on the constant. It is not $(x - k)^2$, which would have a middle term.



$(x+7)(x-7) = x^2 - 49$: the conjugate pair's middle terms cancel, giving a difference of squares with roots ± 7.

Worked example

Multiply $(x+7)(x-7)$.

Read. The two binomials $(x+7)$ and $(x-7)$ are a conjugate pair – same terms, opposite signs – with $k = 7$. This is the difference-of-squares pattern.

Plan. Use $(x+k)(x-k) = x^2 - k^2$. When we FOIL, the outer and inner terms are opposites, so the middle cancels and only $x^2 - k^2$ remains.

Solve.

$(x+7)(x-7) = x^2 - 7x + 7x - 49$	FOIL the conjugate pair
$(x+7)(x-7) = x^2 - 49$	the middle terms $-7x + 7x$ cancel

Check. The middle term cancels as expected, leaving no x-term, and the constant is $-k^2 = -49$ (with a minus sign, not $+49$).

Answer: $x^2 - 49$.

Approach Read · Plan · Solve · Check

Read. Conjugate pair (x + k)(x - k).

Plan. FOIL; the middle terms cancel.

Solve. $x^2 - k^2$.

Check. Did the middle cancel? Is it minus k^2?

Common traps — every wrong answer is one of these

wrong sign.
Writes $x^2 + k^2$ instead of $x^2 - k^2$.

kept a middle term.
Treats it like $(x - k)^2$, adding a $-2kx$ term.

did not square k.
Leaves $-k$ instead of $-k^2$.

Practice

1. *Multiply $(x + 4)(x - 4)$.*

Which is correct?

(A) $x^2 - 8x + 16$
(B) $x^2 - 4$
(C) $x^2 - 16$
(D) $x^2 + 16$

2. *Multiply $(x + 5)(x - 5)$.*

Which is correct?

(A) $x^2 + 25$
(B) $x^2 - 25$
(C) $x^2 - 5$
(D) $x^2 - 10x + 25$

3. *Multiply $(x + 2)(x - 2)$.*

Which is correct?

(A) $x^2 + 4$
(B) $x^2 - 4$
(C) $x^2 - 4x + 4$
(D) $x^2 - 2$

4. *Multiply $(x + 6)(x - 6)$.*

Which is correct?

(A) $x^2 - 6$
(B) $x^2 + 36$

(C) x^2 - 12x + 36

(D) x^2 - 36

5. *Multiply (x + 6)(x - 6).*

Which is correct?

(A) x^2 - 6

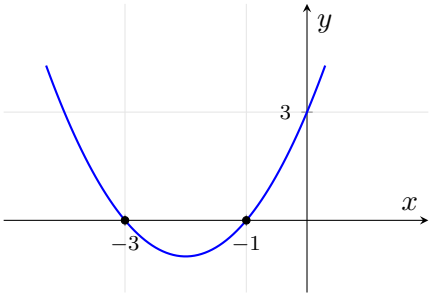
(B) x^2 - 36

(C) x^2 - 12x - 36

(D) x^2 + 36

Sum & product of roots

The skill. For x^2 + bx + c, the sum of the roots is -b and the product is c (Vieta's formulas). The sum takes the NEGATIVE of the middle coefficient; the product is the constant term as-is.



For $x^2 + 4x + 3$ the roots -1 and -3 (the intercepts) satisfy Vieta's formulas: sum $= -4 = -b$, product $= 3 = c$.

Worked example

For $x^2 + 4x + 3$, find the sum and product of its roots.

Read. For $x^2 + 4x + 3$ we read off $b = 4$ and $c = 3$. Vieta's formulas relate these coefficients directly to the sum and product of the roots.

Plan. Vieta's formulas for $x^2 + bx + c$ give the sum of the roots as $-b$ (note the negative) and the product as c (taken as-is). Substitute $b = 4$ and $c = 3$.

Solve.

$\text{sum} = -b$	Vieta: the sum of the roots is $-b$
$\text{sum} = -4$	substitute $b = 4$
$\text{product} = c$	Vieta: the product of the roots is c
$\text{product} = 3$	substitute $c = 3$

Check. Factoring gives roots -1 and -3. Their sum is $-1 + (-3) = -4$ and their product is $(-1)(-3) = 3$, matching both formulas.

Answer: sum = −4, product = 3.

Approach Read · Plan · Solve · Check

Read. Use Vieta's: sum = -b, product = c.

Plan. Read b and c; negate b for the sum.

Solve. sum = -b, product = c.

Check. Did I negate b? Is the product c (not -c)?

Common traps — every wrong answer is one of these

sum sign error.
Uses +b instead of -b for the sum.

product sign error.
Uses -c instead of c for the product.

swapped sum & product.
Reports the constant as the sum and the coefficient as the product.

Practice

1. *For the quadratic $x^2 + 3x + 2$, what are the sum and product of its roots?*

Which is correct?

(A) sum = -3, product = 2

(B) sum = 2, product = 3

(C) sum = -3, product = -2

(D) sum = 3, product = 2

2. *For the quadratic $x^2 + 5x + 6$, what are the sum and product of its roots?*

Which is correct?

(A) sum = 6, product = 5

(B) sum = -5, product = -6

(C) sum = 5, product = 6

(D) sum = -5, product = 6

3. *For the quadratic $x^2 + 7x + 10$, what are the sum and product of its roots?*

Which is correct?

(A) sum = 10, product = 7

(B) sum = -7, product = -10

(C) sum = 7, product = 10

(D) sum = -7, product = 10

4. *For the quadratic $x^2 + 7x + 2$, what are the sum and product of its roots?*

Which is correct?

(A) sum = -7, product = 2

(B) sum = 2, product = 7

(C) sum = 7, product = 2

(D) sum = -7, product = -2

5. *For the quadratic $x^2 + 5x + 6$, what are the sum and product of its roots?*

Which is correct?

(A) sum 5, product 6

(B) sum -5, product 6

(C) sum -5, product -6

(D) sum 6, product -5

Exponent rules & properties

The skill. Exponent rules: multiply same-base powers by ADDING exponents ($x^a * x^b = x^(a+b)$); divide by SUBTRACTING ($x^a / x^b = x^(a-b)$); raise a power to a power by MULTIPLYING ($(x^a)^b = x^(ab)$). A power of a product distributes: $(xy)^a = x^a * y^a$, and a coefficient is raised too: $(2x)^3 = 8x^3$. Also $x^0 = 1$, $x^(-a) = 1/x^a$ (reciprocal, not negative), and $x^(1/n)$ is the nth root.

$$\frac{x^5 \cdot x^2}{x^3} = x^{5+2-3} = x^4$$

Same-base powers: multiply by adding exponents and divide by subtracting, so $5 + 2 - 3 = 4$ gives x^4.

Worked example

Simplify $\frac{x^5 \cdot x^2}{x^3}$.

Read. The expression $\frac{x^5 \cdot x^2}{x^3}$ has a single base x throughout: a multiplication in the numerator, then a division by x^3.

Plan. Use the exponent rules for a common base: add exponents to multiply, then subtract the denominator's exponent to divide.

Solve.

$$\frac{x^5 \cdot x^2}{x^3} = \frac{x^{5+2}}{x^3}$$ multiply the numerator by adding exponents

$$\frac{x^5 \cdot x^2}{x^3} = \frac{x^7}{x^3}$$ simplify $5 + 2 = 7$

$$\frac{x^5 \cdot x^2}{x^3} = x^{7-3}$$ divide by subtracting the exponent

$$\frac{x^5 \cdot x^2}{x^3} = x^4$$ simplify $7 - 3 = 4$

Check. Combining all three exponents at once gives the same result: $5 + 2 - 3 = 4$, so the simplified form is x^4.

Answer: x^4.

Approach Read · Plan · Solve · Check

Read. Identify the operation: are same-base powers multiplied, divided, or nested (power of a power)?

Plan. Add exponents to multiply, subtract to divide, multiply to nest. Apply powers to coefficients too. Convert negative exponents to reciprocals and fractional exponents to roots.

Solve. Apply the matching rule; do not distribute an exponent over addition.

Check. Add vs multiply correct? Did the coefficient get the power? Negative exponent -> reciprocal (not a negative value)? $x^0 = 1$?

Common traps — every wrong answer is one of these

add/multiply exponent confusion.
Multiplies exponents when they should be added (product rule), or adds when they should be multiplied (power rule), or subtracts incorrectly.

negative-exponent error.
Treats $x^{(-a)}$ as a negative value ($-x^a$) instead of the reciprocal $1/x^a$, or drops the negative entirely.

zero/fractional-exponent error.
Takes x^0 as 0 (instead of 1), or mishandles a fractional exponent (e.g. treating $x^{(1/2)}$ as $x/2$ or as division).

coefficient/distribution error.
Fails to raise a coefficient to the power (e.g. $(2x)^3 = 2x^3$), or distributes an exponent over addition ($(x+y)^2 = x^2 + y^2$).

Practice

1. *Simplify $x^3 * x^4$.*

What is the result?

(A) x
(B) $2x^7$
(C) x^{12}
(D) x^7

2. *Simplify $(x^3)^4$.*

What is the result?

(A) x^7
(B) x^{12}
(C) x^{81}
(D) $4x^3$

3. *Rewrite x^(-2) without a negative exponent.*

What is the result?

(A) $-x^2$
(B) -2x
(C) $1/x^2$
(D) x^2

4. *Evaluate 5^0 + 9^(1/2).*

What is the value?

(A) 3
(B) 4
(C) 5.5
(D) 14

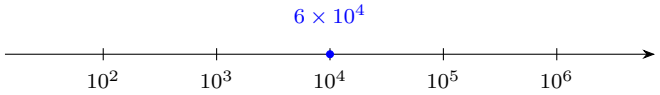
5. *Simplify $(2x^2)^3$.*

What is the result?

(A) $8x^6$
(B) $8x^5$
(C) $2x^6$
(D) $6x^6$

Scientific notation

The skill. Scientific notation writes a number as a x 10^{n}, where the coefficient satisfies 1 <= |a| < 10 and n is an integer. Numbers larger than 10 use a positive exponent; numbers between 0 and 1 use a negative exponent; the exponent counts how many places the decimal moves. To multiply, multiply the coefficients and ADD the exponents; to divide, divide the coefficients and SUBTRACT the exponents; then renormalize so the coefficient is back in the 1-to-10 range. To compare magnitudes, compare exponents first.



Scientific notation places a number by its power of ten: multiplying $(3 \times 10^{-2})(2 \times 10^{6})$ adds the exponents, landing at 6×10^{4}.

Worked example

Compute $(3 \times 10^{-2})(2 \times 10^{6})$.

Read. We multiply two numbers written in scientific notation, 3×10^{-2} and 2×10^{6}. Each is a coefficient times a power of ten.

Plan. Multiply the two coefficients together and add the two powers of ten. Then check that the resulting coefficient stays in the range $1 \le |a| < 10$; renormalize if it does not.

Solve.

$$(3 \times 10^{-2})(2 \times 10^{6}) = (3 \cdot 2) \times 10^{-2+6} \qquad \text{multiply coefficients, add exponents}$$

$$(3 \times 10^{-2})(2 \times 10^{6}) = 6 \times 10^{4} \qquad \text{simplify } 3 \cdot 2 = 6 \text{ and } -2+6 = 4$$

Check. The coefficient 6 already lies in $[1, 10)$, so no renormalizing is needed. The exponents were added $(-2 + 6 = 4)$, not multiplied, giving 6×10^{4}.

Answer: 6×10^{4}.

Approach Read · Plan · Solve · Check

Read. Is the task to convert, to operate (multiply/divide), to normalize, or to compare magnitudes?

Plan. Convert: move the decimal n places (right if n > 0, left if n < 0). Operate: combine coefficients, add/subtract exponents. Normalize: adjust so 1 <= |a| < 10, shifting the exponent. Compare: largest exponent wins.

Solve. Carry out the move or the exponent arithmetic, then renormalize.

Check. Is the coefficient between 1 and 10? Is the exponent's SIGN right for the size of the number? Did I ADD exponents when multiplying (not multiply them)? Did I count decimal places correctly?

Common traps — every wrong answer is one of these

decimal place/count error.
Moves the decimal the wrong number of places when converting between standard and scientific notation, or is off by one or more places.

exponent-sign error.
Uses a positive power of ten for a number less than 1 (or a negative power for a large number), reversing the sign of the exponent.

operation exponent error.
When multiplying or dividing numbers in scientific notation, multiplies the exponents (or otherwise mishandles them) instead of adding when multiplying and subtracting when dividing.

normalization error.
Leaves the coefficient outside the 1-to-10 range, or fails to adjust the exponent after the coefficient product or quotient spills over or under that range.

Practice

1. *Consider the number 3.2 x 10^4.*

What is this in standard form?

(A) 3,200
(B) 32,000
(C) 0.00032
(D) 320

2. *Consider the number 0.00056.*

What is this in scientific notation?

(A) 5.6 x 10^{-4}
(B) 5.6 x 10^4
(C) 56 x 10^{-5}
(D) 5.6 x 10^{-3}

3. *Compute (2 x 10^3)(4 x 10^5).*

What is the product in scientific notation?

(A) 8 x 10^{15}
(B) 6 x 10^8
(C) 8 x 10^2
(D) 8 x 10^8

4. *A calculation produces the value 50 x 10^6.*

Written in proper scientific notation, this is:

(A) 5 x 10^7

(B) 5 x 10^6

(C) 0.5 x 10^8

(D) 50 x 10^6

5. *Compare 6 x 10^8 with 3 x 10^5.*

How many times larger is 6 x 10^8 than 3 x 10^5?

(A) 200

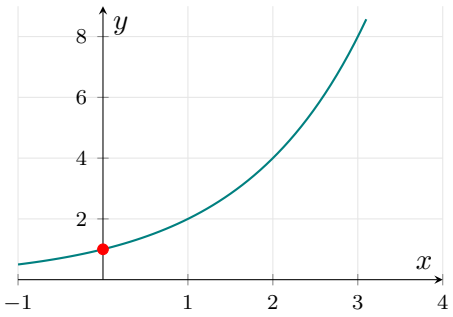
(B) 2 x 10^{40}

(C) 2,000 (that is, 2 x 10^3)

(D) 2 x 10^{13}

Exponential functions (y = a*b^x)

The skill. In y = a*b^x, a is the initial value (y when x = 0) and b is the base, the constant factor multiplied for each unit increase in x. If b > 1 the function grows; if 0 < b < 1 it decays. Exponential change is a constant RATIO between successive values (multiply by b), not a constant difference (which is linear). Evaluate by computing b^x first, then multiplying by a.



Exponential $y = a\,b^x$ multiplies by the base b each step; the marked point $(0, a)$ is the initial value.

Worked example

A biologist models a bacteria count by $y = 6 \cdot 3^x$, where x is the number of days. What is the count at $x = 3$?

Read. The model is $y = 6 \cdot 3^x$, so the initial value is $a = 6$ (the count at $x = 0$) and the base is $b = 3$ (the count triples each day). We want the count at $x = 3$ days.

Plan. Evaluate an exponential by working the power first: raise the base to x to get b^x, then multiply that result by the coefficient a. Do not fold a into the base or multiply a by the

exponent.

Solve.

$y = 6 \cdot 3^x$	start with the model
$y = 6 \cdot 3^3$	substitute $x = 3$
$y = 6 \cdot 27$	evaluate the power $3^3 = 27$
$y = 162$	multiply $6 \cdot 27$

Check. The base $b = 3$ means a constant ratio, so the counts run $6, 18, 54, 162$ for $x = 0, 1, 2, 3$ – each is three times the previous. The value grows because $b > 1$, and 162 is the fourth term, matching our answer.

Answer: 162.

Approach Read · Plan · Solve · Check

Read. Identify the initial value a, the base b, and whether the situation multiplies (exponential) or adds (linear) each step.

Plan. Read a as the value at x = 0 and b as the per-step factor; b > 1 grows, 0 < b < 1 decays. To evaluate, raise b to the power first, then multiply by a.

Solve. Compute b^x, then multiply by a (do not fold a into the base).

Check. Constant ratio (not difference)? a and b in the right roles? Growth vs decay matches b? b^x done before multiplying by a?

Common traps — every wrong answer is one of these

linear/exponential confusion.
Treats an exponential pattern as linear: adds a constant difference each step instead of multiplying by the constant ratio b.

base/coefficient role confusion.
Swaps the initial value a and the base b, or reads the wrong part as the starting value or growth factor.

growth/decay direction error.
Misjudges direction: treats b > 1 as decay or 0 < b < 1 as growth.

evaluation error.
Computes $a*b^x$ as $(a*b)^x$ or as a*b*x, folding the coefficient into the base or treating the exponent as a multiplier.

Practice

1. *An exponential function is $y = 5 * 2^x$.*

What is the initial value (the value when x = 0)?

(A) 5

(B) 2

(C) 10

(D) 7

2. *Consider $y = 3 * 2^x$, $y = 3 * (0.5)^x$, and $y = 2 * 3^x$.*

Which represents exponential decay?

(A) $y = 2 * 3^x$

(B) $y = 3 * 2^x$

(C) None of them decay

(D) $y = 3 * (0.5)^x$

3. *Let $y = 4 * 3^x$.*

What is y when x = 2?

(A) 36

(B) 144

(C) 24

(D) 10

4. *An exponential table shows x = 0, 1, 2 giving y = 3, 6, 12.*

What is y when x = 3?

(A) 36

(B) 24

(C) 18

(D) 15

5. *A population starts at 200 and doubles each year.*

Which function gives the population after x years?

(A) $y = 400^x$

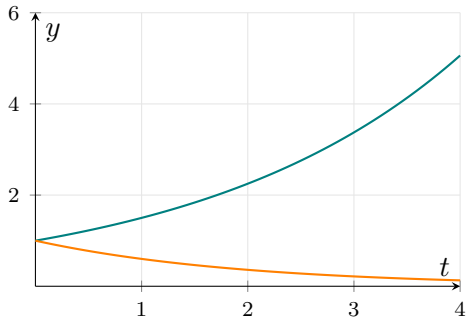
(B) $y = 200 + 2x$

(C) $y = 2 * 200^x$

(D) $y = 200 * 2^x$

Exponential growth & decay in context

The skill. A quantity growing at rate r per period is modeled by y = a(1 + r)^{t}; decaying at rate r by y = a(1 - r)^{t}. The base is 1 + r (growth) or 1 - r (decay), NOT the raw rate r. This is compound (multiplicative) change: each period multiplies by the base, unlike simple/linear change a(1 + rt). Half-life or doubling uses the number of periods as the exponent: y = a(1/2)^(t/h).



Growth uses base $1 + r > 1$ (rising, teal); decay uses base $1 - r < 1$ (falling, orange).

Worked example

A colony of 900 insects declines by 20% each year. How many remain after 2 years?

Read. The colony starts at $a = 900$ insects and declines by 20% each year, so the decay rate is $r = 0.20$. We want the number remaining after $t = 2$ years.

Plan. For decay, the per-year multiplier is the base $1 - r$, not the raw rate r. Here $1 - r = 1 - 0.20 = 0.80$, so use the compound model $y = a(1 - r)^t$: raise the base to the number of years, then multiply by the initial amount.

Solve.

$y = 900(1 - 0.20)^2$	write the decay model with $r = 0.20$
$y = 900(0.80)^2$	simplify the base $1 - 0.20 = 0.80$
$y = 900(0.64)$	square the base: $0.80^2 = 0.64$
$y = 576$	multiply $900 \cdot 0.64$

Check. Because the base $0.80 < 1$, the amount should fall each year: $900 \to 720 \to 576$. After two years 576 remain, which matches, and it is less than the starting 900 as expected for decay.

Answer: 576.

Approach Read · Plan · Solve · Check

Read. Identify the initial amount, the rate, whether it grows or decays, and the number of periods.

Plan. Convert the rate to a base: 1 + r for growth, 1 - r for decay. Use an exponent (multiplicative), not a linear a(1 + rt). For half-life, the exponent is t divided by the period

length.

Solve. Raise the base to the number of periods, then multiply by the initial amount.

Check. Base = 1 +/- r (not raw r)? Right direction? Compounded (exponent) rather than added linearly? Correct number of periods?

Common traps — every wrong answer is one of these

rate-to-base error.
Uses the raw rate r as the base instead of 1 + r (or 1 - r), e.g. modeling 8% growth as $a(0.08)^t$ rather than $a(1.08)^t$; or miscounts the number of periods in the exponent.

growth/decay direction error.
Uses 1 + r for a decline or 1 - r for growth, reversing the base direction.

simple vs compound error.
Models compound change with a linear/additive form a(1 + rt) instead of the exponential $a(1 + r)^t$.

Practice

1. *A quantity starts at 500 and grows 8% per year.*

Which models its value after t years?

(A) $500(0.92)^t$
(B) $500(1.08)^t$
(C) $500(1.8)^t$
(D) $500(0.08)^t$

2. *A car worth $20,000 loses 15% of its value each year.*

Which models its value after t years?

(A) $20000(0.85)^t$
(B) 20000(1 - 0.15t)
(C) $20000(1.15)^t$
(D) $20000(0.15)^t$

3. *You invest $1000 at 5% interest compounded annually.*

Which gives the balance after t years?

(A) 1000(1.05t)
(B) $1000(1.05)^t$

(C) $1000(0.05)^t$

(D) $1000(1 + 0.05t)$

4. *A population of 200 grows 10% per year.*

What is the population after 2 years?

(A) 240

(B) 220

(C) 2

(D) 242

5. *An 80 mg sample halves every 3 hours.*

How much remains after 9 hours?

(A) 10 mg

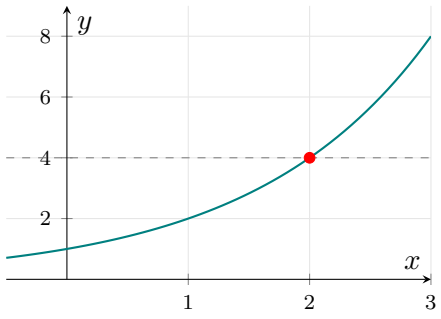
(B) 20 mg

(C) 35 mg

(D) 40 mg

Solving exponential equations by matching bases

The skill. If two powers with the SAME base are equal, their exponents are equal: $b^m = b^n$ implies $m = n$ (for $b > 0$, b not 1). To solve an exponential equation, rewrite both sides as powers of a common base, then set the exponents equal and solve. This often means rewriting numbers as powers ($81 = 3^4$, $8 = 2^3$, $1/8 = 2^{-3}$, $9 = 3^2$). An exponential is repeated multiplication, not multiplication by the exponent, so $3^x = 81$ is NOT solved by dividing 81 by 3.



Rewriting both sides with a common base turns $b^m = b^n$ into $m = n$; here $2^x = 4$ gives $x = 2$.

Worked example

Solve $8^x = 16$.

Read. This is an exponential equation, $8^x = 16$. Both 8 and 16 are powers of 2, so we can

rewrite each side with the common base 2 and then compare exponents.

Plan. Write $8 = 2^3$ and $16 = 2^4$. Use the power-of-a-power rule $(2^3)^x = 2^{3x}$ so the left side becomes a single power of 2. Once both sides share the base 2, set the exponents equal and solve the resulting linear equation.

Solve.

$$\begin{aligned} 8^x &= 16 && \text{start with the given equation} \\ (2^3)^x &= 2^4 && \text{rewrite } 8 = 2^3 \text{ and } 16 = 2^4 \\ 2^{3x} &= 2^4 && \text{apply the power rule } (2^3)^x = 2^{3x} \\ 3x &= 4 && \text{equal bases, so equate the exponents} \\ x &= \frac{4}{3} && \text{divide both sides by 3} \end{aligned}$$

Check. Substitute back: $8^{4/3} = (2^3)^{4/3} = 2^4 = 16$, the original right side. The solution satisfies the equation, so $x = \frac{4}{3}$ is correct.

Answer: $x = \frac{4}{3}$.

Approach Read · Plan · Solve · Check

Read. Can both sides be written as powers of one common base? Identify that base.

Plan. Rewrite each side as base^(exponent). Set the exponents equal. Solve the resulting (often linear) equation. Handle fractions and reciprocals with negative exponents.

Solve. Convert, equate exponents, solve.

Check. Did I match bases before equating exponents (not divide)? Did I rewrite the numbers to the correct powers? Did I use a negative exponent for a fraction less than 1?

Common traps — every wrong answer is one of these

base-matching failure.
Sets exponents equal without first rewriting to a common base, sets the bases equal instead of the exponents, or divides the two sides instead of matching bases.

exponential-as-linear error.
Treats the exponential as linear: solves $b^x = N$ by computing N/b or N - b instead of writing N as a power of b.

rewriting error.
Rewrites a number as the wrong power of the base, e.g. taking $32 = 2^4$ or $81 = 3^3$, producing a wrong exponent.

negative/fractional-exponent error.
Mishandles a fraction or reciprocal target, e.g. solving $2^x = 1/8$ as x = 3 instead of x = -3, or mismanaging a fractional exponent.

Practice

1. *Solve $3^x = 81$.*

What is x?

(A) 4
(B) 3
(C) 27
(D) 78

2. *Solve $2^x = 32$.*

What is x?

(A) 16
(B) 6
(C) 5
(D) 4

3. *Solve $4^x = 8$.*

What is x?

(A) 1
(B) 3/2
(C) 2
(D) 12

4. *Solve $2^x = 1/8$.*

What is x?

(A) -3
(B) -4
(C) 3
(D) 1/3

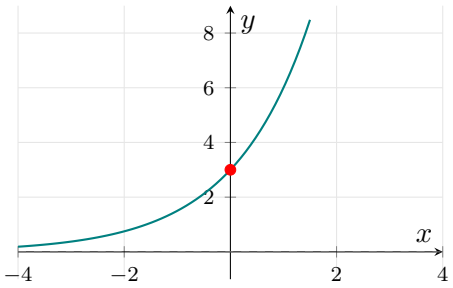
5. *Solve $9^x = 27$.*

What is x?

(A) 2/3
(B) 3
(C) 3/2
(D) 1

Graphs of exponential functions

The skill. For f(x) = a·b^x, the y-intercept is at x = 0: f(0) = a·b^0 = a·1 = a, giving the point (0, a). The horizontal asymptote is y = 0. Because b^0 = 1, the starting value is a, not b and not a·b.



The y-intercept is $(0, a)$ since $b^0 = 1$; the curve approaches the horizontal asymptote $y = 0$.

Worked example

What is the y-intercept of $f(x) = 7 \cdot 2^x$?

Read. The y-intercept is the point where the graph crosses the y-axis, which happens at $x = 0$. So we need the value $f(0)$ for $f(x) = 7 \cdot 2^x$.

Plan. Substitute $x = 0$ into the function. The key fact is that any nonzero base raised to the 0 power is 1, so $2^0 = 1$, leaving just the coefficient.

Solve.

$f(0) = 7 \cdot 2^0$	substitute $x = 0$
$f(0) = 7 \cdot 1$	use $2^0 = 1$
$f(0) = 7$	multiply

Check. The intercept equals the coefficient $a = 7$, giving the point $(0, 7)$. It is not the base $b = 2$ and not the product $a \cdot b = 14$, because the base contributes a factor of 1 at $x = 0$.

Answer: $(0, 7)$.

Approach Read · Plan · Solve · Check

Read. The y-intercept is f(0).

Plan. Substitute x = 0; use b^0 = 1.

Solve. f(0) = a·1 = a.

Check. Did I use b^0 = 1? Is the intercept a (not b, not a·b)?

Common traps — every wrong answer is one of these

used the base.
Takes the y-intercept as the base b instead of the coefficient a.

multiplied a·b.
Reports a·b, forgetting that $b^0 = 1$.

assumed origin.
Assumes the graph passes through (0, 0).

Practice

1. *The function $f(x) = 2 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 2)
(B) (0, 4)
(C) (0, 8)
(D) (0, 0)

2. *The function $f(x) = 5 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 20)
(B) (0, 5)
(C) (0, 0)
(D) (0, 4)

3. *The function $f(x) = 3 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 12)
(B) (0, 0)
(C) (0, 4)
(D) (0, 3)

4. *The function $f(x) = 2 \cdot 3^x$. What is its y-intercept?*

Which is correct?

(A) (0, 2)
(B) (0, 3)

(C) (0, 6)
(D) (0, 0)

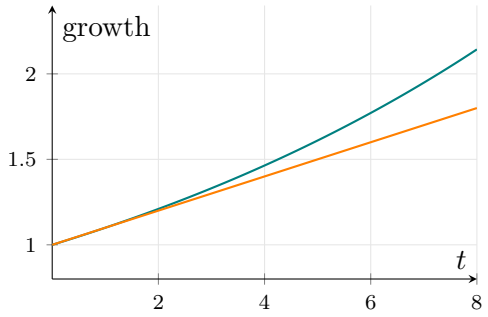
5. *The function $f(x) = 5 \cdot 3^x$. What is its y-intercept?*

Which is correct?

(A) (0, 0)
(B) (0, 3)
(C) (0, 5)
(D) (0, 15)

Compound interest

The skill. Compound interest: $A = P(1 + r)^t$, where r is the rate as a DECIMAL (percent ÷ 100) and t is the number of periods. Each period multiplies by $(1 + r)$, so it is exponential, not simple (additive) interest.



Compound interest $(1 + r)^t$ (curved, teal) outpaces simple interest $1 + rt$ (straight, orange) as periods grow.

Worked example

You invest $1000 at 10% interest compounded annually. What is the balance after 2 years?

Read. The principal is $P = 1000$ dollars, the annual rate is $10\% = 0.10$ as a decimal, and the time is $t = 2$ years. Interest is compounded annually, so use $A = P(1 + r)^t$.

Plan. Convert the percent to a decimal ($10\% = 0.10$), add 1 to get the growth base $1 + r = 1.10$, raise that base to the number of years t, and finally multiply by the principal P.

Solve.

$A = 1000(1 + 0.10)^2$	write the compound-interest model
$A = 1000(1.10)^2$	simplify the base $1 + 0.10 = 1.10$
$A = 1000(1.21)$	square the base: $1.10^2 = 1.21$
$A = 1210$	multiply $1000 \cdot 1.21$

Check. This is compound interest (a power), not simple interest. Simple interest would give $1000(1 + 0.10 \cdot 2) = 1200$ dollars, so the compounded balance of 1210 dollars being a bit larger is exactly what we expect.

Answer: \$1210.

Approach Read · Plan · Solve · Check

Read. Compound growth: $A = P(1 + r)^t$.

Plan. Convert percent to a decimal; raise to the power t.

Solve. $P(1 + rpct/100)^t$.

Check. Did I divide the percent by 100? Is it a power (not times t)?

Common traps — every wrong answer is one of these

percent not converted.
Uses the whole percent as the rate instead of percent ÷ 100.

simple interest.
Adds r·t once instead of raising (1 + r) to the power t.

dropped the 1 +.
Uses r^t as the base factor, omitting the 1 +.

Practice

1. *You invest \$500 at 10% interest compounded annually. Which expression gives the value after 3 years?*

Which is correct?

(A) $500(1 + 10)^3$

(B) $500(1 + (10/100)\cdot 3)$

(C) $500(1 + 10/100)^3$

(D) $500(10/100)^3$

2. *You invest \$500 at 5% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $500(1 + 5)^2$

(B) $500(1 + (5/100)\cdot 2)$

(C) $500(1 + 5/100)^2$

(D) $500(5/100)^2$

3. *You invest $1000 at 20% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $1000(1 + 20/100)^2$

(B) $1000(1 + (20/100)\cdot 2)$

(C) $1000(20/100)^2$

(D) $1000(1 + 20)^2$

4. *You invest $2000 at 5% interest compounded annually. Which expression gives the value after 4 years?*

Which is correct?

(A) $2000(1 + (5/100)\cdot 4)$

(B) $2000(1 + 5)^4$

(C) $2000(5/100)^4$

(D) $2000(1 + 5/100)^4$

5. *You invest $800 at 5% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $800(0.05)^2$

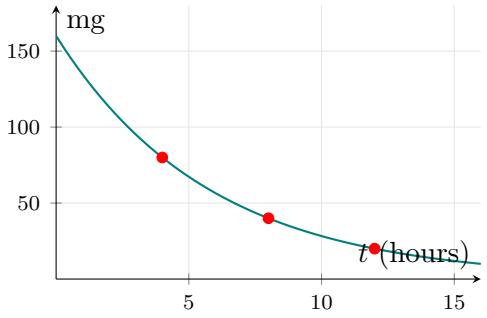
(B) $800(1 + 5)^2$

(C) $800(1 + 0.05\cdot 2)$

(D) $800(1.05)^2$

Half-life & doubling time

The skill. For half-life decay, the amount after time t is P·(1/2)^(t/h), where h is the half-life and t/h is the number of half-lives. The exponent DIVIDES elapsed time by the half-life; the base is 1/2 for decay (2 for doubling).



Each half-life the amount halves; the exponent is t/h, the number of half-lives elapsed.

Worked example

A 160 mg sample decays with a half-life of 4 hours. How much remains after 12 hours?

Read. The sample starts at $P = 160$ mg, the half-life is $h = 4$ hours, and the elapsed time is $t = 12$ hours. We want the amount remaining at $t = 12$.

Plan. The exponent is the number of half-lives, found by dividing elapsed time by the half-life: $\frac{t}{h}$. Because the sample halves each period, the base is $\frac{1}{2}$, giving the model $P\left(\frac{1}{2}\right)^{t/h}$.

Solve.

$A = 160\left(\frac{1}{2}\right)^{12/4}$	write the half-life model
$A = 160\left(\frac{1}{2}\right)^{3}$	the number of half-lives is $\frac{12}{4} = 3$
$A = 160 \cdot \frac{1}{8}$	evaluate $\left(\frac{1}{2}\right)^{3} = \frac{1}{8}$
$A = 20$	multiply $160 \cdot \frac{1}{8}$

Check. Halving step by step, $160 \to 80 \to 40 \to 20$ across the three half-lives in 12 hours. The result 20 mg matches, and it is smaller than the start, as decay requires.

Answer: 20 mg.

Approach Read · Plan · Solve · Check

Read. Decay by half-life: P·(1/2)^(t/h).

Plan. Number of half-lives = t/h.

Solve. Raise 1/2 to that power.

Check. Is the exponent t/h (divided)? Is the base 1/2 for decay?

Common traps — every wrong answer is one of these

multiplied in the exponent.
Uses t·h instead of t/h as the number of half-lives.

linear not exponential.
Models the decay as multiplication by t/h instead of a power.

growth instead of decay.
Uses base 2 (growth) instead of 1/2 (decay).

Practice

1. *A sample of 100 grams decays with a half-life of 4 years. Which expression gives the amount after 8 years?*

Which is correct?

(A) 100(1/2)^(8·4)
(B) 100(1/2)·(8/4)
(C) 100(1/2)^(8/4)
(D) 100(2)^(8/4)

2. *A sample of 80 grams decays with a half-life of 2 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 80(1/2)·(20/2)
(B) 80(1/2)^(20/2)
(C) 80(1/2)^(20·2)
(D) 80(2)^(20/2)

3. *A sample of 80 grams decays with a half-life of 4 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 80(1/2)^(20·4)
(B) 80(2)^(20/4)
(C) 80(1/2)·(20/4)
(D) 80(1/2)^(20/4)

4. *A sample of 240 grams decays with a half-life of 4 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 240(1/2)^(20/4)
(B) 240(1/2)^(20·4)
(C) 240(2)^(20/4)
(D) 240(1/2)·(20/4)

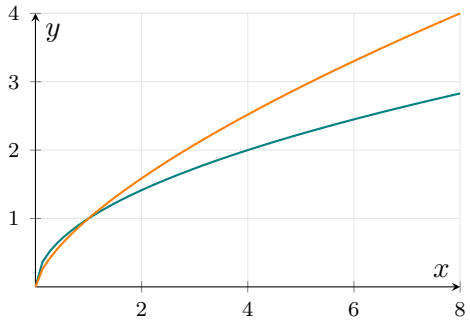
5. *A 200-gram sample decays with a half-life of 5 years. Which expression gives the amount after 15 years?*

Which is correct?

(A) 200·2^(15/5)
(B) 200·(1/2)·3
(C) 200·(1/2)^(15/5)
(D) 200·(1/2)^(15·5)

Radicals & rational exponents (simplify & convert)

The skill. A rational exponent encodes a root and a power: x^(m/n) is the n-th root of x raised to the m (the denominator n is the ROOT, the numerator m is the POWER). So x^(1/2) is the square root of x and x^(2/3) is the cube root of x squared. Radicals follow product and quotient rules: $\sqrt{ab} = \sqrt{a}*\sqrt{b}$ and $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$; simplify by pulling out perfect-square (or perfect-n-th-power) factors. Roots do NOT distribute over addition: $\sqrt{a+b}$ is not $\sqrt{a}+\sqrt{b}$. A negative rational exponent means a reciprocal: x^(-m/n) = 1 / x^(m/n).



A rational exponent is a root-and-power: $x^{m/n} = \sqrt[n]{x^m}$ (denominator = root, numerator = power). Shown: $x^{1/2}$ and $x^{2/3}$.

Worked example

Write $\sqrt[3]{x^5}$ (with $x > 0$) as a single rational exponent, and name the root and power.

Read. The expression $\sqrt[3]{x^5}$ is a cube root of x^5, that is, the fifth power of x taken under a cube root. We want to rewrite it as a single rational exponent.

Plan. A rational exponent $x^{m/n}$ encodes both operations: the denominator n is the root (index) and the numerator m is the power. So the index 3 becomes the denominator and the

power 5 becomes the numerator.

Solve.

$$\sqrt[3]{x^5} = \sqrt[3]{x^5} \qquad \text{index (root) is 3, power is 5}$$

$$\sqrt[3]{x^5} = x^{5/3} \qquad \text{denominator = root 3, numerator = power 5}$$

Check. In $x^{5/3}$ the denominator 3 is the root and the numerator 5 is the power, matching the original. It is a root-and-power, not the product $\frac{5}{3}x$.

Answer: $x^{5/3}$.

Approach Read · Plan · Solve · Check

Read. Decide whether to convert between radical and exponent form, simplify a radical, or evaluate; note any sum under a radical or a negative exponent.

Plan. Convert with denominator = root, numerator = power. Simplify by factoring out perfect powers. Never split a root over a sum. Turn a negative exponent into a reciprocal first.

Solve. Carry out the root/power in the right roles, or extract perfect-power factors.

Check. Root and power in the correct positions (n is the root)? No splitting a root over addition? Negative exponent converted to a reciprocal? Perfect-power factor fully extracted?

Common traps — every wrong answer is one of these

rational-exponent conversion error.
Swaps the root and power in x^(m/n) (uses the numerator as the root or the denominator as the power), or treats the fractional exponent as a coefficient (x^(2/3) as (2/3)x).

radical-of-sum error.
Splits a root over addition or subtraction: treats $\sqrt{a+b}$ as $\sqrt{a}+\sqrt{b}$, or treats a sum under a radical as a product.

simplification error.
Mishandles simplifying a radical: fails to take the root of a perfect-square factor, leaves a simplifiable radical unsimplified, or extracts the wrong factor.

negative rational exponent error.
Drops the reciprocal for a negative exponent (treats x^(-1/2) as $\sqrt{x}$) or treats the negative exponent as negating the value (as $-\sqrt{x}$).

Practice

1. *Consider the expression x^(2/3), where x is positive.*

Which is equivalent?

(A) two-thirds of x

(B) the square root of x cubed

(C) the cube root of x squared

(D) x times two-thirds

2. *Evaluate the square root of (9 + 16).*

What is the value?

(A) 7

(B) 12

(C) 5

(D) 25

3. *Simplify the square root of 50.*

What is the simplified form?

(A) 25 times the square root of 2

(B) 5 times the square root of 2

(C) the square root of 50 (already simplest)

(D) 2 times the square root of 5

4. *Consider x^(-1/2), where x is positive.*

Which is equivalent?

(A) 1 divided by x squared

(B) 1 divided by the square root of x

(C) the negative of the square root of x

(D) the square root of x

5. *Evaluate 8^(2/3).*

What is the value?

(A) 1/4

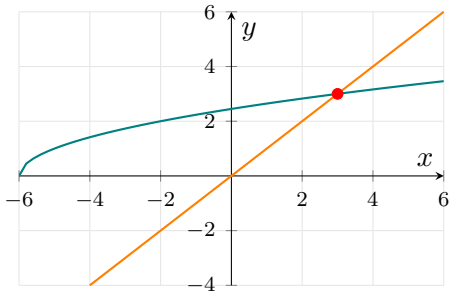
(B) 64

(C) 4

(D) 16/3

Radical & rational equations (solving with extraneous checks)

The skill. To solve a radical equation, ISOLATE the radical first, then square (or raise to the n-th power) both sides; squaring can create EXTRANEOUS solutions, so check every candidate in the ORIGINAL equation and discard any that fail. To solve a rational equation, multiply through by the least common denominator to clear fractions, then solve; exclude any value that makes a denominator zero (an excluded value cannot be a solution even if it satisfies the cleared equation). Squaring a sum expands by FOIL: $(\sqrt{x}+3)^2 = x + 6\sqrt{x} + 9$, not $x + 9$.



Squaring can add false roots; the true solution is the real intersection of $\sqrt{x+6}$ and $y = x$ at $x = 3$, not the extraneous $x = -2$.

Worked example

Solve $\sqrt{x+6} = x$.

Read. This is a radical equation, $\sqrt{x+6} = x$. The radical is already alone on the left, so it is isolated. Squaring can introduce false (extraneous) roots, so we must check every candidate at the end.

Plan. Square both sides to remove the square root. That gives a quadratic; move everything to one side, factor, and read off the candidate solutions. Then substitute each candidate into the original equation and discard any that do not satisfy it.

Solve.

$\sqrt{x+6} = x$	start with the isolated radical
$x + 6 = x^2$	square both sides
$0 = x^2 - x - 6$	move all terms to one side
$0 = (x-3)(x+2)$	factor the quadratic
$x = 3$ or $x = -2$	set each factor equal to 0

Check. Test both in the original. For $x = 3$: $\sqrt{3+6} = \sqrt{9} = 3$, which equals the right side, so it is valid. For $x = -2$: $\sqrt{-2+6} = \sqrt{4} = 2$, but the right side is -2, and $2 \neq -2$, so $x = -2$ is extraneous and is discarded.

Answer: $x = 3$.

Approach Read · Plan · Solve · Check

Read. Identify the equation type (radical or rational) and any denominators or sums that need care.

Plan. Radical: isolate the radical, then square; plan to check for extraneous roots. Rational: find the LCD, clear fractions, and note excluded values (denominator = 0).

Solve. Solve the resulting (often quadratic) equation, then test each candidate in the original.

Check. Isolated before squaring? Expanded a squared sum by FOIL (cross term kept)? Checked each candidate in the original and discarded extraneous ones? Excluded any denominator-zero value?

Common traps — every wrong answer is one of these

extraneous-root acceptance.
Solves by squaring or clearing denominators but does not check candidates in the original equation, accepting an extraneous solution (or, conversely, mishandling the check and discarding a valid one).

squaring/expansion error.
Squares before isolating the radical, squares only one side, or expands a squared sum term-by-term (treats $(\sqrt{x}+3)^2$ as x+9), dropping the cross term, or misgroups the radicand.

excluded-value error.
Fails to exclude a value that makes a denominator zero in a rational equation, reporting it as a solution, or otherwise mishandles the least common denominator.

Practice

1. *Solve the equation: the square root of (x + 2) equals x.*

What is the solution set?

(A) No solution
(B) x = 2
(C) x = 2 or x = -1
(D) x = -1

2. *Solve: the square root of x, plus 3, equals 7.*

What is x?

(A) 4
(B) 40
(C) 46

(D) 16

3. *Solve the rational equation: x / (x - 2) = 2 / (x - 2).*

What is the solution?

(A) x = 2 and x = -2

(B) x = 2

(C) No solution

(D) All real numbers

4. *Solve: the square root of (2x + 3) equals x.*

What is the solution set?

(A) x = 1 or x = 3

(B) x = -1

(C) x = 3 or x = -1

(D) x = 3

5. *Solve the rational equation: x + 6/x = 5.*

What is the solution set?

(A) x = 2 or x = 3

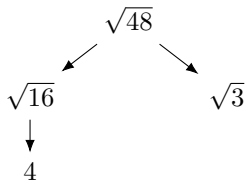
(B) x = 0, 2, or 3

(C) x = 5

(D) x = 2 only

Simplify radicals (perfect-square factors)

The skill. To simplify $\sqrt{n}$, factor out the largest perfect square: $\sqrt{k^2 \cdot m} = k\sqrt{m}$. Only the perfect-square part leaves the root as its square root; the rest stays underneath. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.



Factor out the largest perfect square: $\sqrt{48} = \sqrt{16}\,\sqrt{3} = 4\sqrt{3}$.

Worked example

Simplify $\sqrt{48}$.

Read. We want $\sqrt{48}$ in simplest radical form, so we look for the largest perfect-square factor of 48.

Plan. Write 48 as a perfect square times what remains: $48 = 16 \cdot 3$, where 16 is a perfect square. Then use the product rule $\sqrt{k^2 m} = \sqrt{k^2}\,\sqrt{m} = k\sqrt{m}$ to pull the square root of 16 outside.

Solve.

$\sqrt{48} = \sqrt{16 \cdot 3}$	factor out the perfect square 16
$\sqrt{48} = \sqrt{16}\,\sqrt{3}$	split with the product rule
$\sqrt{48} = 4\sqrt{3}$	take $\sqrt{16} = 4$

Check. The leftover 3 under the root has no perfect-square factor, so the radical is fully simplified. Squaring back, $4^2 \cdot 3 = 16 \cdot 3 = 48$, the original radicand, confirming $4\sqrt{3}$.

Answer: $4\sqrt{3}$.

Approach Read · Plan · Solve · Check

Read. What is the largest perfect square dividing the radicand?

Plan. $\sqrt{k^2 \cdot m} = \sqrt{k^2} \cdot \sqrt{m} = \mathrm{k}\sqrt{m}$.

Solve. Pull out $\sqrt{k^2} = \mathrm{k}$; leave m inside.

Check. Is the leftover m free of perfect-square factors? Did only k leave the root?

Common traps — every wrong answer is one of these

radical mis-handling.
Removes the radical entirely (e.g. $\sqrt{50}$ -> 10) or uses the square k^2 outside instead of its root k.

incomplete simplification.
Pulls a factor out front but leaves the whole original radicand under the root.

left the square under the root.
Writes $\sqrt{k^2 * m}$ as $\mathrm{k}^2 * \sqrt{m}$ (fails to square-root the perfect-square factor).

Practice

1. *Simplify* $\sqrt{45}$.

Which is correct?

(A) 15

(B) $3\sqrt{5}$
(C) $3\sqrt{45}$
(D) $9\sqrt{5}$

2. *Simplify* $\sqrt{80}$.

Which is correct?

(A) $16\sqrt{5}$
(B) $4\sqrt{80}$
(C) $4\sqrt{5}$
(D) 20

3. *Simplify* $\sqrt{20}$.

Which is correct?

(A) $2\sqrt{20}$
(B) 10
(C) $2\sqrt{5}$
(D) $4\sqrt{5}$

4. *Simplify* $\sqrt{54}$.

Which is correct?

(A) $3\sqrt{54}$
(B) $9\sqrt{6}$
(C) 18
(D) $3\sqrt{6}$

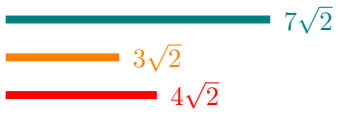
5. *Simplify* $\sqrt{72}$.

Which is correct?

(A) $2\sqrt{18}$
(B) 12
(C) $6\sqrt{2}$
(D) $36\sqrt{2}$

Add & subtract like radicals

The skill. Like radicals (same index and radicand) combine like like terms: $a\sqrt{k} + b\sqrt{k} = (a+b)\sqrt{k}$. The radicand does NOT change and coefficients are not multiplied. Unlike radicals cannot be combined.



Like radicals combine like like terms: $7\sqrt{2} - 3\sqrt{2} = (7-3)\sqrt{2} = 4\sqrt{2}$*; the radicand stays* $\sqrt{2}$.

Worked example

Simplify $7\sqrt{2} - 3\sqrt{2}$**.**

Read. Both terms, $7\sqrt{2}$ and $3\sqrt{2}$, contain the same radical $\sqrt{2}$, so they are like radicals and can be combined – just like combining like terms $7a - 3a$.

Plan. Treat $\sqrt{2}$ as a common factor. Subtract the coefficients 7 and 3, and keep the radical $\sqrt{2}$ unchanged; the number under the root does not change.

Solve.

$7\sqrt{2} - 3\sqrt{2} = (7-3)\sqrt{2}$	factor out the common $\sqrt{2}$
$7\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$	subtract the coefficients $7 - 3 = 4$

Check. The radicand stays 2, and only the coefficients were combined ($7 - 3 = 4$). We did not add or subtract the numbers under the root, so $4\sqrt{2}$ is correct.

Answer: $4\sqrt{2}$.

Approach Read · Plan · Solve · Check

Read. Do the terms share the same radical $\sqrt{k}$?

Plan. If yes, add the coefficients and keep $\sqrt{k}$.

Solve. (a+b)$\sqrt{k}$.

Check. Did I keep the radicand the same? Did I add (not multiply) coefficients?

Common traps — every wrong answer is one of these

added the radicands.
Adds the numbers under the root ($\sqrt{k} + \sqrt{k}$ -> $\sqrt{2}$k) instead of keeping $\sqrt{k}$.

multiplied the coefficients.
Multiplies the coefficients instead of adding them.

false 'cannot combine'.
Treats like radicals as if they cannot be combined.

Practice

1. *Simplify* $3\sqrt{5} + 2\sqrt{5}$.

Which is correct?

(A) Cannot be combined

(B) $6\sqrt{5}$

(C) $5\sqrt{5}$

(D) $5\sqrt{10}$

2. *Simplify* $3\sqrt{5} + 3\sqrt{5}$.

Which is correct?

(A) $9\sqrt{5}$

(B) Cannot be combined

(C) $6\sqrt{5}$

(D) $6\sqrt{10}$

3. *Simplify* $4\sqrt{3} + 5\sqrt{3}$.

Which is correct?

(A) $9\sqrt{3}$

(B) Cannot be combined

(C) $9\sqrt{6}$

(D) $20\sqrt{3}$

4. *Simplify* $4\sqrt{2} + 2\sqrt{2}$.

Which is correct?

(A) $6\sqrt{4}$

(B) Cannot be combined

(C) $6\sqrt{2}$

(D) $8\sqrt{2}$

5. *Simplify* $4\sqrt{3} + 3\sqrt{3}$.

Which is correct?

(A) $7\sqrt{6}$

(B) cannot be combined

(C) $12\sqrt{3}$

(D) $7\sqrt{3}$

Multiply radicals & rationalize denominators

The skill. $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$. To rationalize $\mathrm{n}/\sqrt{k}$, multiply by $\sqrt{k}/\sqrt{k}$: $\mathrm{n}/\sqrt{k} = \mathrm{n}\sqrt{k}/\mathrm{k}$, because $\sqrt{k}\cdot\sqrt{k} = \mathrm{k}$. The radical moves to the numerator and the denominator becomes rational.

$$\frac{4}{\sqrt{2}} \longrightarrow \frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \longrightarrow \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Multiply by $\sqrt{k}/\sqrt{k}$ to clear the radical: $\sqrt{k}\cdot\sqrt{k} = k$ moves the root to the numerator.

Worked example

Rationalize the denominator: $\dfrac{4}{\sqrt{2}}$.

Read. The expression $\dfrac{4}{\sqrt{2}}$ has a radical, $\sqrt{2}$, in the denominator. Rationalizing means rewriting it so no radical remains in the denominator.

Plan. Multiply the fraction by $\dfrac{\sqrt{2}}{\sqrt{2}}$, which equals 1 and so does not change the value. Because $\sqrt{2}\cdot\sqrt{2} = 2$, the denominator becomes the rational number 2 while the radical moves to the numerator. Then simplify the resulting fraction.

Solve.

$$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \qquad \text{multiply by } \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} \qquad \text{use } \sqrt{2}\cdot\sqrt{2} = 2 \text{ in the denominator}$$

$$\frac{4}{\sqrt{2}} = 2\sqrt{2} \qquad \text{reduce } \frac{4}{2} = 2$$

Check. The denominator is now the rational number 2 (in fact fully cleared after reducing), and $\sqrt{2}$ ended up in the numerator, so $2\sqrt{2}$ is the rationalized form.

Answer: $2\sqrt{2}$.

Approach Read · Plan · Solve · Check

Read. Is there a radical in the denominator?

Plan. Multiply top and bottom by $\sqrt{k}$.

Solve. $\mathrm{n}\cdot\sqrt{k}$ over $\sqrt{k}\cdot\sqrt{k} = \mathrm{k}$.

Check. Is the denominator now rational? Did $\sqrt{k}$ end up on top?

Common traps — every wrong answer is one of these

radical lost or misplaced.
Removes the radical without putting $\sqrt{k}$ in the numerator, or drops the n.

dropped the denominator.
Puts $\sqrt{k}$ on top but loses the rational denominator k.

left the radical in the denominator.
Reports n/$\sqrt{k}$ unchanged or rationalizes the numerator only, leaving an irrational denominator.

Practice

1. *Rationalize the denominator: 1/$\sqrt{2}$.*

Which is correct?

(A) $\sqrt{2}/2$
(B) $1\sqrt{2}$
(C) $1\sqrt{2}/2$
(D) $1/2$

2. *Rationalize the denominator: 1/$\sqrt{7}$.*

Which is correct?

(A) $\sqrt{7}/7$
(B) $1\sqrt{7}/7$
(C) $1/7$
(D) $1\sqrt{7}$

3. *Rationalize the denominator: 3/$\sqrt{5}$.*

Which is correct?

(A) $3\sqrt{5}/5$
(B) $3/5$
(C) $3\sqrt{5}$
(D) $\sqrt{5}/5$

4. *Rationalize the denominator: 1/$\sqrt{3}$.*

Which is correct?

(A) $1/3$
(B) $\sqrt{3}/3$
(C) $1\sqrt{3}$
(D) $1\sqrt{3}/3$

5. *Rationalize the denominator:* $3/\sqrt{5}$.

Which is correct?

(A) $3/\sqrt{5}$

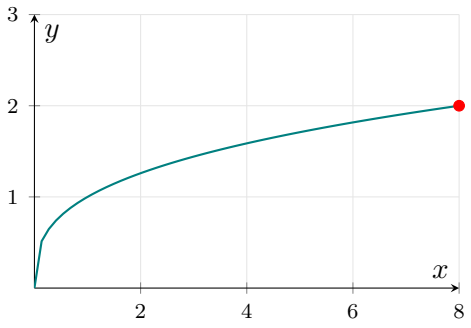
(B) $3\sqrt{5}$

(C) $3/5$

(D) $3\sqrt{5}/5$

nth roots & cube roots

The skill. The cube root undoes cubing: $\sqrt[3]{(c^3)} = c$. In general the nth root of x^n is x. It is not squaring, not division by the index, and not the radicand itself.



The cube root undoes cubing: $\sqrt[3]{c^3} = c$*; the marked point shows* $\sqrt[3]{8} = 2$.

Worked example

Simplify $\sqrt[3]{216}$**.**

Read. The cube root $\sqrt[3]{216}$ asks: what number, when cubed, gives 216? The cube root undoes cubing.

Plan. Find the value c satisfying $c^3 = 216$ by testing small whole numbers. Since $5^3 = 125$ is too small and $7^3 = 343$ is too large, try $c = 6$.

Solve.

$\sqrt[3]{216} = c$	let c be the cube root, so $c^3 = 216$
$6^3 = 216$	test $c = 6$: $6 \cdot 6 \cdot 6 = 216$
$\sqrt[3]{216} = 6$	the cube root of 216 is 6

Check. Cubing back, $6^3 = 216$, the original radicand. The answer is a genuine cube root, not $\dfrac{216}{3} = 72$ and not the square $6^2 = 36$.

Answer: 6.

Approach Read · Plan · Solve · Check

Read. What number cubed gives the radicand?

Plan. Find c with c^3 = radicand.

Solve. $\sqrt[3]{(c^3)} = c$.

Check. Does c^3 equal the radicand? (Not c^2, not radicand/3.)

Common traps — every wrong answer is one of these

wrong root/operation.
Squares, leaves the radicand, or otherwise fails to take the cube root.

multiplied by the index.
Multiplies by the index 3 instead of taking the root.

divided by the index.
Computes the nth root of x as x divided by n instead of the true root.

Practice

1. *Simplify the cube root of 64.*

Which is correct?

(A) 64
(B) 16
(C) 12
(D) 4

2. *Simplify the cube root of 125.*

Which is correct?

(A) 15
(B) 25
(C) 125
(D) 5

3. *Simplify the cube root of 8.*

Which is correct?

(A) 2
(B) 4
(C) 8

(D) 6

4. *Simplify the cube root of 27.*

Which is correct?

(A) 81
(B) 3
(C) 9
(D) 729

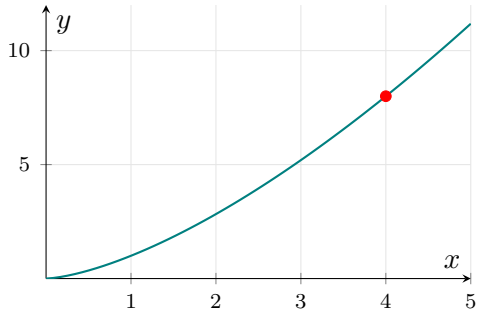
5. *Simplify the cube root of 125.*

Which is correct?

(A) 375
(B) about 41.7
(C) 5
(D) 15625

Operations with rational exponents

The skill. A rational exponent m/n means: take the nth root, then raise to the mth power. base^(p/q) = (q-th root of base)^p. Both the root (denominator) and the power (numerator) must be applied.



Apply both root and power: $x^{p/q} = (\sqrt[q]{x})^p$; the marked point shows $4^{3/2} = (\sqrt{4})^3 = 8$.

Worked example

Evaluate $32^{2/5}$.

Read. We must evaluate $32^{2/5}$. In the exponent $\frac{2}{5}$, the denominator 5 is the root and the numerator 2 is the power, so this means the fifth root of 32, then squared.

Plan. Take the fifth root first (it keeps the numbers small), then apply the power. That is, compute $32^{1/5}$ and square the result: $\left(32^{1/5}\right)^2$. Both the root and the power must be applied.

Solve.

$32^{2/5} = \left(32^{1/5}\right)^2$ split into root (denominator) then power (numerator)

$32^{2/5} = 2^2$ take the fifth root: $32^{1/5} = 2$

$32^{2/5} = 4$ square the result: $2^2 = 4$

Check. Both operations were applied. Writing $32 = 2^5$ confirms it: $(2^5)^{2/5} = 2^{5 \cdot 2/5} = 2^2 = 4$.
Answer: 4.

Approach Read · Plan · Solve · Check

Read. Which part is the root (denominator) and which is the power (numerator)?

Plan. Take the qth root of the base, then raise to p.

Solve. (base^(1/q))^p.

Check. Did I apply BOTH the root and the power?

Common traps — every wrong answer is one of these

skipped the root.
Raises to the power but never takes the root.

skipped the power.
Takes the root but never raises to the power.

used the exponent as a plain multiplier.
Multiplies the base by m/n (or applies m or n alone) instead of splitting into root then power.

Practice

1. *Evaluate 27^(2/3).*

Which is correct?

(A) 3
(B) 9
(C) 729
(D) 27

2. *Evaluate 4^(3/2).*

Which is correct?

(A) 64
(B) 4

(C) 8

(D) 2

3. *Evaluate 9^(3/2).*

Which is correct?

(A) 27

(B) 9

(C) 3

(D) 729

4. *Evaluate 8^(2/3).*

Which is correct?

(A) 8

(B) 2

(C) 4

(D) 64

5. *Evaluate 16^(3/4).*

Which is correct?

(A) 4096

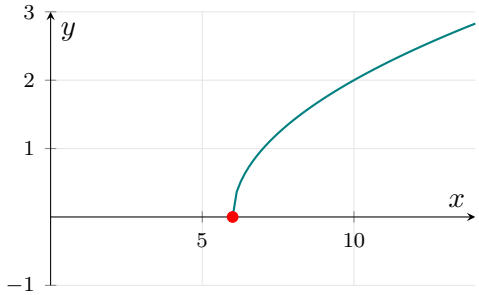
(B) 12

(C) 2

(D) 8

Radical functions: domain & graphs

The skill. For f(x) = $\sqrt{x-a}$, the inside must be ≥ 0 (a square root cannot take a negative). So x - a $\geq$ 0, giving the domain x $\geq$ a. Solving the inequality does not flip its direction here, and the sign of a follows from x - a $\geq$ 0.



$\sqrt{x-a}$ needs $x-a \geq 0$, so the domain is $x \geq a$; the graph starts at $(a, 0)$.

Worked example

What is the domain of $f(x) = \sqrt{x-6}$?

Read. For $f(x) = \sqrt{x-6}$, the expression under the square root cannot be negative, because a real square root of a negative number does not exist. So the radicand $x - 6$ must be at least 0.

Plan. Set the radicand greater than or equal to zero, $x - 6 \geq 0$, and solve for x. Adding a positive constant to both sides does not change the direction of the inequality.

Solve.

$x - 6 \geq 0$	the radicand cannot be negative
$x \geq 6$	add 6 to both sides

Check. At $x = 6$ the radicand is 0 and $\sqrt{0} = 0$ is defined, so 6 belongs to the domain. Any $x < 6$ makes $x - 6$ negative and the root undefined, so the domain is exactly $x \geq 6$.

Answer: $x \geq 6$.

Approach Read · Plan · Solve · Check

Read. What must be true for the inside of the root?

Plan. Set inside ≥ 0 and solve.

Solve. x - a ≥ 0 -> x $\geq$ a.

Check. Is the inequality direction right? Is the sign of a correct?

Common traps — every wrong answer is one of these

flipped the inequality.
Reports x $\leq$ a instead of x $\geq$ a.

sign error on a.
Uses x $\geq$ -a instead of x $\geq$ a.

ignored the restriction.
Claims the domain is all real numbers.

Practice

1. *What is the domain of $f(x) = \sqrt{x-3}$?*

Which is correct?

(A) all real numbers

(B) x $\geq$ 3

(C) x ≥ -3

(D) x ≤ 3

2. *What is the domain of* $f(x) = \sqrt{x-4}$*?*

Which is correct?

(A) x ≥ 4

(B) all real numbers

(C) x ≥ -4

(D) x ≤ 4

3. *What is the domain of* $f(x) = \sqrt{x-5}$*?*

Which is correct?

(A) all real numbers

(B) x ≤ 5

(C) x ≥ 5

(D) x ≥ -5

4. *What is the domain of* $f(x) = \sqrt{x-2}$*?*

Which is correct?

(A) x ≥ -2

(B) all real numbers

(C) x ≥ 2

(D) x ≤ 2

5. *What is the domain of* $f(x) = \sqrt{x+2}$*?*

Which is correct?

(A) x ≤ -2

(B) x ≥ -2

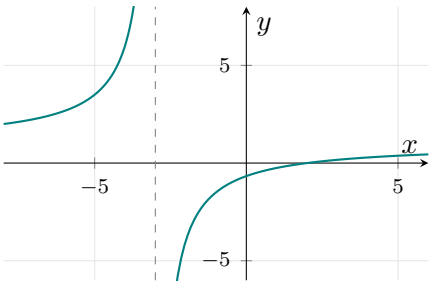
(C) all real numbers

(D) x ≥ 2

Rational expressions: simplify & operate

The skill. A rational expression is a ratio of polynomials. Simplify by FACTORING the numerator and denominator and canceling COMMON FACTORS (whole factors that multiply), never common terms (parts that add). To multiply, factor and cancel across, then multiply numerators and denominators; to divide, multiply by the reciprocal of the second expression. To add or subtract,

first rewrite over a common denominator. The domain excludes every value that makes any denominator zero, including values that later cancel.



Factor, then cancel common factors: $\frac{x^2-4}{x^2+5x+6} = \frac{x-2}{x+3}$*; excluded values come from the original denominator.*

Worked example

Simplify $\dfrac{x^2-4}{x^2+5x+6}$ **and state the domain restrictions.**

Read. We must simplify $\dfrac{x^2-4}{x^2+5x+6}$ and state which x-values are excluded. The domain restrictions come from wherever the *original* denominator is zero, so we note those before canceling anything.

Plan. Factor the numerator as a difference of squares and factor the denominator as a quadratic. Then cancel any factor common to both – canceling whole factors, never individual terms. Finally, list the excluded values from the original denominator, including a factor that cancels.

Solve.

$$\frac{x^2-4}{x^2+5x+6} = \frac{(x-2)(x+2)}{x^2+5x+6}$$ factor the numerator (difference of squares)

$$\frac{x^2-4}{x^2+5x+6} = \frac{(x-2)(x+2)}{(x+2)(x+3)}$$ factor the denominator quadratic

$$\frac{x^2-4}{x^2+5x+6} = \frac{x-2}{x+3}$$ cancel the common factor $(x+2)$

Check. The original denominator factors as $(x+2)(x+3)$, which is zero at $x=-2$ and $x=-3$. Both must be excluded from the domain, even though $(x+2)$ canceled – the restriction still comes from the original expression. So the simplified form is $\dfrac{x-2}{x+3}$ with $x \neq -2$ and $x \neq -3$.

Answer: $\dfrac{x-2}{x+3}$, with $x \neq -2$ and $x \neq -3$.

Approach Read · Plan · Solve · Check

Read. Identify the operation (simplify, multiply, divide, add/subtract) and every denominator that restricts the domain.

Plan. Factor first. Cancel only common factors. Divide by multiplying by the reciprocal. Add/subtract over a common denominator. List excluded values from the ORIGINAL

denominators.

Solve. Carry out factoring, canceling, and combining; keep the excluded values.

Check. Canceled factors (not terms)? Used a common denominator before adding? Inverted the correct fraction when dividing? Excluded all original denominator-zero values?

Common traps — every wrong answer is one of these

cancel-terms-not-factors error.
Cancels across addition/subtraction instead of canceling whole factors: e.g. (3x+6)/3 -> x+6, or cancels an x^2 in a sum, rather than factoring first and canceling a common factor.

no-common-denominator error.
Adds or subtracts by combining numerators and denominators straight across (a/b + c/d treated as (a+c)/(b+d)) instead of using a common denominator.

division/reciprocal error.
Divides without inverting (multiplies straight across), or inverts the wrong fraction (or both).

domain/excluded-value error.
Omits values that make a denominator zero, uses the numerator's zeros instead, or drops a restriction for a factor that cancels but was still excluded in the original.

Practice

1. *Simplify (3x + 6) / 3.*

What is the simplified form?

(A) x + 6
(B) x + 2
(C) x
(D) 3x + 2

2. *Simplify 1/x + 1/y as a single fraction.*

What is the result?

(A) x + y
(B) 2 / (x + y)
(C) 1 / (x + y)
(D) (x + y) / (xy)

3. *Simplify (a/b) divided by (c/d).*

What is the result?

(A) bc / ad

(B) ac / bd

(C) ad / bc

(D) bd / ac

4. *Consider the expression (x + 1) / (x^2 - 4).*

Which values of x are excluded from the domain?

(A) x = 4

(B) None; all real numbers are allowed

(C) x = 2 and x = -2

(D) x = -1

5. *Simplify (x^2 - x - 6) / (x^2 - 9) and state the domain restriction.*

Which is correct?

(A) (x + 2)/(x + 3), with no restriction

(B) (x + 2)/(x + 3), with x not equal to -3 only

(C) (-x - 6)/(-9), by canceling the x^2 terms

(D) (x + 2)/(x + 3), with x not equal to 3 and x not equal to -3

Multiply & divide rational expressions

The skill. To divide rational expressions, multiply by the reciprocal of the divisor (flip the second fraction), then cancel common factors. Dividing is NOT multiplying straight across, and only the divisor is flipped.

$$\frac{10}{x} \div \frac{4}{x} \xrightarrow{\text{flip}} \frac{10}{x} \cdot \frac{x}{4} = \frac{5}{2}$$

Divide by multiplying by the reciprocal of the divisor: flip only the second fraction, then cancel.

Worked example

Divide and simplify: $\frac{10}{x} \div \frac{4}{x}$.

Read. This is a division of two rational expressions, $\frac{10}{x} \div \frac{4}{x}$. To divide by a fraction, we multiply by its reciprocal, so we will flip the divisor (the second fraction).

Plan. Rewrite the division as multiplication by the reciprocal of $\frac{4}{x}$, which is $\frac{x}{4}$. Only the divisor is flipped – the first fraction stays as it is. Then multiply across and cancel common factors.

Solve.

$$\frac{10}{x} \div \frac{4}{x} = \frac{10}{x} \cdot \frac{x}{4} \qquad \text{multiply by the reciprocal of the divisor}$$

$$\frac{10}{x} \div \frac{4}{x} = \frac{10x}{4x} \qquad \text{multiply numerators and denominators}$$

$$\frac{10}{x} \div \frac{4}{x} = \frac{10}{4} \qquad \text{cancel the common factor } x$$

$$\frac{10}{x} \div \frac{4}{x} = \frac{5}{2} \qquad \text{reduce } \frac{10}{4}$$

Check. Only the divisor was flipped, and the factor x appears in both numerator and denominator so it cancels. Reducing $\frac{10}{4}$ gives $\frac{5}{2}$, the final answer.

Answer: $\frac{5}{2}$.

Approach Read · Plan · Solve · Check

Read. Is this a division? Then flip the divisor.

Plan. Multiply by the reciprocal; cancel common factors.

Solve. (a/x)(x/b) -> a/b.

Check. Did I flip only the second fraction? Did the x's cancel?

Common traps — every wrong answer is one of these

multiplied without flipping.
Multiplies the two fractions straight across instead of using the reciprocal.

inverted the result.
Flips the whole answer instead of just the divisor.

cancelled then multiplied.
Cancels the variable but multiplies the remaining coefficients.

Practice

1. *Divide: (5/x) ÷ (3/x). Simplify.*

Which is correct?

(A) 15
(B) 3/5
(C) $15/x^2$
(D) 5/3

2. *Divide: (2/x) ÷ (3/x). Simplify.*

Which is correct?

(A) 2/3
(B) 3/2
(C) 6
(D) $6/x^2$

3. *Divide: (3/x) ÷ (5/x). Simplify.*

Which is correct?

(A) $15/x^2$
(B) 5/3
(C) 3/5
(D) 15

4. *Divide: (3/x) ÷ (7/x). Simplify.*

Which is correct?

(A) $21/x^2$
(B) 21
(C) 7/3
(D) 3/7

5. *Divide: (8/x) ÷ (2/x). Simplify.*

Which is correct?

(A) 4
(B) $16/x^2$
(C) 1/4
(D) 16

Add & subtract rational expressions

The skill. With a common denominator, add fractions by adding numerators and keeping the denominator: a/x + b/x = (a+b)/x. Denominators are not added, numerators are not multiplied.

$$\frac{5}{x}+\frac{6}{x}=\frac{5+6}{x}=\frac{11}{x}$$

With a common denominator, add the numerators and keep the denominator unchanged.

Worked example

Add and simplify: $\frac{5}{x} + \frac{6}{x}$.

Read. Both fractions, $\frac{5}{x}$ and $\frac{6}{x}$, already share the common denominator x, so they are ready to be added directly.

Plan. When fractions have the same denominator, add the numerators and keep that denominator unchanged. Do not add the denominators together and do not multiply the numerators.

Solve.

$$\frac{5}{x} + \frac{6}{x} = \frac{5+6}{x}$$ add numerators over the common denominator

$$\frac{5}{x} + \frac{6}{x} = \frac{11}{x}$$ simplify $5 + 6 = 11$

Check. The denominator stayed x (we did not get $2x$), and the numerators were added, not multiplied, giving $\frac{11}{x}$.

Answer: $\frac{11}{x}$.

Approach Read · Plan · Solve · Check

Read. Do the fractions share a denominator?

Plan. Add the numerators; keep the common denominator.

Solve. (a+b)/x.

Check. Did the denominator stay x? Did I add (not multiply) numerators?

Common traps — every wrong answer is one of these

added the denominators.
Adds the denominators (x + x) instead of keeping the common one.

multiplied the numerators.
Multiplies the numerators instead of adding.

multiplied the denominators.
Multiplies the denominators (x·x) instead of keeping x.

Practice

1. *Add: 4/x + 5/x. Simplify.*

Which is correct?

(A) 9/(2x)

(B) 20/x

(C) 9/x

(D) $9/x^2$

2. *Add: 3/x + 5/x. Simplify.*

Which is correct?

(A) 8/x

(B) 8/(2x)

(C) 15/x

(D) $8/x^2$

3. *Add: 3/x + 1/x. Simplify.*

Which is correct?

(A) 4/(2x)

(B) 4/x

(C) 3/x

(D) $4/x^2$

4. *Add: 2/x + 5/x. Simplify.*

Which is correct?

(A) $7/x^2$

(B) 7/x

(C) 10/x

(D) 7/(2x)

5. *Add: 3/x + 7/x. Simplify.*

Which is correct?

(A) $10/x^2$

(B) 21/x

(C) 10/(2x)

(D) 10/x

Complex (compound) fractions

The skill. A fraction divided by a fraction: (1/a)/(1/b) = (1/a)·(b/1) = b/a. Dividing by 1/b multiplies by b, so flip the bottom fraction and multiply.

$$\frac{2/3}{4/9} = \frac{2}{3} \cdot \frac{9}{4} = \frac{3}{2}$$

A fraction over a fraction: flip the bottom fraction and multiply.

Worked example

Simplify the complex fraction: $\dfrac{2/3}{4/9}$.

Read. The expression $\dfrac{2/3}{4/9}$ is a complex fraction: a fraction divided by a fraction, here $\dfrac{2}{3}$ divided by $\dfrac{4}{9}$.

Plan. Dividing by a fraction is the same as multiplying by its reciprocal, so flip the bottom fraction $\dfrac{4}{9}$ to $\dfrac{9}{4}$ and multiply. Only the bottom (divisor) is inverted; then multiply across and reduce.

Solve.

$$\frac{2/3}{4/9} = \frac{2}{3} \cdot \frac{9}{4} \qquad \text{multiply by the reciprocal of the bottom}$$

$$\frac{2/3}{4/9} = \frac{18}{12} \qquad \text{multiply numerators and denominators}$$

$$\frac{2/3}{4/9} = \frac{3}{2} \qquad \text{reduce } \frac{18}{12} \text{ by 6}$$

Check. Only the bottom fraction was inverted. Multiplying back confirms it: $\dfrac{3}{2} \cdot \dfrac{4}{9} = \dfrac{12}{18} = \dfrac{2}{3}$, the original numerator, so $\dfrac{3}{2}$ is correct.

Answer: $\dfrac{3}{2}$.

Approach Read · Plan · Solve · Check

Read. This is a fraction over a fraction.

Plan. Flip the bottom fraction and multiply.

Solve. (1/a)(b/1) = b/a.

Check. Did I flip the BOTTOM only? Is the result b/a?

Common traps — every wrong answer is one of these

inverted the result.
Reports a/b instead of b/a.

multiplied the denominators.
Multiplies denominators to get 1/(ab).

multiplied straight across.
Multiplies numerators and denominators without flipping.

Practice

1. *Simplify the complex fraction: (1/4) / (1/5).*

Which is correct?

(A) 4/5
(B) 5/4
(C) 1/20
(D) 20

2. *Simplify the complex fraction: (1/4) / (1/3).*

Which is correct?

(A) 4/3
(B) 3/4
(C) 12
(D) 1/12

3. *Simplify the complex fraction: (1/2) / (1/5).*

Which is correct?

(A) 10
(B) 2/5
(C) 5/2
(D) 1/10

4. *Simplify the complex fraction: (1/3) / (1/5).*

Which is correct?

(A) 15
(B) 1/15
(C) 5/3
(D) 3/5

5. *Simplify the complex fraction: (3/4) / (2/5).*

Which is correct?

(A) 15/8

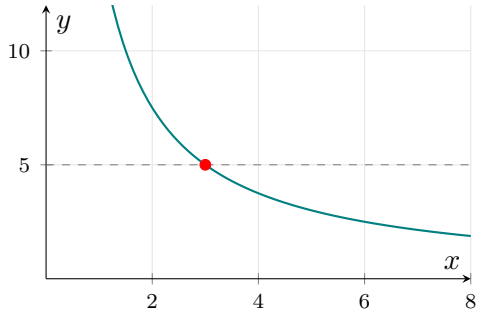
(B) 8/15

(C) 3/20

(D) 3/10

Solve rational equations

The skill. To solve a/x = b, multiply both sides by x to clear the fraction, then solve: a = bx, so x = a/b. Multiplying and dividing must not be swapped, and both sides of the equation are used.



Clear the fraction by multiplying by x: $\frac{15}{x} = 5$ gives $15 = 5x$, so $x = 3$ (the intersection).

Worked example

Solve $\dfrac{15}{x} = 5$ for x.

Read. We solve $\dfrac{15}{x} = 5$ for x. The variable sits in the denominator, so the first move is to clear the fraction by multiplying both sides by x.

Plan. Multiply both sides by x to remove the fraction, which gives a simple linear equation. Then divide both sides by the coefficient of x to isolate x.

Solve.

$\dfrac{15}{x} = 5$	start with the given equation
$15 = 5x$	multiply both sides by x
$\dfrac{15}{5} = x$	divide both sides by 5
$x = 3$	simplify $\dfrac{15}{5}$

Check. Substitute back into the original: $\dfrac{15}{3} = 5$, which matches the right side, so $x = 3$ is correct.

Answer: $x = 3$.

Approach Read · Plan · Solve · Check

Read. Clear the fraction by multiplying by x.

Plan. a/x = b -> a = bx -> x = a/b.

Solve. Divide by b.

Check. Did I divide (not multiply) by b? Does x check in the original?

Common traps — every wrong answer is one of these

multiplied instead of dividing.
Multiplies a and b instead of dividing.

ignored the coefficient.
Reports x = a, dropping the b.

inverted.
Reports b/a instead of a/b.

Practice

1. *Solve 12/x = 3 for x.*

Which is correct?

(A) x = 36
(B) x = 4
(C) x = 3/12
(D) x = 12

2. *Solve 9/x = 3 for x.*

Which is correct?

(A) x = 27
(B) x = 9
(C) x = 3
(D) x = 3/9

3. *Solve 8/x = 2 for x.*

Which is correct?

(A) x = 2/8
(B) x = 16
(C) x = 8

(D) x = 4

4. *Solve 6/x = 2 for x.*

Which is correct?

(A) x = 2/6
(B) x = 6
(C) x = 3
(D) x = 12

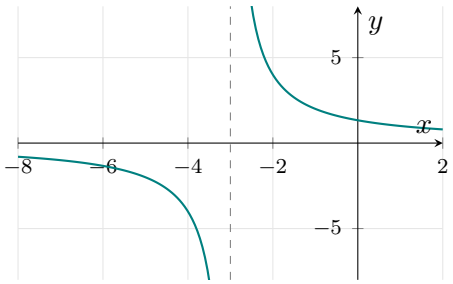
5. *Solve 20/x = 5 for x.*

Which is correct?

(A) 20
(B) 1/4
(C) 100
(D) 4

Excluded values & domain

The skill. A rational expression is undefined where its DENOMINATOR equals zero. For n/(x - a), set x - a = 0 to find the excluded value x = a. Exclusions come from the denominator, not the numerator.



A rational expression is undefined where the denominator is zero; $\frac{4}{x+3}$ excludes $x = -3$ (dashed line).

Worked example

For the expression $\frac{4}{x+3}$, which value of x is excluded from the domain?

Read. For $\frac{4}{x+3}$, the expression is undefined wherever the denominator equals zero, since division by zero is not allowed. The excluded value comes from the denominator, not the numerator.

Plan. Set the denominator equal to zero, $x + 3 = 0$, and solve for x. That value is the one

excluded from the domain.

Solve.

$$x + 3 = 0 \qquad \text{set the denominator equal to zero}$$
$$x = -3 \qquad \text{subtract 3 from both sides}$$

Check. At $x = -3$ the denominator is $-3 + 3 = 0$, making the expression undefined, so $x = -3$ is indeed the excluded value. The numerator 4 plays no role in the exclusion.

Answer: $x = -3$.

Approach Read · Plan · Solve · Check

Read. What makes the denominator zero?

Plan. Set the denominator = 0 and solve.

Solve. x - a = 0 -> x = a.

Check. Did I use the denominator (not numerator)? Right sign?

Common traps — every wrong answer is one of these

sign error.
Reports x = -a instead of x = a.

used the numerator.
Excludes the numerator's value instead of the denominator's.

missed the exclusion.
Claims there is no excluded value.

Practice

1. *For the expression 3/(x - 4), what value of x is excluded from the domain?*

Which is correct?

(A) x = -4
(B) x = 4
(C) none
(D) x = 3

2. *For the expression 2/(x - 3), what value of x is excluded from the domain?*

Which is correct?

(A) x = 3
(B) x = -3

(C) x = 2

(D) none

3. *For the expression 1/(x - 5), what value of x is excluded from the domain?*

Which is correct?

(A) x = 1

(B) none

(C) x = -5

(D) x = 5

4. *For the expression 2/(x - 5), what value of x is excluded from the domain?*

Which is correct?

(A) x = 5

(B) none

(C) x = 2

(D) x = -5

5. *For the expression 5/(x + 6), what value of x is excluded from the domain?*

Which is correct?

(A) x = 5

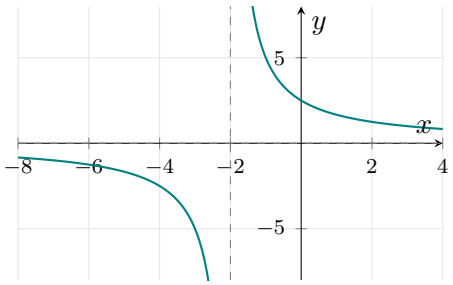
(B) none

(C) x = -6

(D) x = 6

Rational functions: asymptotes & holes

The skill. A vertical asymptote occurs where the denominator is zero (and the numerator is not): for f(x) = n/(x - a), it is at x = a. The horizontal asymptote of this form is y = 0; don't confuse the two.



Vertical asymptote where the denominator is zero ($x = -2$); the horizontal asymptote is $y = 0$.

Worked example

Where is the vertical asymptote of $f(x) = \dfrac{5}{x+2}$?

Read. For $f(x) = \dfrac{5}{x+2}$, a vertical asymptote occurs where the denominator is zero while the numerator is not. The numerator 5 is never zero, so we only need the zero of the denominator.

Plan. Set the denominator equal to zero, $x + 2 = 0$, and solve for x. The result gives the vertical line $x =$ (that value), which is the vertical asymptote.

Solve.

$x + 2 = 0$	set the denominator equal to zero
$x = -2$	subtract 2 from both sides

Check. This gives the vertical asymptote $x = -2$ (an $x =$ line), where the function blows up. Do not confuse it with the horizontal asymptote, which for this form is $y = 0$.

Answer: $x = -2$.

Approach Read · Plan · Solve · Check

Read. Vertical asymptote = where the denominator is 0.

Plan. Set denominator = 0.

Solve. x - a = 0 -> x = a.

Check. Is this the vertical (x =) asymptote, not the horizontal? Right sign?

Common traps — every wrong answer is one of these

sign error.
Reports x = -a instead of x = a.

horizontal/vertical mix-up.
Gives the horizontal asymptote y = 0 instead of the vertical one.

used the numerator.
Uses the numerator value instead of the denominator's zero.

Practice

1. *The function f(x) = 1/(x - 3) has a vertical asymptote at:*

Which is correct?

(A) x = 1

(B) x = 3

(C) x = -3

(D) y = 0

2. *The function f(x) = 2/(x - 4) has a vertical asymptote at:*

Which is correct?

(A) x = 4

(B) x = -4

(C) x = 2

(D) y = 0

3. *The function f(x) = 3/(x - 4) has a vertical asymptote at:*

Which is correct?

(A) x = 3

(B) x = -4

(C) x = 4

(D) y = 0

4. *The function f(x) = 2/(x - 3) has a vertical asymptote at:*

Which is correct?

(A) x = -3

(B) x = 2

(C) x = 3

(D) y = 0

5. *The function f(x) = 1/(x + 2) has a vertical asymptote at:*

Which is correct?

(A) y = -2

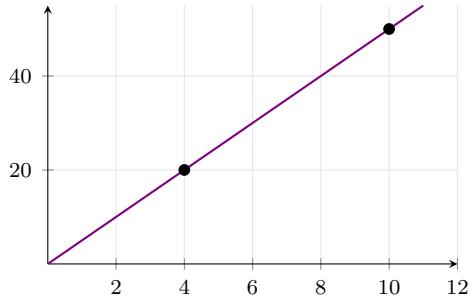
(B) x = 2

(C) x = 1

(D) x = -2

Direct & inverse variation

The skill. In direct variation y = kx, the ratio k = y/x is constant. Find k from a known pair, then use it to predict: y = k·x. Direct variation scales multiplicatively, not additively.



Direct variation $y = kx$ is a straight line through the origin; here $k = 5$, so $(4, 20)$ scales to $(10, 50)$.

Worked example

The quantity y varies directly with x. When $x = 4$, $y = 20$. Find y when $x = 10$.

Read. We are told y varies directly with x and given one pair, $(x, y) = (4, 20)$. We must predict y when $x = 10$. Direct variation means the ratio y/x stays constant, so the same constant k links every pair.

Plan. Direct variation has the form $y = kx$. First find the constant $k = \dfrac{y_1}{x_1}$ from the known pair, then evaluate $y = k \cdot x_2$ at the new input. Two clean stages: find k, then use it.

Solve.

$$
\begin{aligned}
k &= \frac{y_1}{x_1} && \text{direct variation: the ratio is constant} \\
&= \frac{20}{4} && \text{substitute the known pair } (4, 20) \\
&= 5 && \text{divide} \\
y &= k\,x_2 && \text{now predict at the new input} \\
&= 5 \cdot 10 && \text{substitute } k = 5 \text{ and } x_2 = 10 \\
&= 50 && \text{multiply}
\end{aligned}
$$

Check. The ratio holds: $\dfrac{50}{10} = 5 = \dfrac{20}{4}$, the same k. The output scaled multiplicatively by k; we did not add the change in x.

Answer: 50.

Approach Read · Plan · Solve · Check

Read. Direct variation: y = kx. Find k first.

Plan. k = y1/x1, then y = k·x2.

Solve. Multiply k by the new x.

Check. Did I find k before predicting? Did I scale (not add)?

Common traps — every wrong answer is one of these

reused the given y.
Reports the original y instead of scaling to the new x.

added instead of scaling.
Adds the change in x rather than multiplying by k.

scaled without finding k.
Multiplies the given y by the new x without dividing by the old x.

Practice

1. *y varies directly with x. When $x = 2$, $y = 8$. Find y when $x = 5$.*

Which is correct?

(A) 8
(B) 11
(C) 20
(D) 40

2. *y varies directly with x. When $x = 2$, $y = 6$. Find y when $x = 4$.*

Which is correct?

(A) 12
(B) 8
(C) 24
(D) 6

3. *y varies directly with x. When $x = 2$, $y = 6$. Find y when $x = 6$.*

Which is correct?

(A) 6
(B) 36
(C) 18
(D) 10

4. *y varies directly with x. When $x = 3$, $y = 12$. Find y when $x = 5$.*

Which is correct?

(A) 60
(B) 14

(C) 12

(D) 20

5. *y varies directly with x. When $x = 3$, $y = 12$. Find y when $x = 7$.*

Which is correct?

(A) 28

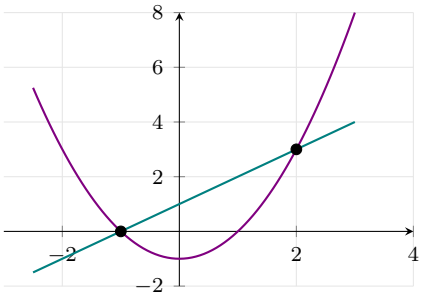
(B) 84

(C) 16

(D) 12

Nonlinear systems: line & parabola

The skill. A system with one linear and one quadratic equation is solved by SUBSTITUTION: solve the linear equation for one variable and substitute into the quadratic, producing a single quadratic equation set equal to zero. The number of real solutions equals the number of intersection points and comes from the discriminant of that quadratic: two (line crosses), one (tangent), or zero (no intersection). Each solution is an ordered pair (x, y): after solving for x, back-substitute to find y. Do not add or subtract the two equations as if both were linear.



A line meets a parabola where $x^2 - 1 = x + 1$: the two crossings are $(-1, 0)$ and $(2, 3)$.

Worked example

Find all intersection points of the system $y = x^2 - 1$ and $y = x + 1$.

Read. The system pairs one quadratic, $y = x^2 - 1$, with one line, $y = x + 1$. Its solutions are the points where the curves meet, so each solution is an ordered pair (x, y), not just an x-value.

Plan. Substitute the line's expression for y into the quadratic so both sides equal y. Move every term to one side to get a quadratic equal to zero, factor to find the x-values, then back-substitute each x into the line to get its y.

Solve.

$$\begin{aligned} x^2 - 1 &= x + 1 && \text{set the two expressions for } y \text{ equal} \\ x^2 - x - 2 &= 0 && \text{move every term to one side} \\ (x-2)(x+1) &= 0 && \text{factor the quadratic} \\ x &= 2,\ -1 && \text{zero-product property: each factor } = 0 \\ y &= x + 1 && \text{back-substitute into the line} \\ &= 3,\ 0 && \text{evaluate at } x = 2 \text{ and } x = -1 \end{aligned}$$

Check. Both roots are kept and paired as points $(2, 3)$ and $(-1, 0)$. Each satisfies both equations: $2 + 1 = 3 = 2^2 - 1$ and $-1 + 1 = 0 = (-1)^2 - 1$.

Answer: $(2, 3)$ and $(-1, 0)$.

Approach Read · Plan · Solve · Check

Read. One equation is linear, one is quadratic; the solutions are the points where the line meets the curve.

Plan. Substitute the linear expression into the quadratic, move everything to one side, and solve the resulting quadratic. Count solutions via the discriminant; find each (x, y) point.

Solve. Solve the quadratic for x (keep ALL roots), then back-substitute each x to get y.

Check. Did I substitute (not add) the equations? Did I keep every root? Did I report ordered pairs (x, y), not bare x-values? Does the count match the discriminant?

Common traps — every wrong answer is one of these

intersection-count error.
Miscounts intersection points, e.g. assuming a linear-quadratic system always has two solutions, or misreading the discriminant of the substituted quadratic.

substitution/setup error.
Adds or subtracts the two equations (elimination, as if both were linear) instead of substituting, or substitutes incorrectly and mishandles a term.

x-only-not-point error.
Solves for x but does not back-substitute for y, reporting bare x-values, a single coordinate, or swapped coordinates instead of ordered-pair points.

dropped-second-solution error.
Finds one intersection point but keeps only one root of the quadratic, dropping the second solution.

Practice

1. *Consider the system $y = x^2 + 1$ and $y = 2x + 4$.*

Substituting to solve, which single equation results?

(A) $x^2 + 1 = 2x$

(B) $x^2 + 1 + 2x + 4 = 0$

(C) $x^2 + 1 = 2x + 4$

(D) $x^2 = 2x + 4$

2. *The system $y = x^2$ and $y = x + 6$.*

What are the intersection points?

(A) (9, 3) and (4, -2)

(B) (3, 3) and (-2, -2)

(C) (3, 9) and (-2, 4)

(D) $x = 3$ and $x = -2$

3. *The system $y = x^2 + 4$ and $y = 2x$.*

How many real solutions does it have?

(A) One

(B) Infinitely many

(C) Zero

(D) Two

4. *The parabola $y = x^2 - 4$ and the line $y = 0$ (the x-axis).*

What are all solution points?

(A) (2, 0) only

(B) (-2, 0) only

(C) (2, 0), since a line meets a parabola at one point

(D) (2, 0) and (-2, 0)

5. *Solve the system $y = x^2 - 2x$ and $y = 3$.*

What are all solution points?

(A) (3, 3) only

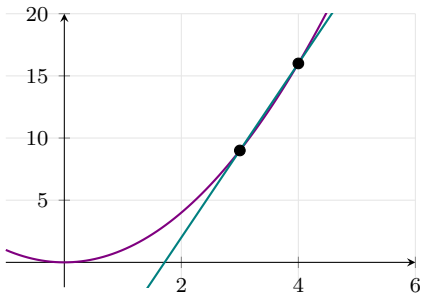
(B) (3, 3) and (-1, 3)

(C) No real solution

(D) $x = 3$ and $x = -1$

Solve line & parabola by substitution

The skill. Substitute the line into the parabola to get one quadratic, move everything to one side, and factor. $x^2 = mx - c$ becomes $x^2 - mx + c = 0$, whose roots are the x-values where the curves meet — usually two.



Setting $x^2 = 7x - 12$ gives $x^2 - 7x + 12 = 0$; the curves meet at $x = 3$ and $x = 4$.

Worked example

Solve the system $y = x^2$ and $y = 7x - 12$ for the x-values of the solutions.

Read. Both equations give y: $y = x^2$ and $y = 7x - 12$. Setting the two expressions equal turns the system into a single quadratic in x, whose roots are the x-coordinates where the line meets the parabola.

Plan. Set $x^2 = 7x - 12$, move every term to one side to form $x^2 - 7x + 12 = 0$, then factor into two binomials and read off both roots.

Solve.

$x^2 = 7x - 12$	set the two expressions for y equal
$x^2 - 7x + 12 = 0$	move every term to one side
$(x - 3)(x - 4) = 0$	factor the quadratic
$x = 3,\ 4$	zero-product property

Check. Both roots satisfy both equations: at $x = 3$, $3^2 = 9 = 7(3) - 12$; at $x = 4$, $4^2 = 16 = 7(4) - 12$. Two intersection points, as expected for a line cutting a parabola.

Answer: $x = 3$ and $x = 4$.

Approach Read · Plan · Solve · Check

Read. Set the expressions equal; make a quadratic = 0.

Plan. $x^2 - mx + c = 0$; factor.

Solve. $(x - r1)(x - r2) = 0$.

Check. Two roots? Do they satisfy both equations?

Common traps — every wrong answer is one of these

read off coefficients.
Reports the coefficients m and c as the solutions instead of solving.

sign error.
Gives the negatives of the roots.

one solution only.
Stops at a single intersection.

Practice

1. *Solve the system $y = x^2$ and $y = 5x - 4$. What are the x-values of the solutions?*

Which is correct?

(A) x = -1 and x = -4
(B) x = 1 and x = 4
(C) x = 1 only
(D) x = 5 and x = 4

2. *Solve the system $y = x^2$ and $y = 4x - 3$. What are the x-values of the solutions?*

Which is correct?

(A) x = 4 and x = 3
(B) x = 1 only
(C) x = 1 and x = 3
(D) x = -1 and x = -3

3. *Solve the system $y = x^2$ and $y = 6x - 8$. What are the x-values of the solutions?*

Which is correct?

(A) x = 6 and x = 8
(B) x = 2 only
(C) x = -2 and x = -4
(D) x = 2 and x = 4

4. *Solve the system $y = x^2$ and $y = 6x - 5$. What are the x-values of the solutions?*

Which is correct?

(A) x = 1 and x = 5
(B) x = -1 and x = -5

(C) x = 1 only

(D) x = 6 and x = 5

5. *Solve the system $y = x^2$ and $y = 6x - 8$. What are the x-values of the solutions?*

Which is correct?

(A) x = 2 and x = 4

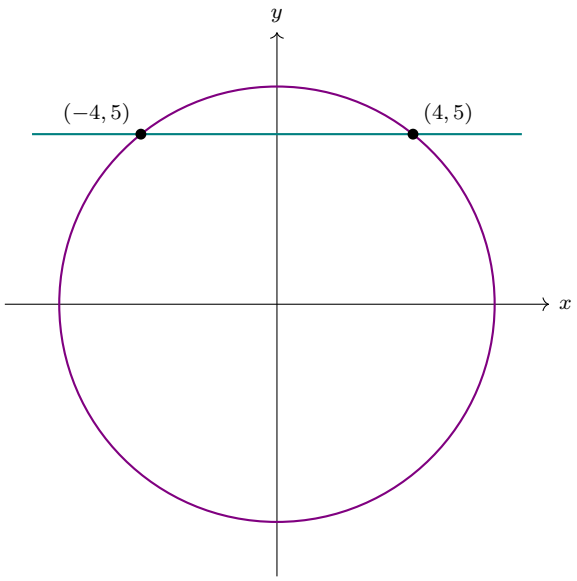
(B) x = -2 and x = -4

(C) x = 2

(D) x = 6 and x = -8

Line & circle systems

The skill. Substitute the line into the circle equation. For $x^2 + y^2 = R^2$ with y = b, we get $x^2 = R^2 - b^2$. Take the square root for the two x-values $\pm\sqrt{R^2 - b^2}$.



The line $y = 5$ cuts the circle $x^2 + y^2 = 41$ at two points, $x = \pm 4$.

Worked example

The circle $x^2+y^2 = 41$ meets the line $y = 5$. Find the x-coordinates of the intersection points.

Read. The line $y = 5$ is horizontal, so every point on it has $y = 5$. Substituting that fixed y into the circle $x^2 + y^2 = 41$ leaves a single equation in x for the crossing points.

Plan. Replace y with 5 to get $x^2 = R^2 - b^2 = 41 - 5^2$, then take the square root of both sides, keeping the $\pm$ so both intersection points appear.

Solve.

$$\begin{aligned} x^2 &= R^2 - b^2 && \text{substitute } y = 5 \text{ into } x^2 + y^2 = R^2 \\ &= 41 - 25 && \text{use } R^2 = 41 \text{ and } b^2 = 5^2 = 25 \\ &= 16 && \text{subtract} \\ x &= \pm\sqrt{16} && \text{take the square root of both sides} \\ &= \pm 4 && \text{simplify the root} \end{aligned}$$

Check. The ± gives two values, as a line should cut a circle in two points. Each checks: $4^2 + 5^2 = 16 + 25 = 41$ and $(-4)^2 + 5^2 = 41$.

Answer: $x = 4$ and $x = -4$.

Approach Read · Plan · Solve · Check

Read. Substitute the line's y into the circle.

Plan. $x^2 = R^2 - b^2$; take $\pm\sqrt{}$.

Solve. $x = \pm\sqrt{R^2 - b^2}$.

Check. Two x-values (±)? Did I subtract b^2 correctly?

Common traps — every wrong answer is one of these

used the line value.
Reports ±b instead of solving for x.

one point only.
Gives a single x instead of ±.

used R^2.
Reports the radius-squared as an x-value.

Practice

1. *The circle $x^2 + y^2 = 52$ meets the line $y = 6$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 4 and x = -4

(B) x = 6 and x = -6

(C) x = 52

(D) x = 4

2. *The circle $x^2 + y^2 = 25$ meets the line $y = 3$. What are the x-coordinates of the intersection*

points?

Which is correct?

(A) x = 4

(B) x = 25

(C) x = 3 and x = -3

(D) x = 4 and x = -4

3. *The circle $x^2 + y^2 = 34$ meets the line $y = 3$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 5 and x = -5

(B) x = 3 and x = -3

(C) x = 5

(D) x = 34

4. *The circle $x^2 + y^2 = 61$ meets the line $y = 6$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 61

(B) x = 6 and x = -6

(C) x = 5

(D) x = 5 and x = -5

5. *The circle $x^2 + y^2 = 25$ meets the line $y = 3$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 3 and x = -3

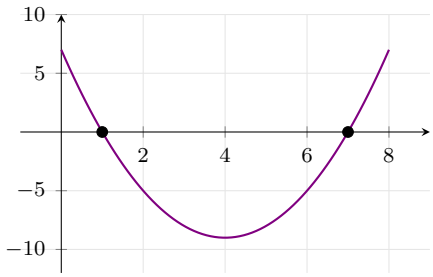
(B) x = 4 and x = -4

(C) x = 5 and x = -5

(D) x = 4

Number of intersections (discriminant)

The skill. After reducing a system to $ax^2 + bx + c = 0$, the discriminant $b^2 - 4ac$ tells how many real intersections: positive = 2, zero = 1, negative = 0. Compute it as $b^2 - 4ac$, subtracting (not adding) and including the 4.



Discriminant $b^2 - 4ac = 36 > 0$, so the parabola crosses the axis twice ($x = 1$ and $x = 7$).

Worked example

A line-parabola system reduces to $x^2 - 8x + 7 = 0$. Find the discriminant $b^2 - 4ac$.

Read. The system has already been reduced to the quadratic $x^2 - 8x + 7 = 0$. Matching it to $ax^2 + bx + c = 0$ gives $a = 1$, $b = -8$, $c = 7$. The discriminant will tell how many real intersections there are.

Plan. Compute $b^2 - 4ac$. Square b, form the product $4ac$, and subtract; keep the factor of 4 and the minus sign. A positive result means two intersections, zero means one, negative means none.

Solve.

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4(1)(7) && \text{substitute } a = 1,\ b = -8,\ c = 7 \\ &= 64 - 28 && \text{square } b \text{ and form } 4ac \\ &= 36 && \text{subtract} \end{aligned}$$

Check. We subtracted $4ac$ rather than adding, and kept the 4. Since $36 > 0$, the parabola crosses the axis twice, matching two real intersection points.

Answer: 36.

Approach Read · Plan · Solve · Check

Read. Identify a, b, c from the reduced quadratic.

Plan. Compute b^2 - 4ac.

Solve. Square b, subtract 4ac.

Check. Did I subtract 4ac (with the 4)?

Common traps — every wrong answer is one of these

added instead of subtracting.
Computes b^2 + 4ac.

dropped the c.
Uses b^2 - 4 instead of b^2 - 4c.

dropped the 4.
Uses b^2 - c instead of b^2 - 4c.

Practice

1. *A line-parabola system reduces to x^2 - 4x + 2 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 12
(B) 8
(C) 24
(D) 14

2. *A line-parabola system reduces to x^2 - 6x + 3 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 32
(B) 33
(C) 24
(D) 48

3. *A line-parabola system reduces to x^2 - 4x + 3 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 28
(B) 12
(C) 13
(D) 4

4. *A line-parabola system reduces to x^2 - 5x + 2 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 33
(B) 17
(C) 23
(D) 21

5. *A line-parabola system reduces to x^2 - 6x + 5 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 31

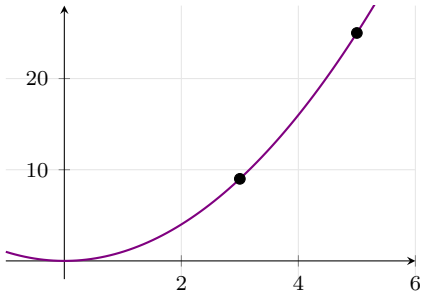
(B) 56

(C) 16

(D) 36

Report intersection points (x and y)

The skill. An intersection is a POINT (x, y), not just an x-value. After finding each x, substitute back (here y = x^2) to get y, then pair them: (x, y). Coordinates are ordered (x first, then y).



Each solution is a point: $x = 3, 5$ on $y = x^2$ give $(3, 9)$ and $(5, 25)$.

Worked example

A line-parabola system has solutions $x = 3$ and $x = 5$. Using $y = x^2$, give the intersection points.

Read. The x-values $x = 3$ and $x = 5$ solve the system, but an intersection is a full point (x, y). We still need the matching y-coordinate for each, found from $y = x^2$.

Plan. Substitute each x into $y = x^2$ to get its y, then write the results as ordered pairs (x, y) with x first.

Solve.

$y(3) = 3^2$	substitute $x = 3$ into $y = x^2$
$= 9$	square
$y(5) = 5^2$	substitute $x = 5$
$= 25$	square
$(x, y) = (3, 9), (5, 25)$	pair each x with its y

Check. Each y was computed from $y = x^2$, and the coordinates are in (x, y) order, not swapped. The points $(3, 9)$ and $(5, 25)$ lie on the parabola.

Answer: $(3, 9)$ and $(5, 25)$.

Approach Read · Plan · Solve · Check

Read. Each solution is a point (x, y).

Plan. Find y from $y = x^2$ for each x.

Solve. Pair (x, x^2).

Check. Did I compute y? Are coordinates in (x, y) order?

Common traps — every wrong answer is one of these

used x as y.
Pairs (x, x) without computing y.

swapped coordinates.
Writes (y, x) instead of (x, y).

gave only x.
Stops at the x-values, no points.

Practice

1. *A line-parabola system has solutions $x = 1$ and $x = 4$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (1, 1) and (16, 4)
(B) (1, 1) and (4, 16)
(C) (1, 1) and (4, 4)
(D) x = 1 and x = 4

2. *A line-parabola system has solutions $x = 2$ and $x = 4$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) x = 2 and x = 4
(B) (2, 4) and (4, 16)
(C) (4, 2) and (16, 4)
(D) (2, 2) and (4, 4)

3. *A line-parabola system has solutions $x = 1$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (1, 1) and (3, 9)
(B) (1, 1) and (3, 3)
(C) x = 1 and x = 3
(D) (1, 1) and (9, 3)

4. *A line-parabola system has solutions $x = 2$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (4, 2) and (9, 3)
(B) (2, 2) and (3, 3)
(C) x = 2 and x = 3
(D) (2, 4) and (3, 9)

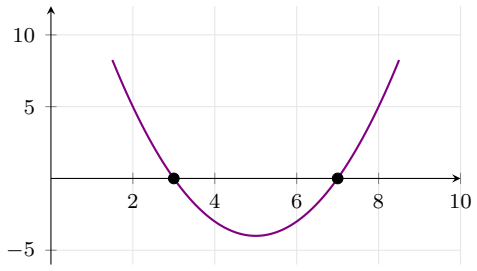
5. *A line-parabola system has solutions $x = 2$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (2, 4) and (3, 9)
(B) (2, 2) and (3, 3)
(C) (4, 2) and (9, 3)
(D) x = 2 and x = 3

Nonlinear systems in context

The skill. Two numbers with a known sum S and product P are the roots of x^2 - Sx + P = 0. The sum enters as -(sum) on the x-term and the product as the constant.



Sum 10 and product 21 are the roots of $x^2 - 10x + 21 = 0$: $x = 3$ and $x = 7$.

Worked example

Two numbers add to 10 and multiply to 21. Which equation do they satisfy?

Read. Two unknown numbers have sum $S = 10$ and product $P = 21$. Numbers with a known

sum and product are exactly the roots of a quadratic, so we can write the equation they satisfy.

Plan. Use the pattern $x^2 - Sx + P = 0$: the sum enters as $-S$ on the x-term and the product as the constant $+P$. Substitute $S = 10$ and $P = 21$.

Solve.

$$x^2 - Sx + P = 0 \qquad \text{roots with sum } S \text{ and product } P$$
$$x^2 - 10x + 21 = 0 \qquad \text{substitute } S = 10 \text{ and } P = 21$$

Check. Factoring gives $(x - 3)(x - 7) = 0$, and indeed $3 + 7 = 10$ and $3 \cdot 7 = 21$, so the sum and product match.

Answer: $x^2 - 10x + 21 = 0$.

Approach Read · Plan · Solve · Check

Read. Sum and product -> a quadratic.

Plan. x² - (sum)x + (product) = 0.

Solve. x² - Sx + P = 0.

Check. Sum negated on x-term? Product is the constant?

Common traps — every wrong answer is one of these

sum sign.
Uses +Sx instead of -Sx.

swapped S and P.
Puts the product on the x-term and the sum as the constant.

product sign.
Uses -P instead of +P.

Practice

1. *Two numbers add to 7 and multiply to 12. Which equation do they satisfy?*

Which is correct?

(A) x² - 12x + 7 = 0
(B) x² + 7x + 12 = 0
(C) x² - 7x + 12 = 0
(D) x² - 7x - 12 = 0

2. *Two numbers add to 6 and multiply to 8. Which equation do they satisfy?*

Which is correct?

(A) x^2 - 6x - 8 = 0

(B) x^2 - 6x + 8 = 0

(C) x^2 - 8x + 6 = 0

(D) x^2 + 6x + 8 = 0

3. *Two numbers add to 8 and multiply to 15. Which equation do they satisfy?*

Which is correct?

(A) x^2 + 8x + 15 = 0

(B) x^2 - 8x - 15 = 0

(C) x^2 - 15x + 8 = 0

(D) x^2 - 8x + 15 = 0

4. *Two numbers add to 9 and multiply to 18. Which equation do they satisfy?*

Which is correct?

(A) x^2 - 9x - 18 = 0

(B) x^2 - 9x + 18 = 0

(C) x^2 + 9x + 18 = 0

(D) x^2 - 18x + 9 = 0

5. *Two numbers add to 9 and multiply to 20. Which equation do they satisfy?*

Which is correct?

(A) x^2 + 9x + 20 = 0

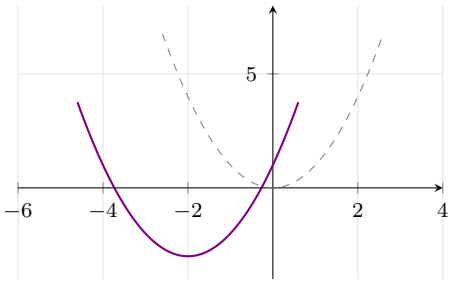
(B) x^2 - 9x - 20 = 0

(C) x^2 - 20x + 9 = 0

(D) x^2 - 9x + 20 = 0

Function transformations

The skill. Starting from y = f(x): a change OUTSIDE the function affects the output (y) and behaves as written, while a change INSIDE f( ) affects the input (x) and behaves OPPOSITE to the sign. So y = f(x) + k shifts vertically by k; y = f(x - h) shifts horizontally RIGHT by h (right for minus, because the input must grow to compensate). y = -f(x) reflects across the x-axis (output sign), while y = f(-x) reflects across the y-axis (input sign). y = a*f(x) stretches vertically when |a| > 1 and compresses when |a| < 1.



$f(x) = x^2$ (dashed) with $g(x) = f(x+2) - 3$: left 2 (inside) and down 3 (outside).

Worked example

Let $g(x) = f(x+2) - 3$. Describe how the graph of g relates to the graph of f.

Read. The rule $g(x) = f(x+2) - 3$ carries two separate changes: an inside change $x + 2$, which acts on the input and is horizontal, and an outside change -3, which acts on the output and is vertical.

Plan. Treat the two changes independently. An inside change reverses its sign and moves the graph horizontally; an outside change is taken as written and moves it vertically.

Solve.

inside $x + 2$ = shift left 2	inside acts on x and reverses the sign
outside -3 = shift down 3	outside acts on y as written

Check. Inside and outside were kept straight: the horizontal move reversed the + to a leftward shift, while the vertical move followed the -3 downward.

Answer: shifted left 2 and down 3.

Approach Read · Plan · Solve · Check

Read. Is the change inside f( ) (acts on x, horizontal, opposite sign) or outside (acts on y, vertical, as written)? Is it a shift, a reflection, or a stretch?

Plan. Outside +k: vertical shift. Inside (x - h): horizontal shift right by h. -f(x): reflect over x-axis. f(-x): reflect over y-axis. a*f(x): vertical stretch (>1) or compress (<1).

Solve. Apply each piece; for combined forms handle the inside change (horizontal, opposite) and outside change (vertical, as written) separately.

Check. Did I move horizontally opposite to the inside sign? Did I keep inside vs outside straight (x vs y)? Right reflection axis? Stretch vs compress matched to |a|?

Common traps — every wrong answer is one of these

horizontal-shift direction error.
Reads y = f(x - h) as a shift LEFT by h (following the minus sign literally) instead of right, reversing the horizontal direction of an inside change.

inside/outside confusion.
Mixes up which change affects which variable: e.g. treating an outside +k as a horizontal shift, or an inside change as vertical, or confusing add with scale.

reflection-axis error.
Swaps the reflection axes, taking -f(x) to reflect across the y-axis and f(-x) across the x-axis, or confuses a reflection with an inverse.

stretch/compression direction error.
Inverts vertical stretch and compression for a*f(x), e.g. treating a > 1 as a compression, or confuses vertical with horizontal scaling.

Practice

1. *Let g(x) = f(x) + 3.*

The graph of g is the graph of f:

(A) stretched vertically by a factor of 3

(B) shifted up 3 units

(C) shifted left 3 units

(D) shifted right 3 units

2. *Let g(x) = f(x - 2).*

The graph of g is the graph of f:

(A) shifted right 2 units

(B) shifted down 2 units

(C) shifted left 2 units

(D) shifted up 2 units

3. *Let g(x) = -f(x).*

The graph of g is the graph of f reflected across:

(A) the origin

(B) the x-axis

(C) the line y = x

(D) the y-axis

4. *Let g(x) = 3*f(x).*

The graph of g is the graph of f:

(A) shifted up 3 units

(B) stretched vertically by a factor of 3

(C) stretched horizontally by a factor of 3

(D) compressed vertically by a factor of 3

5. *Let g(x) = f(x + 1) - 4.*

The graph of g is the graph of f:

(A) shifted left 1 unit and down 4 units

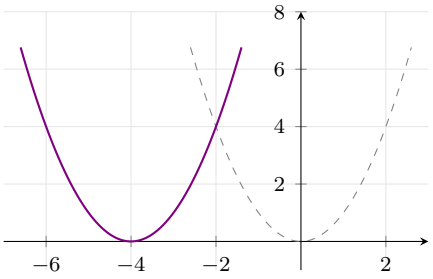
(B) shifted right 1 unit and down 4 units

(C) shifted left 1 unit and up 4 units

(D) shifted left 4 units and down 1 unit

Horizontal shifts

The skill. In g(x)=f(x-h), the change is INSIDE the function, so it moves the graph horizontally. The minus sign means a shift RIGHT by h (counterintuitively); f(x+h) shifts LEFT by h. Inside changes are horizontal and 'backwards'; outside changes are vertical and direct.



The inside change $f(x+4)$ shifts the graph LEFT 4 (opposite the + sign).

Worked example

Let $g(x) = f(x+4)$. How does the graph of g relate to the graph of f?

Read. In $g(x) = f(x+4)$, the $+4$ sits inside $f(\ )$, so it changes the input. Inside changes move the graph horizontally, not vertically.

Plan. Apply the inside-sign rule: an inside change reverses its sign, so $f(x+h)$ shifts LEFT by h while $f(x-h)$ shifts right. Here $h = 4$.

Solve.

inside $x + 4$ = horizontal shift	the $+4$ sits inside $f(\ )$
= left 4	inside changes reverse the sign

Check. The inside sign was reversed, turning $+4$ into a leftward move, and the shift is horizontal rather than vertical.

Answer: shifted left 4 units.

Approach Read · Plan · Solve · Check

Read. Is the change inside f( ) or outside?

Plan. Inside -> horizontal and reversed: f(x-h) is right h.

Solve. Apply the right/left rule.

Check. Did I reverse the inside sign? Inside = horizontal, not vertical?

Common traps — every wrong answer is one of these

horizontal-shift direction error.
Shifts left for f(x-h) instead of right, reversing the counterintuitive inside-sign rule.

inside/outside confusion.
Treats an inside change f(x-h) as a vertical shift instead of a horizontal one.

output-shift error.
Applies an inside (horizontal) change as a vertical shift of the output, up or down.

Practice

1. *Let g(x) = f(x - 3). The graph of g is the graph of f:*

Which is correct?

(A) shifted right 3 units
(B) shifted up 3 units
(C) shifted down 3 units
(D) shifted left 3 units

2. *Let g(x) = f(x - 4). The graph of g is the graph of f:*

Which is correct?

(A) shifted down 4 units
(B) shifted right 4 units
(C) shifted up 4 units
(D) shifted left 4 units

3. *Let g(x) = f(x - 5). The graph of g is the graph of f:*

Which is correct?

(A) shifted left 5 units

(B) shifted right 5 units

(C) shifted up 5 units

(D) shifted down 5 units

4. *Let g(x) = f(x - 2). The graph of g is the graph of f:*

Which is correct?

(A) shifted up 2 units

(B) shifted down 2 units

(C) shifted right 2 units

(D) shifted left 2 units

5. *Let g(x) = f(x - 5). The graph of g is the graph of f:*

Which is correct?

(A) shifted left 5 units

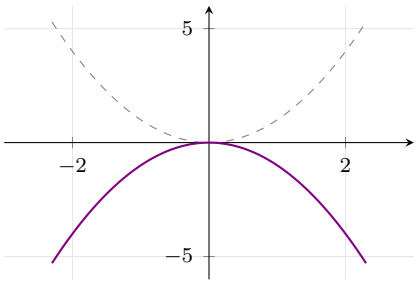
(B) shifted up 5 units

(C) shifted right 5 units

(D) shifted down 5 units

Reflection over the x-axis

The skill. g(x)=-f(x) negates the OUTPUT, reflecting every point over the x-axis: (a,b) -> (a,-b). Compare g(x)=f(-x), which negates the input and reflects over the y-axis. Outside negation = x-axis.



$g(x) = -f(x)$ reflects across the x-axis, negating the output: $(a, b) \to (a, -b)$.

Worked example

The point $(4, -1)$ lies on $y = f(x)$. Under $g(x) = -f(x)$, find the corresponding point on g.

Read. In $g(x) = -f(x)$ the negative sign is outside f, so it acts on the output. Negating the output reflects every point across the x-axis.

Plan. Apply the map $(a, b) \to (a, -b)$: keep the x-coordinate and negate the y-coordinate of the given point $(4, -1)$.

Solve.

$$\begin{aligned} \text{image of } (4,-1) &= (4,-y) && \text{outside } - \text{ negates the output (reflect over } x\text{-axis)} \\ &= (4,-(-1)) && \text{substitute the output } y=-1 \\ &= (4,1) && \text{simplify} \end{aligned}$$

Check. Only the y-coordinate was flipped; $x = 4$ stayed fixed, as an x-axis reflection requires.
Answer: $(4, 1)$.

Approach Read · Plan · Solve · Check

Read. Is the negative outside (-f) or inside (f(-x))?

Plan. Outside negative -> reflect over x-axis: (a,b)->(a,-b).

Solve. Negate the y-coordinate.

Check. Did I flip y (not x)?

Common traps — every wrong answer is one of these

reflection-axis error.
Reflects over the y-axis (negating x) when -f(x) reflects over the x-axis (negating y).

inside/outside confusion.
Leaves the point unchanged or alters the wrong coordinate.

no reflection applied.
Leaves the point unchanged or changes the wrong coordinate, failing to negate the output.

Practice

1. *The point (2, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (2, 2)
(B) (-2, -2)
(C) (2, -2)
(D) (-2, 2)

2. *The point (1, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (1, -2)
(B) (-1, 2)
(C) (1, 2)
(D) (-1, -2)

3. *The point (1, 3) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (-1, -3)
(B) (1, -3)
(C) (-1, 3)
(D) (1, 3)

4. *The point (3, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (3, 2)
(B) (-3, -2)
(C) (3, -2)
(D) (-3, 2)

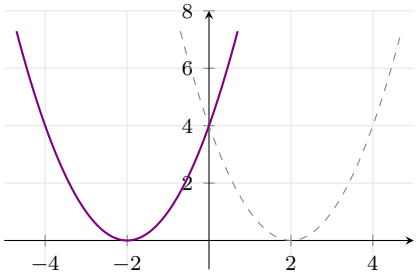
5. *The point (5, -3) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (-5, 3)
(B) (5, -3)
(C) (-5, -3)
(D) (5, 3)

Reflection over the y-axis

The skill. g(x)=f(-x) negates the INPUT, reflecting every point over the y-axis: (a,b) -> (-a,b). Compare -f(x), which reflects over the x-axis. Inside negation = y-axis.



$g(x) = f(-x)$ reflects across the y-axis, negating the input: $(a, b) \to (-a, b)$.

Worked example

The point $(6, 2)$ lies on $y = f(x)$. Under $g(x) = f(-x)$, find the corresponding point on g.

Read. In $g(x) = f(-x)$ the negative sign is inside f, so it acts on the input. Negating the input reflects every point across the y-axis.

Plan. Apply the map $(a, b) \to (-a, b)$: negate the x-coordinate and keep the y-coordinate of the given point $(6, 2)$.

Solve.

$$\begin{aligned} \text{image of } (6,2) &= (-x, 2) && \text{inside } - \text{ negates the input (reflect over } y\text{-axis)} \\ &= (-(6), 2) && \text{substitute the input } x = 6 \\ &= (-6, 2) && \text{simplify} \end{aligned}$$

Check. Only the x-coordinate was flipped; $y = 2$ stayed fixed, as a y-axis reflection requires.
Answer: $(-6, 2)$.

Approach Read · Plan · Solve · Check

Read. Is the negative inside f( )?

Plan. f(-x) -> reflect over y-axis: (a,b)->(-a,b).

Solve. Negate the x-coordinate.

Check. Did I flip x (not y)?

Common traps — every wrong answer is one of these

reflection-axis error.
Reflects over the x-axis (negating y) when f(-x) reflects over the y-axis (negating x).

inside/outside confusion.
Leaves the point unchanged or alters the wrong coordinate.

output-negation error.
Negates the output y instead of the input x, reflecting over the wrong axis.

Practice

1. *The point (3, 2) lies on y = f(x). Under g(x) = f(-x), the corresponding point on g is:*

Which is correct?

(A) (3, 2)

(B) (3, -2)
(C) (-3, -2)
(D) (-3, 2)

2. *The point (1, 3) lies on y = f(x). Under g(x) = f(-x), the corresponding point on g is:*

Which is correct?

(A) (-1, -3)
(B) (-1, 3)
(C) (1, 3)
(D) (1, -3)

3. *The point (2, 3) lies on y = f(x). Under g(x) = f(-x), the corresponding point on g is:*

Which is correct?

(A) (-2, 3)
(B) (2, 3)
(C) (2, -3)
(D) (-2, -3)

4. *The point (3, 3) lies on y = f(x). Under g(x) = f(-x), the corresponding point on g is:*

Which is correct?

(A) (3, 3)
(B) (-3, -3)
(C) (3, -3)
(D) (-3, 3)

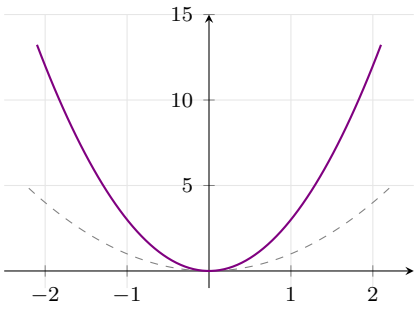
5. *The point (4, 5) lies on y = f(x). Under g(x) = f(-x), the corresponding point on g is:*

Which is correct?

(A) (-4, -5)
(B) (4, 5)
(C) (-4, 5)
(D) (4, -5)

Vertical stretch & compression

The skill. g(x)=a*f(x) multiplies the OUTPUT by a. If a>1 the graph stretches vertically by a factor of a; if 0<a<1 it compresses vertically. Outside multiplication scales vertically.



$g(x) = a\,f(x)$ with $a > 1$ multiplies each output, stretching vertically (shown $\times 3$).

Worked example

Let $g(x) = 5f(x)$, where $5 > 1$. How does the graph of g relate to the graph of f?

Read. In $g(x) = 5f(x)$ the multiplier 5 is outside f, so it scales the output. Multiplying every output value stretches or compresses the graph vertically.

Plan. Use the rule for $a \cdot f(x)$: when $a > 1$ the graph stretches vertically by the factor a; when $0 < a < 1$ it compresses. Here $a = 5 > 1$.

Solve.

outside factor 5 = scales the output	multiplication outside f is vertical
= vertical stretch by 5	a factor above 1 stretches

Check. The multiply acted vertically, not horizontally, and since $5 > 1$ the graph stretched rather than compressed.

Answer: stretched vertically by a factor of 5.

Approach Read · Plan · Solve · Check

Read. Is the multiplier outside f( )?

Plan. a>1 -> vertical stretch by a.

Solve. Apply the factor.

Check. Outside multiply = vertical scale; a>1 = stretch?

Common traps — every wrong answer is one of these

stretch/compression direction error.
Calls a>1 a compression (or a horizontal change) instead of a vertical stretch by a.

inside/outside confusion.
Treats the outside multiplier as a horizontal change.

moved instead of scaled.
Shifts the graph up or down by a instead of multiplying the output by a.

Practice

1. *Let g(x) = 3*f(x), where 3 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed vertically by a factor of 3
(B) stretched vertically by a factor of 3
(C) stretched horizontally by a factor of 3
(D) shifted up 3 units

2. *Let g(x) = 2*f(x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 2
(B) stretched horizontally by a factor of 2
(C) compressed vertically by a factor of 2
(D) shifted up 2 units

3. *Let g(x) = 4*f(x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 4
(B) compressed vertically by a factor of 4
(C) shifted up 4 units
(D) stretched horizontally by a factor of 4

4. *Let g(x) = 4*f(x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched horizontally by a factor of 4
(B) stretched vertically by a factor of 4
(C) shifted up 4 units
(D) compressed vertically by a factor of 4

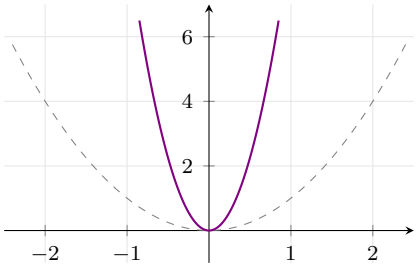
5. *Let g(x) = (1/2)*f(x). The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 2
(B) compressed horizontally
(C) shifted down 2 units
(D) compressed vertically by a factor of 1/2

Horizontal stretch & compression

The skill. g(x)=f(b*x) multiplies the INPUT by b. If b>1 the graph compresses horizontally by a factor of 1/b (counterintuitively); if 0<b<1 it stretches. Inside multiplication scales horizontally and is reversed.



$g(x) = f(bx)$ with $b > 1$ compresses horizontally by $1/b$ (shown $b = 3$).

Worked example

Let $g(x) = f(5x)$, where $5 > 1$. How does the graph of g relate to the graph of f?

Read. In $g(x) = f(5x)$ the multiplier 5 is inside f, so it scales the input. Inside scaling is horizontal and behaves in reverse, so a large multiplier squeezes the graph.

Plan. Use the rule for $f(bx)$: when $b > 1$ the graph compresses horizontally by the factor $\frac{1}{b}$; when $0 < b < 1$ it stretches. Here $b = 5$.

Solve.

$$\begin{aligned} \text{inside factor } 5 &= \text{scales the input} && \text{multiplication inside } f \text{ is horizontal} \\ &= \text{compress by } \frac{1}{5} && \text{inside scaling is reversed, so } 5 > 1 \text{ squeezes} \end{aligned}$$

Check. The multiply acted horizontally and in reverse, so $b > 1$ produced a compression by $\frac{1}{5}$, not a stretch.

Answer: compressed horizontally by a factor of $\frac{1}{5}$.

Approach Read · Plan · Solve · Check

Read. Is the multiplier inside f( )?

Plan. b>1 -> horizontal compression by 1/b.

Solve. Apply 1/b.

Check. Inside multiply = horizontal and reversed?

Common traps — every wrong answer is one of these

stretch/compression direction error.
Calls f(bx) with b>1 a horizontal STRETCH instead of a compression by 1/b.

inside/outside confusion.
Treats the inside multiplier as a vertical change.

confused stretch with shift.
Treats the input multiplier as a horizontal shift rather than a compression or stretch.

Practice

1. *Let g(x) = f(4x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 4 units
(B) compressed horizontally by a factor of 1/4
(C) stretched horizontally by a factor of 4
(D) compressed vertically by a factor of 4

2. *Let g(x) = f(2x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 2 units
(B) stretched horizontally by a factor of 2
(C) compressed vertically by a factor of 2
(D) compressed horizontally by a factor of 1/2

3. *Let g(x) = f(3x), where 3 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed horizontally by a factor of 1/3
(B) stretched horizontally by a factor of 3
(C) shifted right 3 units
(D) compressed vertically by a factor of 3

4. *Let g(x) = f(2x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed vertically by a factor of 2
(B) shifted right 2 units

(C) stretched horizontally by a factor of 2

(D) compressed horizontally by a factor of 1/2

5. *Let g(x) = f((1/2)x). The graph of g is the graph of f:*

Which is correct?

(A) compressed horizontally by a factor of 1/2

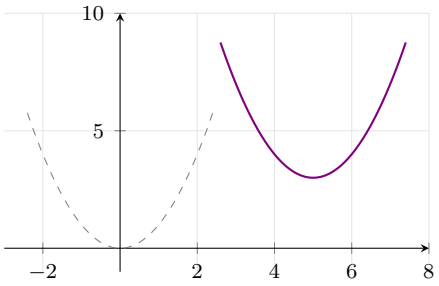
(B) stretched vertically by a factor of 2

(C) shifted left 2 units

(D) stretched horizontally by a factor of 2

Combined transformations

The skill. g(x)=f(x-h)+k combines an inside shift (right h) with an outside shift (up k). Handle inside (horizontal, reversed) and outside (vertical, direct) separately.



$g(x) = f(x-5)+3$: right 5 (inside, reversed) and up 3 (outside, as written).

Worked example

Let $g(x) = f(x-5)+3$. How does the graph of g relate to the graph of f?

Read. The rule $g(x) = f(x-5)+3$ combines two changes: an inside $x-5$, which is horizontal, and an outside $+3$, which is vertical. Handle them one at a time.

Plan. The inside change reverses its sign, so $x-5$ moves the graph right 5; the outside change is taken as written, so $+3$ moves it up 3.

Solve.

inside $x-5$ = shift right 5	inside reverses the sign
outside $+3$ = shift up 3	outside acts as written

Check. The inside term reversed to a rightward shift while the outside term followed $+3$ upward, keeping horizontal and vertical straight.

Answer: shifted right 5 and up 3.

Approach Read · Plan · Solve · Check

Read. Separate inside (horizontal) from outside (vertical).

Plan. x-h -> right h; +k -> up k.

Solve. Combine both shifts.

Check. Inside reversed? Outside direct?

Common traps — every wrong answer is one of these

horizontal-shift direction error.
Shifts left instead of right for the inside x-h term.

inside/outside confusion.
Swaps which change is horizontal vs vertical, or the directions.

vertical-shift direction error.
Moves the graph down for +k (or up for -k), reversing the outside vertical shift.

Practice

1. *Let g(x) = f(x - 3) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 3 and up 2
(B) shifted left 3 and down 2
(C) shifted right 3 and down 2
(D) shifted left 3 and up 2

2. *Let g(x) = f(x - 2) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 2 and up 2
(B) shifted right 2 and down 2
(C) shifted left 2 and down 2
(D) shifted left 2 and up 2

3. *Let g(x) = f(x - 4) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 4 and down 2
(B) shifted right 4 and up 2
(C) shifted left 4 and down 2

(D) shifted left 4 and up 2

4. *Let g(x) = f(x - 2) + 5. The graph of g is the graph of f:*

Which is correct?

(A) shifted left 2 and up 5
(B) shifted left 2 and down 5
(C) shifted right 2 and up 5
(D) shifted right 2 and down 5

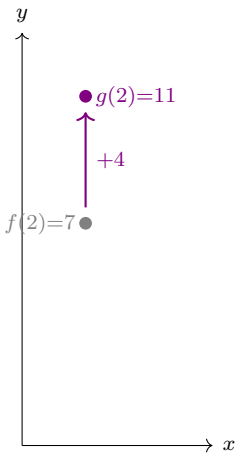
5. *Let g(x) = f(x - 2) + 5. The graph of g is the graph of f:*

Which is correct?

(A) shifted left 2 and up 5
(B) shifted right 2 and up 5
(C) shifted up 2 and right 5
(D) shifted right 2 and down 5

Evaluating transformed functions

The skill. To evaluate a transformed function at a point, apply the rule to the known value. For g(x)=f(x)+k, g(a)=f(a)+k. Substitute the given f-value, then apply the outside operation.



$g(x) = f(x) + 4$ raises each output by 4, so $g(2) = f(2) + 4 = 11$.

Worked example

If $f(2) = 7$ and $g(x) = f(x) + 4$, what is $g(2)$?

Read. We know $f(2) = 7$, and the rule $g(x) = f(x) + 4$ adds 4 to the output of f. To find $g(2)$ we reuse the known value $f(2)$.

Plan. Write $g(2) = f(2) + 4$: substitute the given f-value first, then apply the outside $+4$ to that output.

Solve.

$$\begin{aligned} g(2) &= f(2) + 4 && \text{apply the outside } +4 \text{ to the output} \\ &= 7 + 4 && \text{substitute } f(2) = 7 \\ &= 11 && \text{add} \end{aligned}$$

Check. The $+4$ was applied to the output $f(2) = 7$, not to the input 2, giving $g(2) = 11$.
Answer: 11.

Approach Read · Plan · Solve · Check

Read. What is g in terms of f, and what f-value is given?

Plan. g(a)=f(a)+k: use f(a), then add k.

Solve. Substitute and compute.

Check. Did I apply the +k to the f-value?

Common traps — every wrong answer is one of these

dropped-transformation error.
Reports f(a) without applying the +k transformation.

operation/sign error.
Subtracts k or applies it to the input instead of the output.

used the input, not the output.
Uses the input a in place of f(a), e.g. computes g(a)=a+k instead of f(a)+k.

Practice

1. *If $f(3) = 6$ and $g(x) = f(x) + 2$, what is $g(3)$?*

Which is correct?

(A) 5

(B) 8

(C) 6

(D) 4

2. *If $f(1) = 6$ and $g(x) = f(x) + 2$, what is $g(1)$?*

Which is correct?

(A) 6

(B) 3

(C) 4

(D) 8

3. *If f(3) = 5 and g(x) = f(x) + 3, what is g(3)?*

Which is correct?

(A) 2

(B) 6

(C) 8

(D) 5

4. *If f(3) = 8 and g(x) = f(x) + 2, what is g(3)?*

Which is correct?

(A) 10

(B) 6

(C) 8

(D) 5

5. *If f(5) = 10 and g(x) = f(x) - 3, what is g(5)?*

Which is correct?

(A) 7

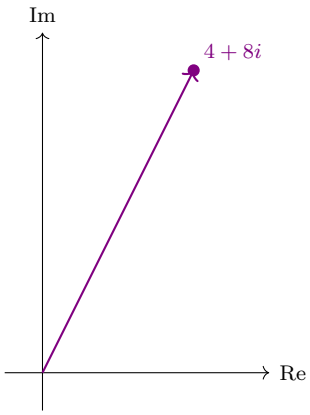
(B) 2

(C) 13

(D) 10

Complex numbers: arithmetic with i

The skill. The imaginary unit i is defined by $i^2 = -1$, so $i = \sqrt{-1}$ and $\sqrt{-9} = 3i$. A complex number a + bi has a real part a and an imaginary part b. Add or subtract by combining real parts with real parts and imaginary parts with imaginary parts, treating them as unlike terms. Multiply by distributing (FOIL), then replace every i^2 with -1 and recombine. The real and imaginary parts never merge into a single number.



A complex number $a+bi$ is the point (a,b) in the plane; here $(3+i)(2+2i)=4+8i$.

Worked example

Multiply $(3+i)(2+2i)$.

Read. The expression $(3+i)(2+2i)$ is a product of two complex numbers. Like any two binomials, they multiply by distributing every term (FOIL), with the twist that $i^2=-1$.

Plan. Form all four products, combine the like i terms, then replace i^2 with -1 and recombine the real parts. Keep real and imaginary parts separate throughout.

Solve.

$$\begin{aligned}(3+i)(2+2i) &= 6+6i+2i+2i^2 && \text{distribute all four products (FOIL)}\\ &= 6+8i+2i^2 && \text{combine the like } i \text{ terms}\\ &= 6+8i+2(-1) && \text{replace } i^2 \text{ with } -1\\ &= 4+8i && \text{combine the real parts}\end{aligned}$$

Check. Every i^2 became -1, and the real part 4 and imaginary part 8 stayed separate rather than merging into one number.

Answer: $4+8i$.

Approach Read · Plan · Solve · Check

Read. Is this addition/subtraction (combine like parts), multiplication (distribute then use $i^2 = -1$), or a square root of a negative (write as a multiple of i)?

Plan. Add/subtract: real with real, imaginary with imaginary. Multiply: FOIL, then substitute $i^2 = -1$, then recombine real and imaginary. $\sqrt{-n} = \sqrt{n}$*i.

Solve. Carry out the operation, applying $i^2 = -1$ wherever an i^2 appears.

Check. Did I replace i^2 with -1? Did I keep real and imaginary parts separate? Did I distribute all four products when multiplying? Is a square root of a negative written with i?

Common traps — every wrong answer is one of these

i-squared sign error.
Fails to use $i^2 = -1$: treats i^2 as $+1$, as 0, or leaves it as i, producing a wrong real part.

combine real/imaginary error.
Merges real and imaginary parts as if they were like terms, e.g. writing $3 + 2i$ as $5i$ or as 5.

multiplication/FOIL error.
Mishandles the distribution when multiplying complex numbers, omitting cross terms or products.

square-root-of-negative error.
Mishandles sqrt of a negative number: returns a negative real, forgets to take the root, or calls it undefined instead of writing it as a multiple of i.

Practice

1. *Recall that i is the imaginary unit.*

What is i^2?

(A) -i
(B) 1
(C) -1
(D) i

2. *Add the complex numbers (3 + 2i) and (1 + 4i).*

What is the sum?

(A) 6 + 4i
(B) 10i
(C) 4 + 6i
(D) 10

3. *Multiply (2 + i)(3 + i).*

What is the product?

(A) 5
(B) 7 + 5i
(C) 6 + 5i
(D) 5 + 5i

4. *Simplify* $\sqrt{-9}$.

What is the result?

(A) undefined

(B) 9i

(C) 3i

(D) -3

5. *Expand* $(1 + i)^2$.

What is the result?

(A) 2 + 2i

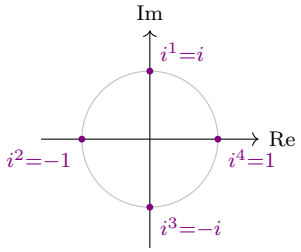
(B) 0

(C) 1 + 2i

(D) 2i

Powers of i & complex division

The skill. The powers of i repeat every four: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, then the cycle restarts. To evaluate i^n, divide n by 4 and use the remainder. The conjugate of a + bi is a - bi (flip only the sign of the imaginary part), and $(a + bi)(a - bi) = a^2 + b^2$, a real number. To divide complex numbers, multiply the numerator and denominator by the conjugate of the denominator, which makes the denominator real.



Powers of i cycle every four steps around the unit circle, so $i^7 = i^3 = -i$.

Worked example

Evaluate i^7.

Read. We must evaluate i^7. The powers of i cycle every four steps (i, -1, $-i$, 1), so only the remainder of the exponent modulo 4 matters.

Plan. Divide 7 by 4 and keep the remainder, then map it with $0 \to 1$, $1 \to i$, $2 \to -1$, $3 \to -i$.

Solve.

$$\begin{aligned} 7 &= 4 \cdot 1 + 3 && \text{divide 7 by 4, remainder 3} \\ i^7 &= i^3 && \text{powers of } i \text{ repeat every four} \\ &= -i && \text{remainder 3 maps to } -i \end{aligned}$$

Check. Splitting confirms it: $i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$.
Answer: $-i$.

Approach Read · Plan · Solve · Check

Read. Is this a power of i (use the cycle), a conjugate, or a division (multiply by the conjugate)?

Plan. Power: take n mod 4 and map 0->1, 1->i, 2->-1, 3->-i. Conjugate: flip the imaginary sign. Division: multiply top and bottom by the denominator's conjugate, then simplify.

Solve. Apply the cycle, form the conjugate, or carry out the conjugate multiplication and simplify with i^2 = -1.

Check. Did I use the remainder mod 4 correctly? Did the conjugate flip only the imaginary part? Did I multiply by the CONJUGATE (not the denominator itself) when dividing?

Common traps — every wrong answer is one of these

power-cycle error.
Maps i^n to the wrong value, e.g. i^3 = i or i^4 = -1, by mishandling the period-4 cycle or its remainder.

conjugate error.
Forms the wrong conjugate (flips the real part or both parts) or mishandles the conjugate product, leaving an i term where the result should be real.

division-method error.
Divides without multiplying by the denominator's conjugate, multiplies by the denominator itself, or forgets to divide through after simplifying.

Practice

1. *Recall the powers of the imaginary unit.*

What is i^3?

(A) -1

(B) i

(C) 1

(D) -i

2. *Use the period-4 cycle of powers of i.*

What is i^{10}?

(A) -1

(B) i

(C) 1

(D) -i

3. *Consider the complex number 3 - 5i.*

What is its conjugate?

(A) -3 + 5i

(B) -3 - 5i

(C) 5 - 3i

(D) 3 + 5i

4. *Multiply (2 + 3i)(2 - 3i).*

What is the product?

(A) -5

(B) 4 - 6i

(C) 13

(D) 4 + 9i

5. *Simplify (1 + 2i) / (1 - i).*

What is the result in the form a + bi?

(A) (1 + 2i)/2

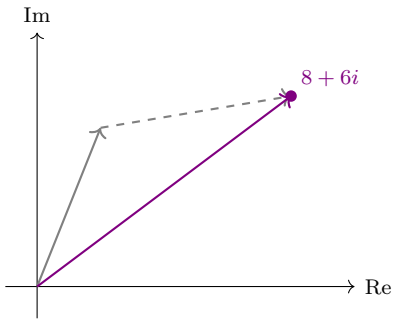
(B) -1 + 3i

(C) -1/2 + (3/2)i

(D) It cannot be simplified

Add & subtract complex numbers

The skill. Complex numbers add component-wise: (a+bi)+(c+di) = (a+c)+(b+d)i. Real parts combine with real, imaginary with imaginary. They are never merged into one number, and the i-parts are added, not multiplied.



Adding complex numbers adds their vectors tip-to-tail: $(2+5i)+(6+1i)=8+6i$.

Worked example

Simplify $(2+5i)+(6+1i)$**.**

Read. In $(2+5i)+(6+1i)$ the real parts are 2 and 6, and the imaginary parts are 5 and 1. Complex numbers add component-wise, like combining unlike terms.

Plan. Add real to real and imaginary to imaginary using $(a+c)+(b+d)i$, keeping the two kinds of part in their own slots.

Solve.

$$\begin{aligned}(2+5i)+(6+1i) &= (2+6)+(5+1)i && \text{group real with real, imaginary with imaginary}\\ &= 8+6i && \text{add within each group}\end{aligned}$$

Check. The real and imaginary parts stayed separate, and the i-coefficients were added $(5+1=6)$, not multiplied.

Answer: $8+6i$.

Approach Read · Plan · Solve · Check

Read. Which are the real parts and which are the imaginary parts?

Plan. Add real to real, imaginary to imaginary.

Solve. (a+c) + (b+d)i.

Check. Did I keep real and imaginary separate? Did I add (not multiply) the i-parts?

Common traps — every wrong answer is one of these

merged real & imaginary.
Adds all four numbers into a single real value.

swapped the parts.
Puts the imaginary sum in the real slot and vice versa.

multiplied the i-parts.
Multiplies the imaginary coefficients instead of adding.

Practice

1. *Simplify (3 + 1i) + (5 + 4i).*

Which is correct?

(A) 8 + 5i
(B) 5 + 8i
(C) 13
(D) 8 + 4i

2. *Simplify (3 + 2i) + (5 + 4i).*

Which is correct?

(A) 14
(B) 8 + 6i
(C) 8 + 8i
(D) 6 + 8i

3. *Simplify (4 + 3i) + (2 + 2i).*

Which is correct?

(A) 6 + 5i
(B) 11
(C) 5 + 6i
(D) 6 + 6i

4. *Simplify (4 + 1i) + (1 + 3i).*

Which is correct?

(A) 5 + 3i
(B) 9
(C) 4 + 5i
(D) 5 + 4i

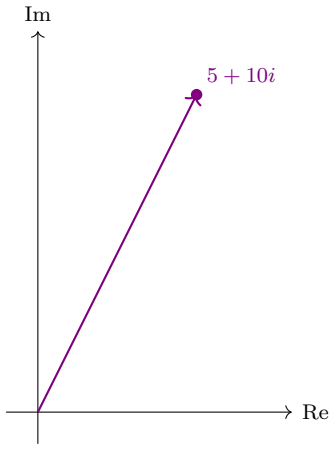
5. *Simplify (7 + 2i) + (1 + 5i).*

Which is correct?

(A) 8 + 10i
(B) 7 + 8i
(C) 8 + 7i
(D) 15

Multiply complex numbers (FOIL with i)

The skill. Multiply complex numbers by FOIL, then replace i^2 with -1: (a+bi)(c+di) = ac + adi + bci + bd·i^2 = (ac - bd) + (ad + bc)i. The i^2 term flips sign, so the real part is ac - bd, not ac + bd.



FOIL, then replace $i^2 = -1$: $(2+i)(4+3i) = 5+10i$, the point $(5, 10)$.

Worked example

Simplify $(2+i)(4+3i)$.

Read. The product $(2+i)(4+3i)$ multiplies two binomials. FOIL gives four terms, and the last one carries i^2, which must be replaced by -1.

Plan. Compute all four products, combine the two middle (i) terms, then substitute $i^2 = -1$ and recombine the real parts.

Solve.

$$\begin{aligned}(2+i)(4+3i) &= 8+6i+4i+3i^2 && \text{distribute all four products (FOIL)}\\ &= 8+10i+3i^2 && \text{combine the like } i \text{ terms}\\ &= 8+10i+3(-1) && \text{replace } i^2 \text{ with } -1\\ &= 5+10i && \text{combine the real parts}\end{aligned}$$

Check. The i^2 term became -1, flipping the real part to $8-3=5$, and both cross terms $6i$ and $4i$ were kept in the imaginary part.

Answer: $5+10i$.

Approach Read · Plan · Solve · Check

Read. FOIL the two binomials; watch the i^2 term.

Plan. Multiply all four; replace i^2 = -1.

Solve. (ac - bd) + (ad + bc)i.

Check. Did i^2 become -1? Did I include both middle terms?

Common traps — every wrong answer is one of these

i-squared as +1.
Leaves the last term as +bd (real = ac + bd) instead of subtracting.

straight-across product.
Multiplies real·real and i·i only, skipping the cross terms.

one cross term.
Uses only one of the two middle products for the imaginary part.

Practice

1. *Simplify (2 + 3i)(3 + 1i).*

Which is correct?

(A) 3 + 2i
(B) 3 + 11i
(C) 6 + 3i
(D) 9 + 11i

2. *Simplify (3 + 3i)(3 + 1i).*

Which is correct?

(A) 6 + 3i
(B) 6 + 12i
(C) 12 + 12i
(D) 9 + 3i

3. *Simplify (3 + 1i)(1 + 1i).*

Which is correct?

(A) 2 + 4i
(B) 3 + 1i
(C) 2 + 3i
(D) 4 + 4i

4. *Simplify (1 + 2i)(1 + 1i).*

Which is correct?

(A) -1 + 1i
(B) -1 + 3i

(C) 3 + 3i

(D) 1 + 2i

5. *Simplify (1 + 2i)(4 + 3i).*

Which is correct?

(A) 10 + 11i

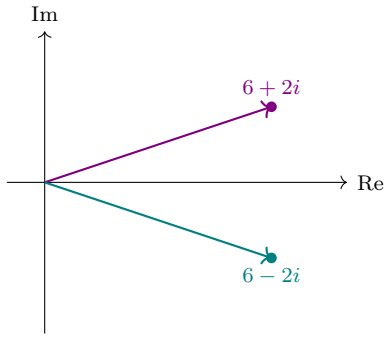
(B) 4 + 6i

(C) -2 + 11i

(D) -2 + 3i

Complex conjugates & division

The skill. A number times its conjugate is real: (a+bi)(a-bi) = a^2 - $(bi)^2$ = a^2 + b^2, because i^2 = -1 flips the sign of the -b^2i^2 term. This is the key step for dividing complex numbers (multiply by the conjugate).



A number and its conjugate mirror across the real axis; their product $6^2 + 2^2 = 40$ is real.

Worked example

Find the product $(6 + 2i)(6 - 2i)$**.**

Read. The factors $(6 + 2i)$ and $(6 - 2i)$ form a conjugate pair $(a + bi)(a - bi)$ with $a = 6$ and $b = 2$. Their product is guaranteed to be a real number.

Plan. Use the identity $(a + bi)(a - bi) = a^2 + b^2$, which follows because the two cross terms cancel and $-b^2i^2$ becomes $+b^2$.

Solve.

$$\begin{aligned} (6 + 2i)(6 - 2i) &= 6^2 + 2^2 && \text{conjugate pair: } (a + bi)(a - bi) = a^2 + b^2 \\ &= 36 + 4 && \text{square } a = 6 \text{ and } b = 2 \\ &= 40 && \text{add} \end{aligned}$$

Check. The $-b^2i^2$ term turned into $+b^2$, so the parts added instead of subtracting, and the result 40 is real with no leftover i.

Answer: 40.

Approach Read · Plan · Solve · Check

Read. This is (a+bi)(a-bi) — a conjugate pair.

Plan. Use difference of squares; the middle terms cancel.

Solve. $a^2 - b^2i^2 = a^2 + b^2$.

Check. Did the $-b^2i^2$ become $+b^2$? Is the result real?

Common traps — every wrong answer is one of these

i-squared as +1.
Computes $a^2 - b^2$ instead of $a^2 + b^2$.

dropped the b^2 term.
Reports only a^2.

left an imaginary unit.
Keeps an i on an otherwise real result.

Practice

1. *What is the product (4 + 3i)(4 - 3i)?*

Which is correct?

(A) 16
(B) 7
(C) 25i
(D) 25

2. *What is the product (4 + 1i)(4 - 1i)?*

Which is correct?

(A) 16
(B) 17i
(C) 15
(D) 17

3. *What is the product (2 + 3i)(2 - 3i)?*

Which is correct?

(A) -5

(B) 13

(C) 13i

(D) 4

4. *What is the product (3 + 1i)(3 - 1i)?*

Which is correct?

(A) 10

(B) 8

(C) 9

(D) 10i

5. *What is the product (5 + 2i)(5 - 2i)?*

Which is correct?

(A) 21

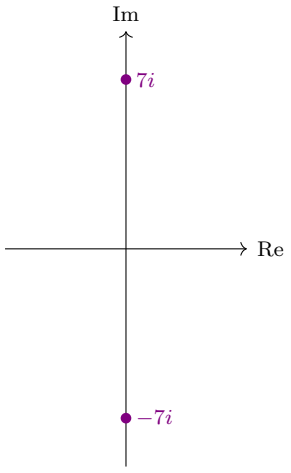
(B) 25 - 4i

(C) 25

(D) 29

Complex solutions of quadratics

The skill. When a quadratic has a negative under the square root, the solutions are complex. For $x^2 + k = 0$: $x^2 = -k$, so $x = \pm\sqrt{-k} = \pm\sqrt{k}\cdot i$. A 'no real solution' quadratic still has complex solutions.



$x^2 + 49 = 0$ has no real roots; its complex roots $x = \pm 7i$ sit on the imaginary axis.

Worked example

Solve $x^2 + 49 = 0$ (allow complex solutions).

Read. The equation $x^2 + 49 = 0$ forces x^2 to equal a negative number. No real number squares to a negative, so the solutions are complex, written with i.

Plan. Isolate $x^2 = -49$, take the square root of both sides, and write the root of the negative as a multiple of i, keeping the $\pm$ for both solutions.

Solve.

$x^2 + 49 = 0$	start from the equation
$x^2 = -49$	subtract 49 from both sides
$x = \pm\sqrt{-49}$	take the square root of both sides
$= \pm\sqrt{49}\,i$	write the negative root with i
$= \pm 7i$	simplify $\sqrt{49} = 7$

Check. Substituting back: $(7i)^2 = 49i^2 = -49$, and $-49 + 49 = 0$. Both $\pm$ roots are kept and each includes the imaginary unit i.

Answer: $x = \pm 7i$.

Approach Read · Plan · Solve · Check

Read. Is x^2 set equal to a negative?

Plan. x^2 = -k -> x = $\pm\sqrt{-k} = \pm\sqrt{k}$ i.

Solve. Take $\sqrt{k}$, attach i, keep $\pm$.

Check. Did I include i? Did I take the square root of k (not use k itself)? Both $\pm$ roots?

Common traps — every wrong answer is one of these

dropped the imaginary unit.
Solves x^2 = k, giving real $\pm\sqrt{k}$ and no i.

stopped at 'no solution'.
Says there is no solution instead of finding complex roots.

forgot the square root.
Uses k instead of $\sqrt{k}$ for the coefficient of i.

Practice

1. *Solve x^2 + 25 = 0 (allow complex solutions).*

Which is correct?

(A) x = 5i or x = -5i

(B) x = 5 or x = -5

(C) x = 25i or x = -25i

(D) No real or complex solution

2. *Solve $x^2 + 16 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 4i or x = -4i

(B) No real or complex solution

(C) x = 16i or x = -16i

(D) x = 4 or x = -4

3. *Solve $x^2 + 9 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 9i or x = -9i

(B) x = 3i or x = -3i

(C) x = 3 or x = -3

(D) No real or complex solution

4. *Solve $x^2 + 4 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 2 or x = -2

(B) x = 2i or x = -2i

(C) x = 4i or x = -4i

(D) No real or complex solution

5. *Solve $x^2 + 9 = 0$ (allow complex solutions).*

Which is correct?

(A) x = ±3

(B) x = ±3i

(C) x = ±9i

(D) no real solution

DOMAIN 3

Problem-Solving & Data Analysis

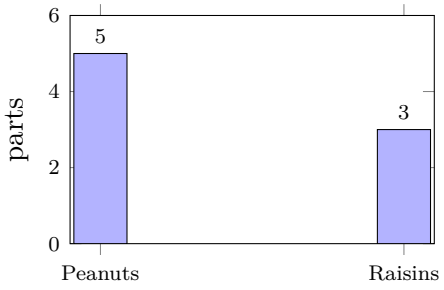
Reading quantities and the stories they tell

"It is a capital mistake to theorize before one has data."
— Arthur Conan Doyle

About 15% of the section. Ratios, rates, proportions, percentages, units, and one- and two-variable data, probability, and inference.

Ratios & equivalent ratios

The skill. A ratio a:b compares two quantities. Equivalent ratios scale BOTH parts by the same factor (multiply, not add). A part-to-part ratio a:b has a+b total parts; do not read a:b as 'a out of b'. Simplify by dividing both parts by their common factor, and keep the order the question states.



A part-to-part ratio 5 : 3: five peanut parts to three raisin parts, 8 parts in all.

Worked example

A trail mix is made with peanuts and raisins in a ratio of 5 : 3. A batch contains 20 peanuts. How many raisins are in the batch?

Read. The ratio peanuts:raisins is 5 : 3, so every batch holds 5 peanut parts for each 3 raisin parts. We are told the batch has 20 actual peanuts and asked how many raisins accompany them. Because 5 : 3 is part-to-part, the numbers count parts, not totals.

Plan. The 20 real peanuts correspond to the 5 peanut parts, so first find the multiplier that turns 5 parts into 20 peanuts. Then apply that same multiplier to the 3 raisin parts, since equivalent ratios scale both parts by the same factor.

Solve.

$$\frac{20}{5} = 4 \qquad \text{multiplier: real peanuts over peanut parts}$$

$$3 \times 4 = 12 \qquad \text{scale the raisin parts by that multiplier}$$

Check. The result gives 20 : 12, and dividing both parts by 4 returns 5 : 3, the original ratio. The scaling was done by multiplying, not by adding the same amount to each part, so the ratio is preserved.

Answer: 12.

Approach Read · Plan · Solve · Check

Read. Identify what is being compared and whether the ratio is part-to-part or part-to-whole.

Plan. To scale, find the multiplier between a known part and its ratio number; multiply every part by it. For a total, split it into a+b parts.

Solve. Multiply (never add) to scale; divide both parts by the GCF to simplify; keep the stated order.

Check. Did you scale by multiplying? Is the ratio part-part or part-whole? Is the order and the simplification correct?

Common traps — every wrong answer is one of these

additive scaling.
Scales a ratio by adding the same amount to both parts instead of multiplying by a common factor.

part-part vs part-whole confusion.
Reads a part-to-part ratio a:b as 'a out of b' (part-whole), e.g. taking a/b of a total instead of a/(a+b).

order/inversion error.
Writes or applies the ratio in the wrong order, or assigns a quantity to the wrong part.

simplification error.
Leaves a ratio unsimplified when simplest form is asked, or reduces the two parts by different factors.

Practice

1. *The ratio of cats to dogs is 2:3. There are 8 cats.*

How many dogs are there?

(A) 12
(B) 16
(C) 9
(D) 16/3 (about 5.3)

2. *The ratio of boys to girls in a class is 3:5, and there are 40 students.*

How many are boys?

(A) 25
(B) 24
(C) 15
(D) 8

3. *A recipe uses 6 cups of flour to 4 cups of sugar.*

What is the ratio of sugar to flour in simplest form?

(A) 6:4
(B) 4:6
(C) 3:2
(D) 2:3

4. *A mix has sand:cement:gravel = 2:3:5. A batch uses 30 kg of cement.*

How much gravel is used?

(A) 5 kg
(B) 75 kg
(C) 32 kg
(D) 50 kg

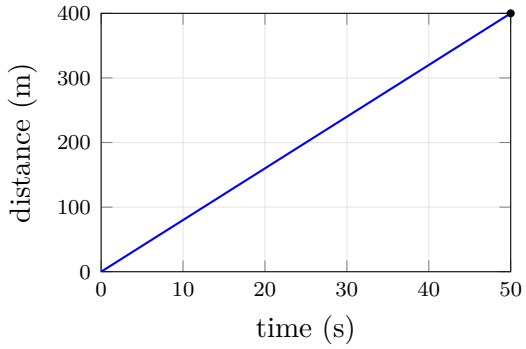
5. *A photo 4 in wide by 6 in tall is enlarged proportionally to be 15 in tall.*

How wide is the enlargement?

(A) 9 in
(B) 10 in
(C) 22.5 in
(D) 13 in

Unit rates

The skill. A unit rate is a quantity per ONE unit of another (miles per hour, dollars per item). Compute it by dividing the first quantity by the second, and keep the order the question asks (miles per hour is miles/hours, not hours/miles). To compare options, put them on the same per-unit basis; do not compare raw totals or sizes.



Constant speed: the slope, 8 meters per one second, is the unit rate.

Worked example

A runner covers 400 meters in 50 seconds. What is the runner's speed in meters per second?

Read. The runner covers a distance of 400 meters in a time of 50 seconds. We are asked for the speed in meters per second, which is the distance travelled during one single second.

Plan. A unit rate is the first quantity divided by the second. Since we want meters per second, meters go on top and seconds on the bottom; dividing reduces the rate to a per-one-second amount.

Solve.

$$\frac{400}{50} = 8 \qquad \text{meters divided by seconds gives meters per second}$$

Check. Reversing the arithmetic, 8 meters per second times 50 seconds gives back 400 meters, and the rate is stated per ONE second, in the order the question asked.

Answer: 8 meters per second.

Approach Read · Plan · Solve · Check

Read. Identify the two quantities and which one is 'per one' (the denominator).

Plan. Divide to get the rate per single unit; attach the correct unit label. To compare, reduce every option to the same per-unit rate.

Solve. Divide in the asked order; reduce to per-one; scale up if a different amount is requested.

Check. Is the rate per ONE unit, in the right order, with the correct units? For comparisons, did you normalize before judging?

Common traps — every wrong answer is one of these

rate inversion.
Computes the reciprocal rate (hours per mile instead of miles per hour) by dividing in the wrong order.

failure to reduce to per-one.
Reports a total or a per-multiple amount instead of dividing down to a single unit.

units/label error.
Right number, wrong or missing unit label (e.g. '50' or '50 miles' instead of '50 miles per hour').

compare-without-normalizing.
Compares raw totals or sizes instead of per-unit rates when judging which option is better.

Practice

1. *A car travels 150 miles in 3 hours.*

What is its speed in miles per hour?

(A) 150 miles per hour
(B) 0.02 hours per mile
(C) 50 miles
(D) 50 miles per hour

2. *6 apples cost $4.50.*

What is the price per apple?

(A) 1.33 apples per dollar
(B) $4.50
(C) $0.90
(D) $0.75 per apple

3. *Store A sells 12 oz for $3.00; Store B sells 20 oz for $4.50.*

Which is the better deal (lower price per ounce)?

(A) Store B (lower price per ounce)
(B) Store A, because $3.00 is less than $4.50
(C) Store B, because it has more ounces
(D) Store A (comparing ounces per dollar inverted)

4. *A printer prints 240 pages in 8 minutes (a rate of 30 pages per minute).*

How many pages does it print in 5 minutes?

(A) 150 pages

(B) 30 pages

(C) 1200 pages

(D) About 0.17 pages

5. *A recipe needs 3 cups of flour for 12 cookies.*

How much flour is needed for 30 cookies?

(A) 120 cups

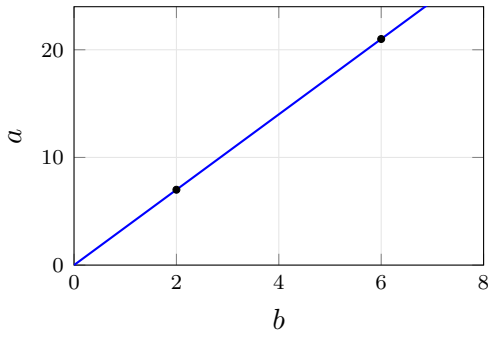
(B) 2.5 cups

(C) 0.25 cups

(D) 7.5 cups

Proportions & solving

The skill. A proportion sets two ratios equal: a/b = c/d. Solve it by cross-multiplication (ad = bc) and then solving the resulting linear equation, or by finding the scale factor between the ratios. Keep corresponding quantities in matching positions (same unit on top in both ratios).



Equivalent ratios $\frac{7}{2} = \frac{21}{6}$ lie on one line through the origin.

Worked example

Solve the proportion $\frac{7}{2} = \frac{21}{x}$.

Read. The proportion $\frac{7}{2} = \frac{21}{x}$ sets two ratios equal, and the unknown x sits in the denominator of the second ratio.

Plan. Cross-multiply: each numerator times the other ratio's denominator. That clears the fractions and leaves a linear equation in x, which we then solve by dividing.

Solve.

$$\begin{aligned} 7x &= 2 \times 21 && \text{cross-multiply across the proportion} \\ 7x &= 42 && \text{multiply out the right-hand side} \\ x &= 6 && \text{divide both sides by 7} \end{aligned}$$

Check. Substituting back, $\frac{21}{6} = \frac{7}{2}$ since both equal 3.5, so the two ratios are indeed equivalent.
Answer: 6.

Approach Read · Plan · Solve · Check

Read. Set up two equal ratios with corresponding quantities in the same positions (units aligned).

Plan. Cross-multiply to get a linear equation, or scale one ratio to the other; then solve for the unknown.

Solve. Carry out the cross-multiplication and solve the linear equation for the unknown.

Check. Are the units aligned across the two ratios? Does the answer's size make sense relative to the known ratio?

Common traps — every wrong answer is one of these

misaligned setup.
Places quantities in non-corresponding positions (e.g. miles over hours on one side, hours over miles on the other), so the proportion is set up inconsistently.

cross-multiply structure error.
Multiplies straight across (a*c = b*d) or multiplies a numerator by its own denominator instead of cross-multiplying.

additive proportion error.
Solves a proportion by adding the difference between terms rather than using the multiplicative relationship.

Practice

1. *Solve the proportion 3/4 = x/20.*

What is x?

(A) 15

(B) 19

(C) 26.67

(D) 60

2. *If 5 pens cost $2, how much do 15 pens cost? (Set up 5/2 = 15/x or 5/15 = 2/x.)*

What is the cost of 15 pens?

(A) $37.50

(B) $12

(C) $2.67

(D) $6

3. *Solve 8/x = 6/9.*

What is x?

(A) 5.33

(B) 11

(C) 12

(D) 0.75

4. *A map uses a scale where 2 inches represents 50 miles. Two cities are 7 inches apart on the map.*

What is the actual distance between the cities?

(A) 14.29 miles

(B) 55 miles

(C) 175 miles

(D) 100 miles

5. *A recipe for 4 servings uses 6 oz of cheese. You want to make 10 servings.*

How much cheese is needed?

(A) 12 oz

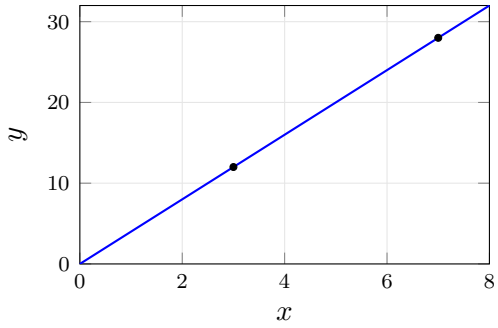
(B) 40 oz

(C) 6.67 oz

(D) 15 oz

Proportional relationships (y = kx)

The skill. Two quantities are proportional when y = kx: the graph is a straight line through the origin and the ratio y/x is a constant k (the constant of proportionality, which is also the unit rate and the slope). Find k as y/x. A relationship with a nonzero intercept (y = mx + b) is NOT proportional.



A proportional relationship $y = 4x$: a line through the origin with constant ratio $k = y/x$.

Worked example

y is proportional to x, and $y = 12$ when $x = 3$. What is y when $x = 7$?

Read. Because y is proportional to x, the graph is a line through the origin and the ratio y/x is a single constant k. We know one point, $(3, 12)$, and want the value of y when $x = 7$.

Plan. First recover the constant of proportionality as $k = y/x$ from the known point. That constant is also the unit rate, so once we have it we write $y = kx$ and substitute $x = 7$.

Solve.

$\frac{12}{3} = 4$	constant of proportionality, y over x
$4 \times 7 = 28$	multiply the constant by the new x

Check. The new point $(7, 28)$ has $28/7 = 4$, the same constant k, confirming both points lie on the one line through the origin.

Answer: 28.

Approach Read · Plan · Solve · Check

Read. Check whether the relationship passes through the origin and has a constant ratio y/x.

Plan. Compute k = y/x from any point (for a proportional relationship every point gives the same k). Write y = kx; apply k by multiplying.

Solve. Divide y by x for k; multiply by k to find a new y, or divide by k to find x.

Check. Is k = y/x (not x/y)? Does the relationship actually go through the origin (otherwise it is not proportional)?

Common traps — every wrong answer is one of these

constant-of-proportionality error.
Computes k as x/y (inverted) or otherwise reads the constant of proportionality incorrectly.

proportional vs non-proportional confusion.
Treats a relationship with a nonzero intercept (or a nonlinear one) as proportional, not checking

the through-origin / constant-ratio condition.

application error.
Misapplies k: divides when multiplication is needed (or vice versa), or applies it in the wrong direction.

Practice

1. *A proportional relationship has the values $x = 2$, $y = 6$ and $x = 5$, $y = 15$.*

What is the constant of proportionality?

(A) It is not proportional, so k is undefined
(B) 1/3
(C) 8
(D) 3

2. *Consider $y = 4x$, $y = 4x + 3$, and $y = x^2$.*

Which represents a proportional relationship?

(A) y = 4x + 3
(B) y = 4x only
(C) Both y = 4x and y = 4x + 3
(D) $y = x^2$

3. *y is proportional to x, and when $x = 8$, $y = 20$.*

Which equation models the relationship?

(A) y = 20x
(B) y = 2.5x
(C) y = 0.4x
(D) y = x + 12

4. *Cost y is proportional to weight x, with constant of proportionality $k = \$1.50$ per pound.*

How much does 12 pounds cost?

(A) \$0.125
(B) \$13.50
(C) \$18
(D) \$8

5. *A proportional relationship passes through the origin and the point (4, 10).*

What is the constant of proportionality (unit rate)?

(A) 2.5

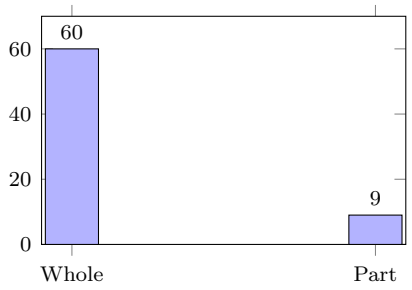
(B) 0.4

(C) 6

(D) It cannot be found without more points

Percentages

The skill. A percent is a number per 100: $25\% = 0.25 = 1/4$. To find a percent OF a number, multiply by its decimal form. To find what percent one number is of another, divide part by whole. To find the whole from a part and percent, divide the part by the decimal percent. Watch the decimal place and which quantity is the base.



15% of 60 is 9: the part is 0.15 times the whole (the base).

Worked example

Find 15% of 60.

Read. We want 15% OF 60, so 60 is the base (the whole) and 15% is the fraction of it we are taking. The answer is a part of 60, so it must come out smaller than 60.

Plan. A percent is a number per 100, so first rewrite 15% as the decimal 0.15 by dividing by 100. Then "of" means multiply, so multiply the base 60 by that decimal.

Solve.

$15\% = 0.15$	a percent is a number per 100
$0.15 \times 60 = 9$	multiply the base by the decimal

Check. Break it up: 10% of 60 is 6 and 5% of 60 is 3, and $6 + 3 = 9$, matching the result.
Answer: 9.

Approach Read · Plan · Solve · Check

Read. Identify the part, the whole (base), and the percent; decide which is unknown.

Plan. Percent of: whole x decimal. What percent: part / whole. Find the whole: part / decimal percent.

Solve. Convert the percent to a decimal (divide by 100), then multiply or divide as planned.

Check. Is the percent converted with the correct decimal place? Is the right quantity used as the base?

Common traps — every wrong answer is one of these

percent-decimal conversion error.
Converts a percent to a decimal with the wrong place value (25% as 2.5 or 0.025), or treats the percent number as if it were already a decimal/whole.

base/part inversion.
Uses the wrong quantity as the base: divides whole by part, or treats the part as the whole.

reverse-percent operation error.
When finding the whole from a part and percent, multiplies instead of dividing (or uses the wrong operation).

Practice

1. *Find 25% of 80.*

What is the result?

(A) 200
(B) 3.2
(C) 2000
(D) 20

2. *What percent of 40 is 10?*

What is the percent?

(A) 25%
(B) 30%
(C) 4%
(D) 400%

3. *Write 0.6 as a percent.*

What is the percent?

(A) 600%

(B) 60%

(C) 6%

(D) 0.6%

4. *30 is 20% of what number?*

What is the number?

(A) 1.5

(B) 150

(C) 6

(D) 600

5. *A shirt is on sale for $45, which is 75% of its original price.*

What was the original price?

(A) $60

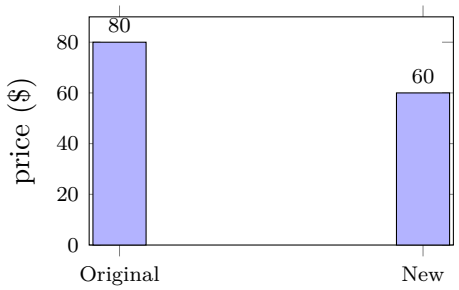
(B) $36

(C) $56.25

(D) $33.75

Percent change

The skill. Percent change = (change / ORIGINAL value) x 100, always over the original, not the new value. To apply a change, multiply by (1 + r) for an increase or (1 - r) for a decrease. Successive percent changes MULTIPLY (a 20% then 30% rise is 1.2 x 1.3 = 1.56, not 50%); they do not add.



A drop from $80 to $60 is a 25% decrease, measured against the original.

Worked example

A price falls from \$80 to \$60. What is the percent decrease?

Read. The price starts at the original value \$80 and falls to the new value \$60. We are asked for the percent decrease, which measures the drop relative to the starting price.

Plan. Percent change is the amount of change divided by the ORIGINAL value, then written as a percent. So first find the dollar change, then divide by 80 (the original, not the new \$60).

Solve.

$$80 - 60 = 20 \qquad \text{change: original minus new price}$$

$$\frac{20}{80} = 0.25 \qquad \text{divide the change by the original}$$

$$0.25 = 25\% \qquad \text{write the decimal as a percent}$$

Check. Applying a 25% decrease, $80 \times (1 - 0.25) = 80 \times 0.75 = 60$, which lands exactly on the new price.

Answer: 25%.

Approach Read · Plan · Solve · Check

Read. Identify the original (base) value and the direction of change.

Plan. Percent change uses the original as the denominator. New value: multiply by (1 +/- r). For several changes, multiply the factors.

Solve. Compute change/original, or chain the multipliers for successive changes.

Check. Is the ORIGINAL value the base? Did you multiply (not add) successive changes? Right direction?

Common traps — every wrong answer is one of these

wrong base.
Uses the new value (or another wrong quantity) as the denominator for percent change instead of the original.

successive-change additive error.
Adds successive percent changes instead of multiplying the factors (20% then 30% treated as 50%, or +20% then -20% treated as returning to the original).

direction/value error.
Applies a change in the wrong direction, adds the raw percent number instead of the percent amount, or reports the change amount instead of the new value.

Practice

1. *A price rises from \$40 to \$50.*

What is the percent increase?

(A) 20%
(B) 125%
(C) 25%
(D) 10%

2. *A \$200 item increases by 15%.*

What is the new price?

(A) \$230
(B) \$215
(C) \$30
(D) \$170

3. *A population grows 10% one year, then 20% the next.*

What is the total percent growth?

(A) 132%
(B) 30%
(C) 32%
(D) 200%

4. *A \$100 item drops 20%, then the sale price rises 20%.*

What is the final price?

(A) \$104
(B) \$96
(C) \$80
(D) \$100

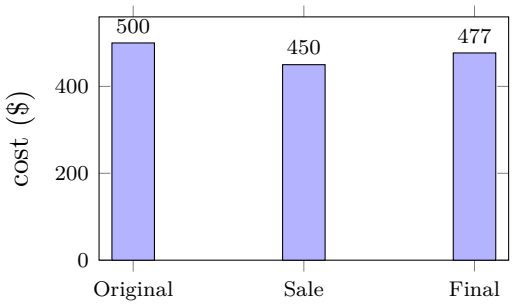
5. *A stock falls from \$50 to \$40. You want it to return to \$50.*

By what percent must it now rise?

(A) 25%
(B) 0% – it returns on its own
(C) 10%
(D) 20%

Apply ratios, rates & percent in context (multi-step)

The skill. Multi-step problems chain rates, proportions, and percents. Work step by step: find each intermediate quantity, carry units, and answer the quantity actually asked. Keep the correct base for each percent step, and do not report an intermediate value as the final answer.



Chained percents: a −10% discount, then a +6% tax applied to the new base.

Worked example

A \$500 phone is discounted 10%, and then 6% sales tax is applied to the sale price. What is the final cost?

Read. The phone starts at \$500. First a 10% discount is taken, then 6% sales tax is charged on the discounted (sale) price, not on the original. We want the final amount paid.

Plan. Handle the two percent steps in order, each with its own base. A 10% discount multiplies by $1 - 0.10 = 0.90$; the 6% tax then multiplies the new sale price by $1 + 0.06 = 1.06$.

Solve.

$500 \times 0.90 = 450$	apply the ten percent discount
$450 \times 1.06 = 477$	add six percent tax to the sale price

Check. The tax alone is $0.06 \times 450 = 27$ dollars, and $450 + 27 = 477$, matching the final cost. The tax base is the \$450 sale price, not the original \$500.

Answer: \$477.

Approach Read · Plan · Solve · Check

Read. List what is given and what is asked, with units; identify the chain of steps.

Plan. Compute each intermediate quantity in order (rate -> amount -> cost, or discount -> tax, etc.), tracking the base for each percent.

Solve. Carry each step through with units; combine successive percents multiplicatively.

Check. Did you answer the asked quantity (not an intermediate)? Right units? Correct base at each percent step?

Common traps — every wrong answer is one of these

multi-step setup error.
Picks the wrong operation or order, or skips a required step (e.g. multiplying distance by price without converting through the rate).

answer-interpretation/units.
Reports the wrong quantity or drops/uses wrong units.

intermediate-value-as-final.
Stops at an intermediate result and reports it as the final answer, or carries the wrong base between steps.

Practice

1. *A car travels 180 miles using 6 gallons of fuel. Fuel costs $4 per gallon.*

What is the fuel cost for a 300-mile trip at the same efficiency?

(A) $40
(B) $1200
(C) $24
(D) 10 gallons

2. *An $80 jacket is marked 25% off, then 8% sales tax is applied to the sale price.*

What is the final cost?

(A) $62.40
(B) $64.80
(C) $60
(D) 64.8 (no units)

3. *3 machines make 600 parts in 4 hours, all at the same constant rate.*

How many parts do 5 machines make in 6 hours?

(A) 1000
(B) 1500
(C) 900
(D) 3000

4. *A town's population is 8000. It grows 25% one year, then falls 20% the next.*

What is the final population?

(A) 10000

(B) 8000

(C) 8400

(D) 6400

5. *A recipe serves 4 and needs 2 cups of flour. You scale it for 10 servings, and flour costs $0.50 per cup.*

What is the flour cost for the scaled recipe?

(A) $2.50

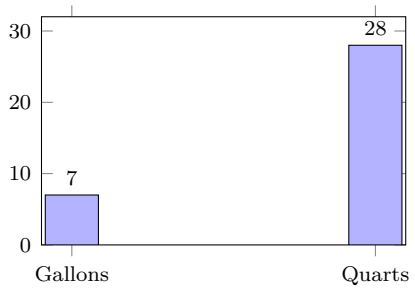
(B) $1.25

(C) 5 cups

(D) $5.00

Unit conversion & dimensional analysis

The skill. Convert units by multiplying by conversion factors written as fractions equal to 1, arranged so the unwanted unit cancels (the unit to remove goes in the denominator). Chain several factors for multi-step conversions, cancelling units along the way. For a compound rate (like miles per hour to feet per second) you must convert BOTH the top and bottom units. Whether a count grows or shrinks tells you if the factor is right side up: going to a smaller unit makes the number larger.



7 gallons = 28 quarts: moving to a smaller unit gives a larger count.

Worked example

There are 4 quarts in a gallon. How many quarts are in 7 gallons?

Read. We start with 7 gallons and want the equivalent number of quarts. The given equivalence is 1 gallon = 4 quarts, and a quart is a smaller unit than a gallon.

Plan. Write the conversion factor as a fraction equal to 1, namely $\frac{4\text{ qt}}{1\text{ gal}}$, with gallons in the denominator so the starting gallons cancel and quarts remain.

Solve.

$$7 \times \frac{4}{1} = 28 \qquad \text{multiply by quarts per gallon so gallons cancel}$$

Check. We moved to a smaller unit, so the count should grow, and 7 gallons becoming 28 quarts is a fourfold increase, exactly as 4 quarts per gallon predicts.

Answer: 28 quarts.

Approach Read · Plan · Solve · Check

Read. What units do I start with and what do I need? Is it a single quantity or a compound rate (two units to convert)?

Plan. Write each conversion factor as a fraction equal to 1, oriented so the unwanted unit cancels. Multiply through the chain. For a rate, convert numerator and denominator units both.

Solve. Cancel units across the chain and multiply the numbers.

Check. Did the unwanted units cancel (factor right side up)? Did I convert EVERY unit in a compound rate? Did the magnitude move the right direction? Are my conversion values correct?

Common traps — every wrong answer is one of these

factor-orientation error.
Uses the conversion factor upside down (divides when multiplication is needed or vice versa), so the units do not cancel and the magnitude moves the wrong way.

incomplete-chain error.
Converts only one unit of a compound rate or stops partway through a multi-step chain, leaving a unit unconverted.

unit-tracking/label error.
Loses track of units across the chain, reports a value with the wrong unit, or omits a needed conversion of the final unit.

conversion-factor value error.
Uses an incorrect equivalence (e.g. 1 hour = 100 minutes, 1 foot = 10 inches) or misremembers a conversion factor.

Practice

1. *There are 60 minutes in an hour.*

How many minutes are in 3 hours?

(A) 0.05

(B) 180

(C) 300

(D) 20

2. *A car travels at 60 miles per hour.*

How far does it travel in 30 minutes?

(A) 120 miles

(B) 30 miles

(C) 1800 miles

(D) 2 miles

3. *A speed is given as 60 miles per hour.*

What is this speed in miles per minute?

(A) 1 mile per minute

(B) 60 miles per minute

(C) 3600 miles per minute

(D) 1/60 mile per minute

4. *There are 12 inches in a foot.*

How many inches are in 4 feet?

(A) 16 inches

(B) 1/3 inch

(C) 40 inches

(D) 48 inches

5. *A faucet drips 2 milliliters per second. There are 1000 milliliters in a liter.*

How many liters drip in one hour?

(A) 7.2 liters

(B) 2 liters

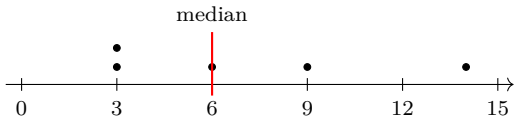
(C) 120 liters

(D) 7200 liters

One-variable data: measures of center & spread

The skill. Mean = sum divided by count; median = the middle value of the ORDERED data (average of the two middle values when the count is even); mode = most frequent value; range = maximum minus minimum (a measure of spread, not center). The mean uses every value, so it is

pulled by outliers; the median is the middle position, so it is resistant to outliers. To find a missing value from a target mean, multiply the mean by the count to get the required total.



Ordered data $3, 3, 6, 9, 14$*; the median is the middle value,* 6.

Worked example

Find the median of $6, 3, 9, 3, 14$**.**

Read. We have the five values $6, 3, 9, 3, 14$ and want the median, which is the middle value once the data are placed in order. The list is not yet sorted, so ordering must come first.

Plan. Sort the five numbers from least to greatest. With an odd count of 5, the median is the single value in the third (middle) position, with two values below it and two above.

Solve. Ordered, the data are $3, 3, 6, 9, 14$. The middle entry, third of five, is 6.

Check. Exactly two values (3 and 3) lie below 6 and two (9 and 14) lie above it, so 6 sits in the true middle position.

Answer: 6.

Approach Read · Plan · Solve · Check

Read. Identify which measure is asked (center: mean/median/mode; spread: range) and whether outliers or an even count are in play.

Plan. Mean: sum / count. Median: order the data, take the middle (average two middles if even). Range: max - min. Missing value: required total = mean x count.

Solve. Carry out the computation on the ordered data; keep center and spread distinct.

Check. Answered the measure actually asked? Ordered before taking the median, and averaged two middles if even? Treated the outlier's larger effect on the mean than the median? Range = max - min (not a center)?

Common traps — every wrong answer is one of these

mean/median confusion.
Reports the mean when the median (or mode) is asked, or vice versa; or averages a subset instead of solving, or assumes a missing value equals the mean.

median-without-ordering error.
Takes the middle of the unordered list, or with an even count picks a single middle value instead of averaging the two middle values.

outlier-resistance error.
Believes an outlier shifts the median as much as the mean, or that the mean is resistant; misjudges which measure an extreme value affects more.

range/spread vs center confusion.
Confuses range (spread) with a center measure, computes range as max + min or as a single value, or reports a spread/extreme value when a center is asked.

Practice

1. *The data set is 4, 8, 8, 10, 20.*

What is the mean?

(A) 16
(B) 20
(C) 10
(D) 8

2. *Consider the data set 7, 2, 9, 4.*

What is the median?

(A) 7
(B) 5.5
(C) 5
(D) 9

3. *A data set of scores is 80, 82, 85, 88, 90. A new score of 20 is then added to the set.*

Which statement is true?

(A) Neither the mean nor the median changes
(B) The mean and median decrease by the same amount
(C) The mean decreases more than the median
(D) The median decreases more than the mean

4. *The data set is 12, 5, 19, 8, 5.*

What is the range?

(A) 9.8
(B) 14
(C) 5

(D) 24

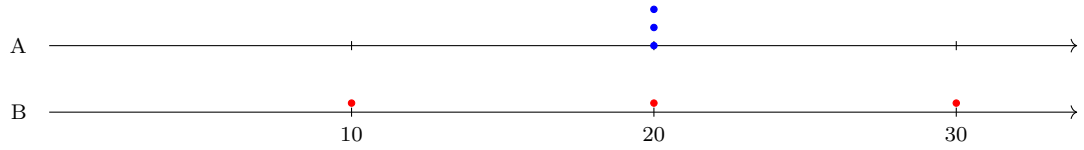
5. *Four numbers have a mean of 10. Three of the numbers are 6, 8, and 12.*

What is the fourth number?

(A) 26
(B) 14
(C) 8.67
(D) 10

Distributions & standard deviation (comparing spread)

The skill. Standard deviation measures how far values typically fall from the mean: larger SD means more spread out, smaller SD means more clustered near the mean (SD = 0 when all values are equal). The SAT compares SD QUALITATIVELY, not by formula. Range and SD both measure spread but differently: two sets with the same range can have different SDs. Distribution shape matters: in a roughly symmetric distribution mean is about equal to median; a right skew (long high tail) pulls the mean ABOVE the median, a left skew pulls it below. Adding the same constant to every value shifts the mean but leaves the spread (range and SD) unchanged.



Same mean 20: Set A is clustered (small SD), Set B is spread out (larger SD).

Worked example

Set A is $20, 20, 20$ and Set B is $10, 20, 30$; both have a mean of 20. Which set has the greater standard deviation?

Read. Both sets have the same mean, 20, so the question is purely about spread: which set's values sit farther, on average, from that shared mean. The PSAT wants this judged qualitatively, not by the SD formula.

Plan. Standard deviation grows as the typical distance of the values from the mean grows. So compare how far each set's values stray from 20: tightly clustered means small SD, widely scattered means large SD.

Solve. Every value in Set A equals 20, so each sits 0 from the mean and the spread is 0. Set B's values, 10 and 30, each sit 10 from the mean, so its values are farther out and Set B is more spread.

Check. Greater typical distance from the mean means a larger standard deviation, so Set B has the larger SD; a set of identical values like Set A must have SD exactly 0.

Answer: Set B.

Approach Read · Plan · Solve · Check

Read. Decide whether the question is about spread (SD/range) or shape (skew), and whether you are comparing sets or transforming one.

Plan. For SD, judge how clustered vs spread the values are around the mean (not just the extremes). For shape, use the tail direction to place the mean relative to the median. For a constant shift, remember spread is unchanged.

Solve. Compare or reason qualitatively; only compute a mean/median when needed to anchor the comparison.

Check. More spread = larger SD (not reversed)? Same range does not force same SD? Right skew => mean above median? A constant shift leaves SD unchanged?

Common traps — every wrong answer is one of these

SD-direction error.
Reverses the meaning of standard deviation: thinks tightly clustered data has a large SD, or that more spread means a smaller SD, or that a higher mean implies a higher SD.

range-as-SD error.
Judges standard deviation solely by the range or the extremes, assuming equal range forces equal SD or that the larger maximum always means the larger SD.

skew/mean-median error.
Gets the skew relationship wrong: thinks a right skew pulls the mean below the median, or that the mean always equals the median regardless of shape.

spread-invariance error.
Thinks adding the same constant to every value changes the standard deviation, or that shifting the center changes the spread; confuses a change in center with a change in spread.

Practice

1. *Set A is 10, 10, 10, 10. Set B is 2, 8, 12, 18.*

Which set has the greater standard deviation?

(A) Set B

(B) Set A

(C) Cannot be determined

(D) They are equal

2. *Set A is 1, 1, 1, 9 and Set B is 1, 4, 6, 9. Both sets have a range of 8.*

Which statement is true?

(A) Standard deviation cannot differ when the range is equal

(B) The two sets can have different standard deviations

(C) The set with the larger maximum has the larger standard deviation

(D) The sets must have the same standard deviation because they have the same range

3. *A data set is skewed to the right (it has a long tail of high values).*

How do the mean and median compare?

(A) The mean is less than the median

(B) The mean equals the median

(C) The mean is greater than the median

(D) The median is greater than the maximum

4. *Every value in a data set is increased by 5.*

What happens to the standard deviation?

(A) It stays the same

(B) It increases by 5

(C) It cannot be determined

(D) It increases, but not by 5

5. *Two classes have the same mean test score. Class A's scores cluster tightly near the mean; Class B's scores are widely spread out.*

Which class has the larger standard deviation?

(A) Class A

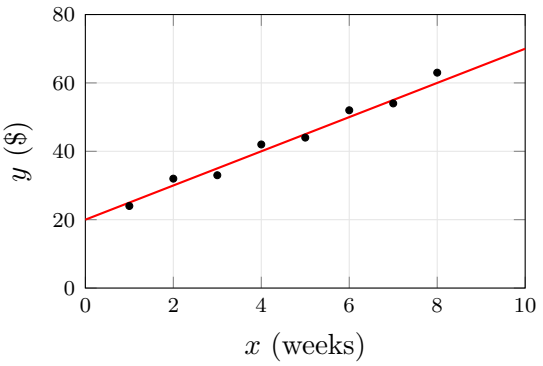
(B) Class B

(C) They are equal, because the means are equal

(D) Cannot be determined without the exact values

Two-variable data: scatterplots & line of best fit

The skill. A scatterplot shows paired (x, y) data; a line (or curve) of best fit summarizes the trend. For a linear fit $y = mx + b$, the slope m is the PREDICTED change in y per unit increase in x (a rate), and b is the predicted y when $x = 0$. Use the model to predict by substituting x. A residual is actual minus predicted (a point above the line has a positive residual). Slope sign gives the association direction; how tightly points hug the line gives strength. Choose the model by the pattern: roughly constant change is linear, repeated multiplication (doubling/halving, curving) is exponential. Association is not causation.



A positive linear association; the line of best fit $y = 5x + 20$ summarizes the trend.

Worked example

A scatterplot of a savings account balance y (in dollars) versus time x (in weeks) has line of best fit $y = 5x + 20$. What balance does the model predict at week 8?

Read. The line of best fit $y = 5x + 20$ models the balance y in dollars after x weeks. The slope 5 is dollars added per week and 20 is the starting balance at week 0. We want the predicted balance at week 8.

Plan. To predict, substitute $x = 8$ into the WHOLE equation, keeping the intercept 20. Compute the slope term first, then add the intercept.

Solve.

$5 \times 8 = 40$	substitute week 8 into the slope term
$40 + 20 = 60$	add the starting-balance intercept

Check. The slope adds \$5 each week, so over 8 weeks it adds \$40 to the \$20 start, giving \$60; dropping the intercept would have wrongly given \$40.

Answer: \$60.

Approach Read · Plan · Solve · Check

Read. Identify what is asked: interpret the slope/intercept, predict a value, compare a point to the prediction (residual), judge association, or choose a model type.

Plan. Slope = change in y per unit x; intercept = value at x = 0. To predict, substitute x into the full equation. Residual = actual - predicted. Pick linear for constant change, exponential for repeated multiplication.

Solve. Substitute and compute; keep slope (a rate) distinct from a single predicted value.

Check. Slope read as a rate (not a total or the intercept)? Prediction used the WHOLE equation? Residual = actual - predicted with the right sign? Model type matches the pattern? Did not claim causation from correlation?

Common traps — every wrong answer is one of these

best-fit slope interpretation error.
Misreads the slope of the line of best fit: treats it as a total, as a single predicted y-value, as the intercept (value at x = 0), or as the x-variable itself, instead of the predicted change in y per unit increase in x.

intercept/prediction misuse.
Predicts incorrectly: drops the intercept (uses only mx) or uses only the intercept, reads the wrong axis, or reports an actual data value as the predicted value (or the predicted value as the residual).

model-type choice error.
Fits a line of best fit (constant slope) to clearly multiplicative/curved data, or forces an exponential model onto linear data; confuses constant additive change with repeated multiplication.

association/causation misread.
Reads a negative association as positive (or vice versa), treats correlation as proof of causation, or claims the model predicts an exact rather than an estimated value.

Practice

1. *A scatterplot of plant height (y, in cm) versus time (x, in days) has line of best fit $y = 1.5x + 4$.*

What does 1.5 best represent?

(A) The predicted height when the time is 0 days
(B) The number of days
(C) The total height of the plant
(D) The predicted increase in height, in cm, for each additional day

2. *The line of best fit is $y = 1.5x + 4$, where y is height in cm and x is time in days.*

What height does the model predict at day 10?

(A) 4 cm
(B) 19 cm
(C) 15 cm
(D) 1.5 cm

3. *A quantity is measured over equal time periods as 4, 8, 16, 32 (it doubles each period).*

Which model best fits the data?

(A) An exponential model
(B) A linear model

(C) A line of best fit with constant slope

(D) No model can fit the data

4. *A scatterplot shows that as weekly hours of TV (x) increase, test score (y) decreases, with line of best fit y = -2x + 90.*

Which is the best interpretation?

(A) Each additional hour of TV is associated with a 2-point increase in score

(B) Watching TV causes scores to drop by exactly 2 points

(C) Test scores have no relationship with hours of TV

(D) Each additional hour of TV is associated with a predicted 2-point decrease in score

5. *The line of best fit for a car's value (y, in thousands of dollars) versus age (x, in years) is y = -1.2x + 18. A particular 3-year-old car actually sold for $16,000.*

What is the residual for this car?

(A) $14,400

(B) $2,000

(C) $1,600 (the actual value is above the predicted value)

(D) -$1,600

Probability & two-way tables

The skill. Probability = favorable outcomes / total outcomes (a part-to-whole ratio between 0 and 1). In a two-way table, a JOINT probability is a single cell over the grand total; a MARGINAL probability is a row or column total over the grand total; a CONDITIONAL probability restricts the denominator to the given subgroup: P(A given B) = (count in A and B) / (total in B). The phrase 'selected from group B' or 'given B' sets the denominator to B's total, NOT the grand total. The complement: P(not A) = 1 - P(A).

	Likes coffee	Does not	Total
Teens	25	15	40
Adults	30	10	40
Total	55	25	80

A two-way table: a conditional probability restricts the denominator to one group's total (e.g. 40 teens).

Worked example

A survey of 80 people records coffee preference by age: among 40 teens, 25 like coffee and 15 do not; among 40 adults, 30 like coffee and 10 do not; overall 55 like coffee and 25 do not. If a teen is selected at random, what is the probability the

teen likes coffee?

Read. The phrase "if a teen is selected" restricts the selection to teens only, so this is a conditional probability. The relevant group is the 40 teens, of whom 25 like coffee.

Plan. For a conditional probability, the denominator is the given subgroup's total, here the 40 teens, and the numerator is the count within that subgroup who like coffee, here 25.

Solve.

$$\frac{25}{40} = 0.625 \qquad \text{teens who like coffee over all teens}$$

Check. The denominator is the subgroup total 40, not the grand total 80; using 80 would wrongly treat this as a joint probability.

Answer: 0.625.

Approach Read · Plan · Solve · Check

Read. Classify the question: joint (cell / grand total), marginal (row or column total / grand total), or conditional (restricted to a given subgroup).

Plan. Identify the correct numerator cell and the correct denominator. For a conditional, the denominator is the given group's total; for a joint, it is the grand total. Use the complement when 'not' is asked.

Solve. Read the right cell and total from the table; form the ratio favorable / total.

Check. Denominator matches the group the selection is drawn from? Conditional uses the subgroup total, not the grand total? Ratio is favorable / total (not inverted)? Complement handled if 'not'?

Common traps — every wrong answer is one of these

conditional-denominator error.
For a conditional probability, divides by the grand total (or the wrong subgroup total) instead of the total of the group the selection is restricted to.

joint/conditional confusion.
Uses a single cell over the grand total (joint) when a conditional is asked, or computes a conditional when a joint is asked.

ratio-inversion/complement error.
Inverts the probability (total / favorable, or favorable / unfavorable), or forgets the complement so that P(not A) is given as P(A).

table-reading error.
Reads the wrong cell, row, or column from the table, or uses a marginal total where a joint count is needed.

Practice

1. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

If one student is selected at random, what is the probability the student plays a sport?

(A) 0.45
(B) 0.55
(C) 100/55
(D) 55/45

2. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

If a Grade 10 student is selected at random, what is the probability the student plays a sport?

(A) 0.5
(B) 0.7
(C) 0.55
(D) 0.35

3. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

Among students who play a sport, what is the probability a randomly selected one is in Grade 9?

(A) 55/20
(B) 20/50 (0.40)
(C) 20/100 (0.20)
(D) 20/55 (about 0.36)

4. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

What is the probability that a randomly selected student is in Grade 9 AND plays a sport?

(A) 0.20
(B) 0.55
(C) 0.50
(D) 0.70

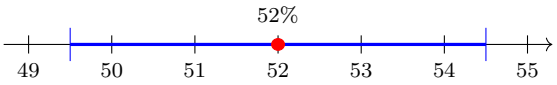
5. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

What is the probability that a randomly selected student does NOT play a sport?

(A) 0.55
(B) 0.30
(C) 0.90
(D) 0.45

Inference, margin of error & evaluating statistical claims

The skill. A sample statistic estimates a population parameter. The margin of error gives a plausible interval around the estimate (estimate plus or minus the margin); a larger random sample produces a SMALLER margin of error. A result generalizes only to the population that was randomly SELECTED from, not beyond it. Causation requires a randomized controlled EXPERIMENT (random ASSIGNMENT to treatment vs control); an observational study can show association but not establish cause. In short: random selection supports generalization; random assignment supports causal claims.



Estimate 52% *with a margin of error of* 2.5*: a plausible interval from* 49.5% *to* 54.5%.

Worked example

A survey estimates that 52% of shoppers prefer online checkout, with a margin of error of 2.5 percentage points. What is the plausible interval for the true level of preference?

Read. The survey gives a point estimate of 52% with a margin of error of 2.5 percentage points. The margin of error defines a band of plausible values around the estimate, and we are asked for that interval.

Plan. Form the interval by taking the estimate plus or minus the margin of error: subtract 2.5 for the lower end and add 2.5 for the upper end.

Solve.

$52\% - 2.5\% = 49.5\%$	lower end: estimate minus the margin
$52\% + 2.5\% = 54.5\%$	upper end: estimate plus the margin

Check. The interval 49.5% to 54.5% is centered on the estimate 52% and spans 2.5 points each way; it marks plausible values, not an absolute guarantee.

Answer: 49.5% to 54.5%.

Approach Read · Plan · Solve · Check

Read. Decide what is claimed: an interval estimate (margin of error), a generalization to a population, or a causal claim.

Plan. For margin of error, form the interval and recall larger samples shrink it. For generalization, check what population was randomly sampled. For causation, check whether there was random assignment (experiment) or only observation.

Solve. State the plausible interval, the population the claim can extend to, and whether cause or only association is supported.

Check. Margin of error is a plausible interval, not an absolute bound? Generalized only to the sampled population? Causation claimed only with random assignment?

Common traps — every wrong answer is one of these

margin-of-error meaning error.
Treats the margin of error as the maximum possible error or an absolute bound, takes the point estimate as exact, or believes a larger sample produces a larger margin of error.

generalization-scope error.
Generalizes the result beyond the population that was randomly sampled, to a different population, or from a biased/non-random sample.

causation-from-observation error.
Concludes a causal relationship from an observational study (no random assignment), treating an observed association as evidence of cause.

selection-vs-assignment confusion.
Conflates random selection with random assignment: thinks random selection licenses causal claims, or that random assignment licenses generalization to a broader population.

Practice

1. *A poll estimates that 60% of voters support a measure, with a margin of error of 3 percentage points.*

Which is the best interpretation?

(A) Exactly 3% of the responses were recorded incorrectly
(B) The true level of support is plausibly between 57% and 63%
(C) Support is definitely 60%
(D) Support absolutely cannot be outside 57% to 63%

2. *A researcher wants the margin of error of an estimate to be smaller.*

Which change would achieve this?

(A) Decrease the sample size

(B) Increase the sample size

(C) Survey fewer people but more carefully

(D) Sample size has no effect on the margin of error

3. *Researchers randomly sampled students at one high school and found that 40% bike to school.*

To which group can this result be appropriately generalized?

(A) All high school students in the country

(B) Students at that high school

(C) All residents of the city

(D) All adults who commute to work

4. *An observational study finds that people who drink more coffee tend to report higher productivity.*

What can be appropriately concluded?

(A) Higher productivity causes people to drink more coffee

(B) There is an association, but the study cannot establish that coffee causes higher productivity

(C) Coffee and productivity are unrelated

(D) Drinking more coffee causes higher productivity

5. *In an experiment, volunteers from one college were randomly ASSIGNED to use either a new study method or the usual one. The new-method group scored higher.*

What is appropriate to conclude?

(A) The new method likely caused the higher scores, but the result may not generalize beyond similar volunteers

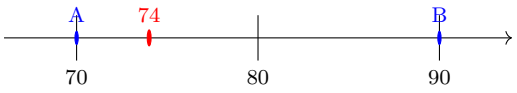
(B) The result generalizes to all adults but no causal claim can be made

(C) The new method caused higher scores for all college students everywhere

(D) No causal claim is possible; this is only an association

Weighted averages & combined means

The skill. A weighted average weights each value by its frequency or group size: it is the sum of (value times weight) divided by the sum of the weights. The simple average of two group means is the overall mean only when the groups are the same size; otherwise you must weight by the counts. The key bridge is total = mean times count: reconstruct each group's total, add the totals, and divide by the total count. To find a missing value or weight that produces a target average, work from the required total, not from the averages directly.



Group means 70 (n=20) and 90 (n=5); the weighted mean 74 leans toward the bigger group.

Worked example

Class A has 20 students averaging 70; Class B has 5 students averaging 90. What is the combined average of all 25 students?

Read. Class A has 20 students averaging 70 and Class B has 5 students averaging 90. The two classes are different sizes, so the combined average is a weighted average, not the plain average of 70 and 90.

Plan. Use total = mean × count to rebuild each class's point total, add the two totals for the grand total of points, then divide by the total of 25 students.

Solve.

$$20 \times 70 = 1400 \qquad \text{total points for class A}$$

$$5 \times 90 = 450 \qquad \text{total points for class B}$$

$$\frac{1400 + 450}{25} = 74 \qquad \text{combined total over all 25 students}$$

Check. The result 74 leans toward 70 because the larger class (20 students) pulls the average toward its mean; the naive average $(70 + 90)/2 = 80$ ignores the sizes and is wrong.

Answer: 74.

Approach Read · Plan · Solve · Check

Read. Are the groups or categories the same size? If not, a simple average of the means is wrong. Identify each value and its weight (count or percent).

Plan. Compute sum(value*weight) / sum(weight). Equivalently use total = mean*count for each group, sum totals, divide by total count. For a missing piece, set required total = target mean * total count and solve.

Solve. Multiply each value by its weight, sum, and divide; or reconstruct and combine totals.

Check. Did I weight by the group sizes (not just average the means)? Did I pair each weight with the right value? Did I divide by the total weight? For a reverse problem, did I work from totals?

Common traps — every wrong answer is one of these

average-of-averages error.
Averages the group means directly, ignoring that the groups have different sizes, instead of weighting by the counts.

weight/value swap.
Pairs each value with the wrong weight, or interchanges which quantity is the weight and which is the value.

total-vs-mean error.
Mishandles total = mean times count: reports a total in place of a mean, forgets to divide by the total count, or adds means or counts incorrectly.

reverse weighted-average error.
When solving for a missing value or weight that yields a target average, reads the new average as the missing value or uses the change in average, instead of working from the required total.

Practice

1. *Class A has 10 students averaging 80. Class B has 30 students averaging 90.*

What is the combined average of all 40 students?

(A) 87.5
(B) 3500
(C) 85
(D) 170

2. *A course grade is 30% homework and 70% exam. A student scores 90 on homework and 80 on the exam.*

What is the weighted course grade?

(A) 87
(B) 83
(C) 170
(D) 85

3. *In a weighted average, the value 4 has weight 3 and the value 10 has weight 1.*

What is the weighted average?

(A) 8.5
(B) 7
(C) 5.5
(D) 22

4. *After 4 tests a student's average is 85. After a 5th test, the average rises to 86.*

What did the student score on the 5th test?

(A) 1

(B) 86

(C) 90

(D) 87

5. *Group X has 5 people with mean 12. Group Y has 15 people with mean 20.*

What is the mean of all 20 people?

(A) 32

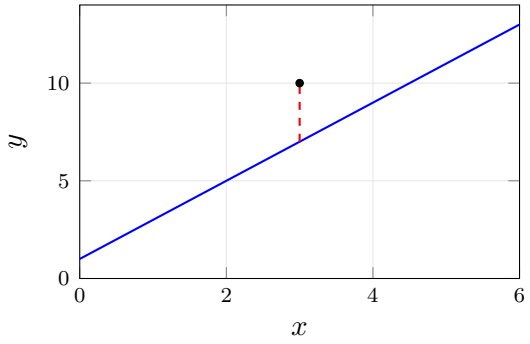
(B) 16

(C) 360

(D) 18

Line of best fit: prediction & residuals

The skill. To predict from a line of best fit y = mx + b, substitute the input for x and evaluate y. A residual is actual minus predicted: a point ABOVE the line has a positive residual (the model underpredicts there); a point below has a negative residual (the model overpredicts). Predicting within the range of the data is interpolation and is reasonable; predicting far outside that range is extrapolation and is less reliable because the pattern may not continue. The slope of the fitted line is the modeled rate of change, with the units of y per unit of x.



Residual = actual − predicted: the point $(3, 10)$ lies above $y = 2x + 1$, so the residual $+3$ is positive.

Worked example

A line of best fit is $y = 2x + 1$. At $x = 3$ the actual observed value is 10. What is the residual at $x = 3$?

Read. The line of best fit is $y = 2x + 1$, and at $x = 3$ the actual observed value is 10. A residual measures how far that actual value sits above or below what the line predicts.

Plan. First predict the value at $x = 3$ by substituting into $y = 2x + 1$. Then take residual = actual − predicted, so a positive result means the point lies above the line.

Solve.

$$2 \times 3 + 1 = 7 \qquad \text{predicted value from the model at x equals 3}$$

$$10 - 7 = 3 \qquad \text{residual: actual minus predicted}$$

Check. The residual +3 is positive, which agrees with the actual point $(3, 10)$ lying above the line $y = 2x + 1$, where the model underpredicts.

Answer: 3.

Approach Read · Plan · Solve · Check

Read. Is the task a prediction (plug in x), a residual (compare actual to predicted), a reliability judgment (interpolation vs extrapolation), or a slope interpretation?

Plan. Prediction: substitute x into y = mx + b. Residual: actual minus predicted, and read its sign as under/over-prediction. Reliability: check whether the input is inside the data range. Slope: state the rate with units.

Solve. Evaluate or subtract as needed; keep the sign of the residual meaningful.

Check. Did I substitute into the correct variable? Is the residual actual minus predicted (above the line = positive)? Is the input inside the data range? Are the slope's units y per x?

Common traps — every wrong answer is one of these

prediction computation error.
Misuses the fitted equation: drops the intercept, evaluates in the wrong order, or solves for x instead of evaluating y.

residual sign/direction error.
Reverses the residual: treats a point above the line as having a negative residual, computes predicted minus actual, or confuses under- and over-prediction.

extrapolation-reliability error.
Treats a prediction far outside the data range as just as reliable as one inside it, not recognizing extrapolation.

slope-as-rate misread.
Misreads the fitted slope's meaning or units, e.g. using the intercept as the rate or inverting the units of the slope.

Practice

1. *A line of best fit is $y = 3x + 5$.*

What does the model predict for y when x = 4?

(A) 12

(B) 27

(C) x = -1/3

(D) 17

2. *A data point lies above the line of best fit.*

What is true of its residual?

(A) It is positive; the model underpredicts this point

(B) It is negative; the model overpredicts this point

(C) It is zero

(D) It cannot be determined

3. *For a particular x, the actual y is 20 and the model predicts 17.*

What is the residual?

(A) 37

(B) -3

(C) 3

(D) about 1.18

4. *A model is built from data with x between 1 and 10, then used to predict y at x = 100.*

How should this prediction be regarded?

(A) As just as reliable as predictions within the data range

(B) As guaranteed accurate because it uses the best-fit line

(C) As an extrapolation that may be unreliable

(D) As impossible to compute

5. *A line of best fit y = 2.5x + 10 models a plant's height in centimeters versus its age in weeks.*

What does the slope mean, and what height does the model give at week 6?

(A) Height grows about 10 cm per week; 25 cm at week 6

(B) Height grows about 2.5 cm per week; 25 cm at week 6

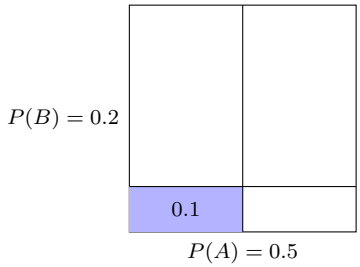
(C) Height grows about 2.5 cm per week; 15 cm at week 6

(D) Height grows about 2.5 weeks per cm; 25 cm at week 6

Compound & conditional probability

The skill. For independent events, the probability that BOTH happen multiplies: P(A and B) = P(A)*P(B). The probability that EITHER happens adds, minus the overlap: P(A or B) = P(A) +

P(B) - P(A and B); for mutually exclusive events the overlap is zero. The complement rule gives P(not A) = 1 - P(A). A conditional probability is P(A given B) = P(A and B) / P(B). In short, 'and' multiplies (for independent events), 'or' adds (subtracting any overlap), and a complement subtracts from 1.



Area model for independent events: $P(A \text{ and } B) = 0.5 \times 0.2 = 0.1$.

Worked example

Two independent events have $P(A) = 0.5$ and $P(B) = 0.2$. What is $P(A$ and $B)$?

Read. The two events are independent, meaning neither affects the other's chance, with $P(A) = 0.5$ and $P(B) = 0.2$. We want the probability that BOTH A and B occur.

Plan. For independent events the word "and" means multiply, so $P(A \text{ and } B) = P(A)\,P(B)$.

Solve.

$$0.5 \times 0.2 = 0.1 \qquad \text{multiply the two probabilities for independent and}$$

Check. The joint probability 0.1 is smaller than either 0.5 or 0.2, which makes sense since requiring both events to happen can only reduce the chance.

Answer: 0.1.

Approach Read · Plan · Solve · Check

Read. Is this an 'and' (both), an 'or' (either), a complement (not), or a conditional (given)? Are the events independent or mutually exclusive?

Plan. And (independent): multiply. Or: add and subtract the overlap. Complement: 1 minus. Conditional: joint over the condition's probability.

Solve. Apply the matching rule with the given probabilities.

Check. Did I multiply for 'and' and add for 'or' (not swapped)? Did I subtract the overlap in 'or'? Is the complement 1 minus? Is the conditional denominator the GIVEN event?

Common traps — every wrong answer is one of these

and/or operation swap.
Uses the wrong combination rule: adds probabilities for 'and', or multiplies them for 'or'.

independence/overlap error.
Treats dependent events as independent, or computes 'or' without subtracting the overlap, double-counting the shared outcomes.

complement error.
Computes P(not A) incorrectly, e.g. as P(A), as -P(A), or as 1/P(A) instead of 1 - P(A).

conditional-denominator error.
Uses the wrong denominator for a conditional probability, e.g. the grand total or the joint probability instead of the probability of the given event.

Practice

1. *A fair coin and a fair six-sided die are each tossed once.*

What is the probability of getting heads AND a 3?

(A) 7/12
(B) 2/3
(C) 1/12
(D) 1/8

2. *A fair six-sided die is rolled once.*

What is the probability of rolling a 2 OR a 5?

(A) 2/5
(B) 1/6
(C) 1/36
(D) 1/3

3. *The probability that it rains tomorrow is 0.3.*

What is the probability that it does NOT rain?

(A) 0.3
(B) -0.3
(C) 0.7
(D) about 3.33

4. *In a group, P(likes tea and coffee) = 0.2 and P(likes coffee) = 0.5.*

What is P(likes tea GIVEN likes coffee)?

(A) 0.4

(B) 0.7

(C) 0.1

(D) 0.2

5. *For two events, P(A) = 0.6, P(B) = 0.3, and P(A and B) = 0.1.*

What is P(A or B)?

(A) 0.1

(B) 0.9

(C) 0.18

(D) 0.8

DOMAIN 4

Geometry & Trigonometry

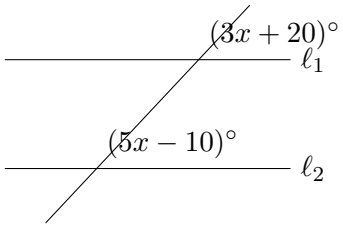
Shape, measure, and the relations that fix them

"There is no royal road to geometry."
— Euclid

About 15% of the section. Lines and angles, triangles and similarity, circles, area and volume, right-triangle trigonometry, and radians.

Angles & lines

The skill. Complementary angles sum to 90 degrees; supplementary angles sum to 180. Angles on a straight line sum to 180; around a point, 360. Vertical angles (across an intersection) are equal. When parallel lines are cut by a transversal, corresponding and alternate (interior or exterior) angles are EQUAL, while same-side (co-interior) angles are SUPPLEMENTARY. Set up an equation from the relationship and solve.



Parallel lines $\ell_1 \parallel \ell_2$ cut by a transversal: corresponding angles are equal.

Worked example

Two parallel lines are cut by a transversal. One angle measures $(3x + 20)^\circ$ and its corresponding angle measures $(5x - 10)^\circ$. Find x and the measure of the angle.

Read. Given: a transversal across parallel lines with corresponding angles $(3x + 20)^\circ$ and $(5x - 10)^\circ$. Asked: x and the angle.

Plan. Corresponding angles at parallel lines are EQUAL, so set the two expressions equal.

Solve.

$3x + 20 = 5x - 10$	set the corresponding angles equal
$20 + 10 = 5x - 3x$	collect the variable and constant terms
$30 = 2x$	simplify each side
$x = 15$	divide both sides by 2
$3(15) + 20 = 65°$	substitute back to find the angle

Check. The corresponding angle: $5(15) - 10 = 65°$. Both agree, and corresponding pairs should be equal, not supplementary.

Answer: $x = 15$, angle $= 65°$.

Approach Read · Plan · Solve · Check

Read. Identify the relationship between the angles (complementary, supplementary, vertical, or a parallel-line pair).

Plan. Pick the rule: sum to 90, sum to 180, equal, or supplementary. Write the equation.

Solve. Solve the resulting equation for the unknown angle or variable.

Check. Did you use 90 vs 180 correctly? For parallel lines, is the pair equal or supplementary?

Common traps — every wrong answer is one of these

complementary/supplementary swap.
Confuses which relationship sums to 90 and which sums to 180.

parallel-line angle-pair error.
Treats equal pairs (corresponding/alternate) as supplementary, or same-side interior pairs (supplementary) as equal.

wrong relationship/setup.
Applies the wrong angle relationship (e.g. treats vertical angles as supplementary, or a linear pair as equal) or sets up the equation incorrectly.

Practice

1. *Angle A and angle B are complementary, and A = 35 degrees.*

What is B?

(A) 35 degrees

(B) 55 degrees

(C) 325 degrees

(D) 145 degrees

2. *Two lines intersect, forming an angle of 70 degrees.*

What is the measure of its vertical angle?

(A) 70 degrees
(B) 290 degrees
(C) 110 degrees
(D) 20 degrees

3. *Two parallel lines are cut by a transversal, and one angle measures 65 degrees.*

What is the measure of its alternate interior angle?

(A) 65 degrees
(B) 130 degrees
(C) 115 degrees
(D) 25 degrees

4. *Two parallel lines are cut by a transversal. Two same-side interior angles are formed, and one measures 110 degrees.*

What is the measure of the other?

(A) 250 degrees
(B) 70 degrees
(C) 110 degrees
(D) 20 degrees

5. *Two angles form a linear pair and measure (2x + 10) degrees and (3x - 5) degrees.*

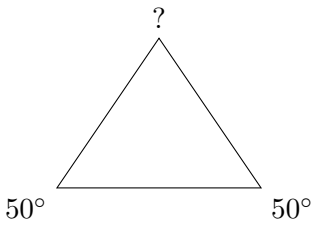
What is x?

(A) 35
(B) 15
(C) 17
(D) 175

Triangles (angle sum, isosceles, exterior angle, inequality)

The skill. The three interior angles of a triangle sum to 180 degrees. An isosceles triangle has two equal sides and two equal base angles (and the converse). An equilateral triangle has all angles 60

degrees. An exterior angle equals the SUM of the two remote (non-adjacent) interior angles. Three lengths form a triangle only if the sum of the two shorter ones exceeds the longest (the triangle inequality).



Isosceles triangle: the two base angles are equal, and all three angles sum to 180°.

Worked example

An isosceles triangle has two equal base angles, each measuring 50°. Find the vertex angle.

Read. Given: an isosceles triangle whose two base angles are each 50°. Asked: the vertex angle.

Plan. The three interior angles sum to 180°; subtract the two base angles from 180°.

Solve.

$$\begin{aligned} \text{vertex} &= 180^\circ - (50^\circ + 50^\circ) && \text{the three interior angles sum to } 180^\circ \\ &= 180^\circ - 100^\circ && \text{add the two equal base angles} \\ &= 80^\circ && \text{subtract from } 180^\circ \end{aligned}$$

Check. $50^\circ + 50^\circ + 80^\circ = 180^\circ$, and I subtracted from 180, not 360.

Answer: 80°.

Approach Read · Plan · Solve · Check

Read. Identify what is given (angles, sides, exterior angle) and which triangle fact applies.

Plan. Use angle sum = 180 for a missing angle; equal sides -> equal base angles; exterior = sum of remote interiors; check the triangle inequality for side sets.

Solve. Set up and solve the equation or test the inequality.

Check. Did you subtract from 180 (not 360)? For isosceles, are the right two angles equal? Is the exterior angle the SUM of the remote pair?

Common traps — every wrong answer is one of these

angle-sum error.
Does not use the 180-degree total correctly: reports the sum of two given angles, or subtracts from 360 instead of 180.

isosceles base-angle error.
Fails to convert equal sides into equal base angles, or mis-assigns which angles (vertex vs. base) are equal.

exterior-angle error.
Treats an exterior angle as equal to one remote interior angle, or as supplementary to their sum, instead of equal to the SUM of the two remote interiors.

triangle-inequality error.
Assumes any three lengths form a triangle, ignoring that the two shorter sides must sum to more than the longest.

Practice

1. *A triangle has two angles measuring 40 degrees and 75 degrees.*

What is the third angle?

(A) 115 degrees
(B) 65 degrees
(C) 40 degrees
(D) 245 degrees

2. *An isosceles triangle has a vertex angle of 40 degrees.*

What is the measure of each base angle?

(A) 70 degrees
(B) 40 degrees
(C) 140 degrees
(D) 100 degrees

3. *An exterior angle of a triangle measures 120 degrees, and the two remote interior angles are equal.*

What is the measure of each remote interior angle?

(A) 240 degrees
(B) 60 degrees
(C) 30 degrees
(D) 120 degrees

4. *Consider four sets of side lengths.*

Which set can form a triangle?

(A) 2, 3, 6
(B) 1, 1, 3
(C) 4, 4, 10
(D) 5, 6, 9

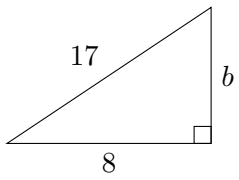
5. *The angles of a triangle measure x, 2x, and 3x degrees.*

What is the largest angle?

(A) 60 degrees
(B) 90 degrees
(C) 180 degrees
(D) 30 degrees

Right triangles & the Pythagorean theorem

The skill. In a right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse (the longest side, opposite the right angle). To find a leg, rearrange: $leg^2 = c^2$ - $other_leg^2$. Common triples: 3-4-5, 5-12-13, 8-15-17. Special triangles: a 45-45-90 has legs equal and hypotenuse = leg * $\sqrt{2}$; a 30-60-90 has sides x (short leg), x*$\sqrt{3}$ (long leg), 2x (hypotenuse).



Right triangle: $a^2 + b^2 = c^2$, where c is the hypotenuse.

Worked example

A right triangle has a hypotenuse of length 17 and one leg of length 8. Find the length of the other leg.

Read. Given: hypotenuse $c = 17$, one leg $= 8$. Asked: the other leg (call it b).

Plan. A leg is found by subtracting: $b^2 = c^2 - a^2$, then take the square root.

Solve.

$b^2 = c^2 - a^2$	rearrange the Pythagorean theorem for a leg
$b^2 = 17^2 - 8^2$	substitute the hypotenuse and known leg
$b^2 = 289 - 64$	square each term
$b^2 = 225$	subtract
$b = 15$	take the positive square root

Check. $8^2 + 15^2 = 64 + 225 = 289 = 17^2$; this is the 8-15-17 triple, and 17 is the longest side.

Answer: 15.

Approach Read · Plan · Solve · Check

Read. Identify the right angle and which side is the hypotenuse; note whether a leg or the hypotenuse is unknown. Check for a special triangle.

Plan. Use $a^2 + b^2 = c^2$ for a hypotenuse, or c^2 - leg^2 for a leg. For special triangles, apply the $\sqrt{2}$ or $\sqrt{3}$ ratio.

Solve. Square, add or subtract, then take the square root; or scale the special-triangle ratio.

Check. Is the hypotenuse the longest side? Did you take the final square root? Right ratio ($\sqrt{2}$ for 45-45-90, $\sqrt{3}$ for the long leg of 30-60-90)?

Common traps — every wrong answer is one of these

hypotenuse/leg confusion.
Applies $a^2 + b^2 = c^2$ to find a LEG (adding the hypotenuse in) instead of subtracting: $leg^2 = c^2$ - $other_leg^2$.

missing square or square-root.
Adds or subtracts the side lengths directly without squaring, or stops at the squared value without taking the final square root.

special-triangle ratio error.
Uses the wrong special-triangle ratio ($\sqrt{3}$ where $\sqrt{2}$ belongs, or vice versa), or assigns the ratio to the wrong side.

Practice

1. *A right triangle has legs of length 3 and 4.*

What is the length of the hypotenuse?

(A) 12

(B) 5

(C) 25

(D) 7

2. *A right triangle has a hypotenuse of 13 and one leg of 5.*

What is the length of the other leg?

(A) 8

(B) about 13.9 (sqrt of 194)

(C) 18

(D) 12

3. *A right triangle has legs of length 6 and 8.*

What is the length of the hypotenuse?

(A) 10

(B) 100

(C) 14

(D) sqrt of 28

4. *A 45-45-90 right triangle has legs of length 5.*

What is the length of the hypotenuse?

(A) $5*\sqrt{2}$

(B) 5

(C) 10

(D) $5*\sqrt{3}$

5. *In a 30-60-90 triangle, the shorter leg has length 4.*

What is the length of the hypotenuse?

(A) $4*\sqrt{3}$

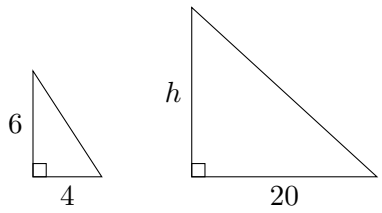
(B) 12

(C) 8

(D) $4*\sqrt{2}$

Similarity & congruence

The skill. Similar figures have the same shape: corresponding angles are equal and corresponding sides are proportional by a constant scale factor k. Congruent figures are similar with k = 1 (same size). To find a missing side, set up a proportion that keeps corresponding sides aligned. Lengths scale by k, but AREAS scale by k^2 (and volumes by k^3).



Similar right triangles: the height-to-shadow ratio is the same for both.

Worked example

A 6-foot-tall person casts a 4-foot shadow at the same time a flagpole casts a 20-foot shadow. The person and the flagpole with their shadows form similar right triangles. How tall is the flagpole?

Read. Given: person height 6 with shadow 4; flagpole shadow 20. Asked: flagpole height h.

Plan. Corresponding sides of similar triangles are proportional: $\frac{\text{height}}{\text{shadow}}$ is the same for both.

Solve.

$\frac{6}{4} = \frac{h}{20}$	set the height-to-shadow ratios equal
$h = 20 \cdot \frac{6}{4}$	multiply both sides by 20
$h = 20 \cdot \frac{3}{2}$	simplify the fraction
$h = 30$	multiply

Check. Scale factor from person to pole is $\frac{20}{4} = 5$; multiplying the height, $6 \cdot 5 = 30$. I multiplied by the factor rather than adding a difference.

Answer: 30 feet.

Approach Read · Plan · Solve · Check

Read. Confirm the figures are similar; identify which sides correspond and the scale factor.

Plan. For a length, write a proportion with corresponding sides in matching positions. For an area, use k^2, not k.

Solve. Solve the proportion (multiply by k), or multiply an area by k^2.

Check. Are corresponding sides aligned (not inverted)? Did you multiply by the scale factor (not add a difference)? Area uses k^2?

Common traps — every wrong answer is one of these

corresponding-side mismatch.
Pairs non-corresponding sides in the proportion, or inverts the ratio, so the setup is inconsistent.

additive vs multiplicative scaling.
Adds the difference between corresponding sides instead of multiplying by the scale factor.

area scale-factor error.
Scales an area by the linear factor k (or k^3) instead of k^2.

Practice

1. *Two triangles are similar. A side of length 4 in the first corresponds to a side of length 6 in the second.*

What length in the second triangle corresponds to a side of length 6 in the first?

(A) 8
(B) 4
(C) 4.5
(D) 9

2. *Triangle ABC is similar to triangle DEF, with AB = 8 corresponding to DE = 12 and BC = 10.*

What is the length of EF?

(A) 14
(B) 30
(C) about 6.7
(D) 15

3. *Two rectangles are similar. The smaller is 3 by 5, and the larger has a short side of 9.*

What is the long side of the larger rectangle?

(A) 45
(B) about 1.7
(C) 11
(D) 15

4. *Two similar triangles have a linear scale factor of 3, and the smaller has an area of 8.*

What is the area of the larger triangle?

(A) 24
(B) 8
(C) 72
(D) 216

5. *Triangle PQR is similar to triangle STU, with PQ = 6 corresponding to ST = 9. The area of triangle PQR is 20.*

What is the area of triangle STU?

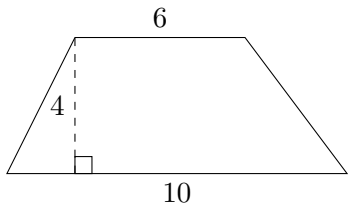
(A) 30
(B) about 13.3

(C) 45

(D) 67.5

Area & perimeter

The skill. Perimeter is the total distance around a figure (linear units); area is the space inside (square units). Rectangle area = base x height, perimeter = 2(l + w). Triangle area = (1/2) x base x perpendicular height. Parallelogram area = base x perpendicular height. Trapezoid area = (1/2)(b1 + b2)h. For composite figures, decompose into rectangles and triangles, then add or subtract regions. Always use the PERPENDICULAR height, not a slanted side.



Trapezoid: area $= \frac{1}{2}(b_1 + b_2)h$ *using the perpendicular height* h*.*

Worked example

A trapezoid has parallel bases of length 6 and 10 and a perpendicular height of 4. Find its area.

Read. Given: parallel bases $b_1 = 6$ and $b_2 = 10$, perpendicular height $h = 4$. Asked: the area.

Plan. Trapezoid area $= \frac{1}{2}(b_1 + b_2)h$.

Solve.

$A = \frac{1}{2}(b_1 + b_2)h$	trapezoid area formula
$A = \frac{1}{2}(6 + 10)(4)$	substitute the two bases and the height
$A = \frac{1}{2}(16)(4)$	add the bases
$A = 32$	multiply

Check. The result is between the two rectangles $6 \times 4 = 24$ and $10 \times 4 = 40$, as an average of the bases should give.

Answer: 32 square units.

Approach Read · Plan · Solve · Check

Read. Decide whether area or perimeter is asked, and identify the base and the PERPENDICULAR height (not a slant side).

Plan. Pick the formula; for composites, split into known shapes and decide add vs. subtract.

Solve. Apply the formula (keep the 1/2 for triangles); combine composite regions.

Check. Area (square units) or perimeter (linear)? Did you use the perpendicular height and keep the 1/2? Did composite regions add or subtract correctly?

Common traps — every wrong answer is one of these

area/perimeter confusion.
Computes perimeter when area is asked (or vice versa), or otherwise confuses the linear and square dimensions.

triangle 1/2 omission.
Uses base x height for a triangle's area, omitting the factor of 1/2.

wrong height.
Uses a slanted side as the height instead of the perpendicular height in a triangle or parallelogram.

composite-figure error.
Adds a removed region instead of subtracting it, ignores a sub-region, or double-counts when combining shapes.

Practice

1. *A rectangle measures 8 cm by 5 cm.*

What is its area?

(A) 40 square cm
(B) 13 cm
(C) 64 square cm
(D) 26 cm

2. *A triangle has a base of 12, a perpendicular height of 5, and a slanted side of 13.*

What is its area?

(A) 30
(B) 78
(C) 156
(D) 60

3. *A parallelogram has a base of 9, a perpendicular height of 4, and a slanted side of 5.*

What is its area?

(A) 28

(B) 36

(C) 45

(D) 18

4. *An L-shaped figure is a 6 by 8 rectangle with a 2 by 3 rectangular notch removed from one corner.*

What is the area of the L-shape?

(A) 48

(B) 54

(C) 42

(D) 6

5. *A figure is a 10 by 6 rectangle with a triangle (base 6, perpendicular height 4) attached along one side.*

What is the total area?

(A) 60

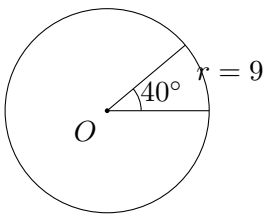
(B) 72

(C) 32

(D) 84

Circles

The skill. For a circle of radius r: circumference C = 2*pi*r (= pi*d), area A = pi*r^2. An arc or sector for a central angle of theta degrees is the fraction theta/360 of the whole: arc length = (theta/360)(2*pi*r), sector area = (theta/360)(pi*r^2). The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$ – note the center coordinates flip sign from the equation, and the right side is r SQUARED.



Sector with central angle θ: arc length = $\frac{\theta}{360}(2\pi r)$.

Worked example

A circle has a radius of 9. A sector of the circle has a central angle of 40°. Find the length of the arc that bounds the sector.

Read. Given: radius $r = 9$, central angle $\theta = 40°$. Asked: the arc length.

Plan. Arc length $= \frac{\theta}{360}(2\pi r)$, the fraction $\frac{\theta}{360}$ of the full circumference.

Solve.

$s = \frac{\theta}{360}(2\pi r)$	arc length is that fraction of the circumference
$s = \frac{40}{360}(2\pi \cdot 9)$	substitute the angle and radius
$s = \frac{1}{9}(18\pi)$	reduce the fraction and the circumference
$s = 2\pi$	multiply

Check. 40° is $\frac{1}{9}$ of 360°, and $\frac{1}{9}$ of the circumference 18π is 2π; I used radius, not diameter, and kept the $\frac{\theta}{360}$ fraction.

Answer: 2π.

Approach Read · Plan · Solve · Check

Read. Identify whether r or d is given, and whether the question wants circumference, area, an arc/sector, or the equation.

Plan. Use 2*pi*r for distance around, pi*r^2 for space inside; multiply by theta/360 for a part; for the equation read center as (h, k) with flipped signs and r = $\sqrt{rightside}$.

Solve. Substitute the radius (not diameter), apply the fraction, or take the square root for r.

Check. Radius or diameter? Circumference vs area formula? Did you include theta/360? Center signs flipped and r vs r^2 correct?

Common traps — every wrong answer is one of these

radius/diameter confusion.
Uses the diameter where the radius is required (or vice versa), e.g. substituting d into pi*r^2 or 2*pi*r.

circumference/area formula swap.
Uses 2*pi*r when area is asked, or pi*r^2 when circumference is asked.

arc/sector fraction error.
Omits the theta/360 fraction (giving the whole circle), or applies the fraction incorrectly.

circle-equation error.
Reads the center with the wrong sign, or treats the right-hand side as r instead of r^2 (forgetting the square root).

Practice

1. *A circle has a radius of 5.*

What is its circumference?

(A) 10*pi
(B) 25*pi
(C) 5*pi
(D) 20*pi

2. *A circle has a radius of 4.*

What is its area?

(A) 16*pi
(B) 8*pi
(C) 64*pi
(D) 4*pi

3. *A circle has a radius of 6, and a sector has a central angle of 60 degrees.*

What is the area of the sector?

(A) 6*pi
(B) 12*pi
(C) 36*pi
(D) 2*pi

4. *A circle has the equation $(x - 3)^2 + (y + 2)^2 = 16$.*

What are its center and radius?

(A) Center (3, -2), radius 4
(B) Center (-3, 2), radius 4
(C) Center (-3, 2), radius 16
(D) Center (3, -2), radius 16

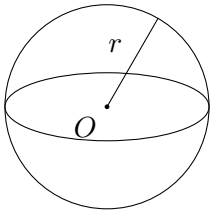
5. *A circle has center (1, -4) and passes through the point (1, 0).*

What is its equation?

(A) $(x - 1)^2 + (y + 4)^2 = 16$
(B) $(x - 1)^2 + (y + 4)^2 = 64$
(C) $(x + 1)^2 + (y - 4)^2 = 16$
(D) $(x - 1)^2 + (y + 4)^2 = 4$

Volume & surface area

The skill. Volume measures space inside a solid (cubic units); surface area is the total area of its faces (square units). Rectangular prism: V = lwh. Cylinder: V = pi*r^2*h. Cone: V = (1/3)pi*r^2*h. Sphere: V = (4/3)pi*r^3, surface area = 4*pi*r^2. Cones and pyramids carry a 1/3; a sphere carries 4/3 and uses r CUBED. Use the radius, not the diameter.



Sphere of radius r: surface area $4\pi r^2$, volume $\frac{4}{3}\pi r^3$.

Worked example

A sphere has a radius of 5. Find its surface area, leaving the answer in terms of π.

Read. Given: sphere with radius $r = 5$. Asked: surface area (a square-unit quantity), not volume.

Plan. Sphere surface area $= 4\pi r^2$.

Solve.

$A = 4\pi r^2$	sphere surface-area formula
$A = 4\pi(5)^2$	substitute the radius
$A = 4\pi \cdot 25$	square the radius
$A = 100\pi$	multiply

Check. Surface area uses r^2 (not r^3), so units are square, and I used the radius 5, not a diameter.

Answer: 100π square units.

Approach Read · Plan · Solve · Check

Read. Identify the solid, whether volume or surface area is asked, and the radius (not diameter).

Plan. Choose the matching formula; remember the 1/3 for cone/pyramid and the 4/3 with r^3 for a sphere.

Solve. Substitute the radius, apply the correct power, and include any fractional coefficient.

Check. Volume (cubic) or surface area (square)? Did you include the 1/3 or 4/3? Right power (r^2 vs r^3) and radius, not diameter?

Common traps — every wrong answer is one of these

volume/surface-area confusion.
Computes surface area when volume is asked (or vice versa), or uses a 2D area formula for a 3D quantity.

missing fractional coefficient.
Omits the 1/3 for a cone or pyramid, or the 4/3 for a sphere (e.g. using the cylinder formula for a cone).

radius/diameter & power error.
Uses the diameter in place of the radius, or the wrong power (r^2 where r^3 belongs).

Practice

1. *A rectangular box measures 4 by 3 by 5.*

What is its volume?

(A) 94
(B) 60
(C) 47
(D) 12

2. *A cylinder has a radius of 3 and a height of 10.*

What is its volume?

(A) 60*pi
(B) 90*pi
(C) 30*pi
(D) 360*pi

3. *A cone has a radius of 3 and a height of 4.*

What is its volume?

(A) 48*pi
(B) 36*pi
(C) 4*pi
(D) 12*pi

4. *A sphere has a radius of 3.*

What is its volume?

(A) 12*pi

(B) 108*pi

(C) 27*pi

(D) 36*pi

5. *A cube has an edge length of 5.*

What is its surface area?

(A) 25

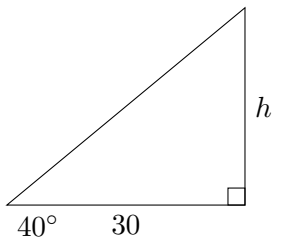
(B) 30

(C) 150

(D) 125

Right-triangle trigonometry

The skill. In a right triangle, relative to an acute angle: sin = opposite/hypotenuse, cos = adjacent/hypotenuse, tan = opposite/adjacent (SOH-CAH-TOA). To find a side, multiply the known side by the right ratio. The two acute angles are complementary, so sin(theta) = cos(90 - theta): the sine of an angle equals the cosine of its complement. (The SAT uses right-triangle trig only – no unit circle.)



Angle of elevation θ: opposite = adjacent $\times \tan\theta$.

Worked example

From a point 30 feet from the base of a tree, the angle of elevation to the top of the tree is $40°$. Using $\tan 40° \approx 0.84$, find the height of the tree. (The tree, ground, and line of sight form a right triangle with the right angle at the base of the tree.)

Read. Given: horizontal distance (adjacent) = 30, angle = $40°$, $\tan 40° \approx 0.84$. Asked: the tree height (opposite side).

Plan. Opposite and adjacent are linked by tangent: $\tan 40° = \frac{\text{opposite}}{\text{adjacent}}$, so opposite = adjacent $\cdot$ $\tan 40°$.

Solve.

$$\begin{aligned} \text{opposite} &= \text{adjacent} \cdot \tan 40^\circ && \text{tangent links opposite and adjacent} \\ h &= 30 \cdot \tan 40^\circ && \text{substitute the adjacent side} \\ h &= 30 \cdot 0.84 && \text{use } \tan 40^\circ \approx 0.84 \\ h &= 25.2 && \text{multiply} \end{aligned}$$

Check. The height (≈ 25.2) is less than the hypotenuse and comparable to the 30-ft distance, which fits a 40° elevation; I multiplied (not divided) by the ratio.

Answer: ≈ 25.2 feet.

Approach Read · Plan · Solve · Check

Read. Mark the given acute angle; label each side as opposite, adjacent, or hypotenuse relative to it.

Plan. Pick the ratio that links the known and unknown sides (SOH-CAH-TOA). For complementary angles, use sin(theta) = cos(90 - theta).

Solve. Form the ratio (right side up), and multiply to solve for a missing side.

Check. Right ratio for what's asked? Opposite/adjacent labeled from the correct angle? Ratio not inverted? Cofunction uses the complement?

Common traps — every wrong answer is one of these

wrong/inverted ratio.
Selects the wrong trig ratio (sin for cos, etc.) or inverts it (hypotenuse/opposite instead of opposite/hypotenuse), including dividing by a ratio when one should multiply.

opposite/adjacent identification error.
Mislabels which leg is opposite versus adjacent (or treats a leg as the hypotenuse) relative to the given angle.

cofunction error.
Treats sin(theta) = cos(theta) instead of sin(theta) = cos(90 - theta), or otherwise mishandles the complementary relationship between the two acute angles.

Practice

1. *In a right triangle, the acute angle theta has an opposite side of 3, an adjacent side of 4, and a hypotenuse of 5.*

   What is sin(theta)?

   (A) 4/5

(B) 5/3
(C) 3/4
(D) 3/5

2. *Same right triangle: opposite 3, adjacent 4, hypotenuse 5, angle theta.*

What is cos(theta)?

(A) 3/5
(B) 4/5
(C) 4/3
(D) 5/4

3. *A right triangle has a 30-degree angle and a hypotenuse of 10. (sin 30 = 0.5, cos 30 is about 0.87.)*

What is the length of the side opposite the 30-degree angle?

(A) 0.5
(B) 20
(C) about 8.7
(D) 5

4. *In a right triangle, sin(theta) = 0.6, and phi is the other acute angle.*

What is cos(phi)?

(A) 0.8
(B) about 1.67
(C) 0.6
(D) 0.4

5. *A 12-foot ladder leans against a wall, making a 60-degree angle with the ground. (sin 60 is about 0.87, cos 60 = 0.5.)*

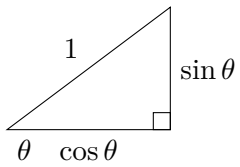
How high up the wall does the ladder reach?

(A) 24 ft
(B) about 10.4 ft
(C) about 0.87 ft
(D) 6 ft

Radian measure & trigonometric relationships

The skill. Radians measure an angle by arc length: a full circle is 2*pi radians = 360 degrees, so 180 degrees = pi radians. Convert with degrees * (pi/180) = radians and radians * (180/pi) =

degrees. Arc length is s = r*theta only when theta is in RADIANS. Sine and cosine are cofunctions: sin(theta) = cos(90 degrees - theta), so an angle's sine equals the cosine of its COMPLEMENT (not its supplement). The Pythagorean identity $\sin^2(\text{theta}) + \cos^2(\text{theta}) = 1$ holds for every angle.



Unit hypotenuse: legs $\cos\theta$ *and* $\sin\theta$ *give* $\sin^2\theta + \cos^2\theta = 1$.

Worked example

An acute angle θ satisfies $\sin\theta = 0.8$. Find $\cos\theta$.

Read. Given: $\sin\theta = 0.8$ with θ acute. Asked: $\cos\theta$.

Plan. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \sin^2\theta$; take the positive root since θ is acute.

Solve.

$\cos^2\theta = 1 - \sin^2\theta$	Pythagorean identity, solved for cosine
$\cos^2\theta = 1 - (0.8)^2$	substitute $\sin\theta = 0.8$
$\cos^2\theta = 1 - 0.64$	square
$\cos^2\theta = 0.36$	subtract
$\cos\theta = 0.6$	take the positive square root since θ is acute

Check. I squared $\sin\theta$ before subtracting, and $(0.6)^2 + (0.8)^2 = 0.36 + 0.64 = 1$.
Answer: $\cos\theta = 0.6$.

Approach Read · Plan · Solve · Check

Read. Is the task a unit conversion, a cofunction/complement relationship, the Pythagorean identity, or an arc length?

Plan. Convert by multiplying by pi/180 or 180/pi. Cofunction: pair an angle with its 90-degree complement. Identity: $\cos^2 = 1 - \sin^2$ (and vice versa), squaring first. Arc length: s = r*theta with theta in radians.

Solve. Apply the relevant rule, keeping units straight (radians for s = r*theta).

Check. Did I use the right conversion factor and keep pi? Complement (90 - theta), not supplement? Did I SQUARE before subtracting in the identity? Was theta in radians for the arc length?

Common traps — every wrong answer is one of these

degree-radian conversion error.
Converts with the wrong factor or direction: multiplies by 180/pi when going from degrees to radians, drops the pi, or otherwise reverses the conversion.

complementary/cofunction error.
Misuses the cofunction relationship: treats sin(theta) = cos(theta), uses the supplement 180 - theta instead of the complement 90 - theta, or applies the value's complement (1 - value) instead of the angle's.

Pythagorean-identity error.
Mishandles $\sin^2 + \cos^2 = 1$: forgets to square (e.g. 1 - sin instead of 1 - $\sin^2$), returns $\sin^2$ in place of $\cos^2$, or sets a single squared term equal to 1.

arc-length radian/degree error.
Uses s = r*theta with theta in degrees instead of radians, or confuses the arc length with the central angle itself.

Practice

1. *Convert 90 degrees to radians.*

What is the result?

(A) 2*pi
(B) 90*pi
(C) pi/2
(D) 180/pi

2. *Convert pi/3 radians to degrees.*

What is the result?

(A) 60*pi degrees
(B) 60 degrees
(C) 30 degrees
(D) 120 degrees

3. *It is given that sin(40 degrees) = 0.64.*

What is cos(50 degrees)?

(A) 0.36
(B) 0.64
(C) It cannot be determined

(D) -0.64

4. *An angle theta satisfies sin(theta) = 3/5.*

What is $\cos^2$(theta)?

(A) 1
(B) 2/5
(C) 9/25
(D) 16/25

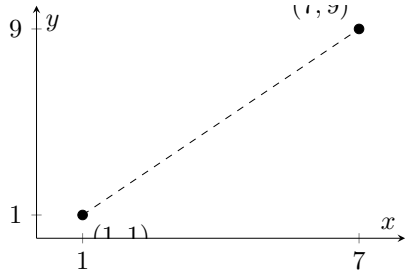
5. *A circle has radius 6. A central angle of pi/3 radians subtends an arc.*

What is the length of that arc?

(A) 360
(B) 12*pi
(C) pi/3
(D) 2*pi

Coordinate geometry: distance & midpoint

The skill. The distance between (x1, y1) and (x2, y2) is $\sqrt{(x2 - x1)^2}$ + (y2 - y1)^2): it is the Pythagorean theorem applied to the horizontal and vertical differences, so you SUBTRACT, square, add, and take the square root. The midpoint is ((x1 + x2)/2, (y1 + y2)/2): you AVERAGE the coordinates, so you ADD and halve. Distance uses differences and ends in a square root; midpoint uses sums and ends in a division by two. Mixing the two operations is the central trap.



Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$*; the midpoint averages the coordinates.*

Worked example

Find the distance between the points $(1, 1)$ **and** $(7, 9)$**.**

Read. Given: two points $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (7, 9)$. Asked: the distance (a single length).

Plan. Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$: subtract, square, add, square-root.

Solve.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	distance formula
$d = \sqrt{(7 - 1)^2 + (9 - 1)^2}$	substitute the two points
$d = \sqrt{6^2 + 8^2}$	subtract within each pair
$d = \sqrt{36 + 64}$	square each difference
$d = \sqrt{100}$	add
$d = 10$	take the square root

Check. The horizontal and vertical legs 6 and 8 form a 6-8-10 right triangle, and I both squared and took the final square root.

Answer: 10.

Approach Read · Plan · Solve · Check

Read. Is this a distance (a single length, uses differences and a square root) or a midpoint (a point, averages the coordinates)?

Plan. Distance: subtract coordinates, square each difference, add, square-root. Midpoint: add matching coordinates and divide each sum by 2. Track signs carefully with negative coordinates.

Solve. Carry out the right operation; pair x with x and y with y.

Check. Distance: did I square AND square-root? Midpoint: did I add then halve (not subtract)? Did I keep signs right on negative coordinates? Did I pair coordinates correctly?

Common traps — every wrong answer is one of these

distance-formula error.
Omits the square or the square root, or fails to square the differences, when computing distance (e.g. reporting the sum of squares without rooting, or adding absolute differences).

distance/midpoint operation swap.
Uses the wrong operation for the task: subtracting coordinates to find a midpoint, or averaging/adding coordinates to find a distance.

midpoint error.
Adds the coordinates without halving, halves only one coordinate, or otherwise mishandles the average when finding a midpoint.

sign/pairing error.
Makes a sign error with negative coordinates (e.g. treating a difference with a negative value as positive) or pairs x with y incorrectly.

Practice

1. *Consider the points (1, 2) and (4, 6).*

What is the distance between them?

(A) $\sqrt{7}$

(B) 7

(C) 25

(D) 5

2. *Consider the points (2, 3) and (8, 7).*

What is the midpoint of the segment joining them?

(A) (5, 5)

(B) (6, 4)

(C) (5, 10)

(D) (10, 10)

3. *Two points have coordinates (x1, y1) and (x2, y2).*

Which expression gives their midpoint?

(A) ((x1 - x2)/2, (y1 - y2)/2)

(B) (x2 - x1, y2 - y1)

(C) $\sqrt{(x2 - x1^2 + (y2 - y1)^2)}$

(D) ((x1 + x2)/2, (y1 + y2)/2)

4. *Consider the points (-2, 1) and (1, 5).*

What is the distance between them?

(A) $\sqrt{37}$

(B) $\sqrt{17}$

(C) $\sqrt{20}$

(D) 5

5. *Let M be the midpoint of the segment with endpoints (0, 0) and (6, 8).*

What is the distance from (0, 0) to M?

(A) 10

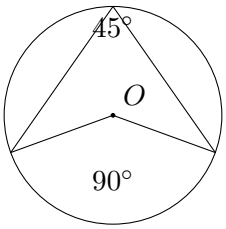
(B) $\sqrt{7}$

(C) 25

(D) 5

Circle angles: central, inscribed & arcs

The skill. A central angle has the same degree measure as the arc it intercepts. An inscribed angle is HALF the measure of the arc it intercepts, and therefore half of a central angle that subtends the same arc. A special case: an angle inscribed in a semicircle (its rays pass through the ends of a diameter) intercepts a 180-degree arc, so it measures 90 degrees. Arc MEASURE in degrees is different from arc LENGTH in units.



An inscribed angle is half the central angle on the same arc.

Worked example

An inscribed angle of a circle measures $45°$. Find the measure of the arc it intercepts, and the measure of a central angle that subtends that same arc.

Read. Given: an inscribed angle of $45°$. Asked: its intercepted arc, and a central angle on the same arc.

Plan. An inscribed angle is HALF its intercepted arc, so the arc is twice the angle; a central angle equals its intercepted arc.

Solve.

$$\begin{aligned} \text{arc} &= 2 \times 45^\circ && \text{an inscribed angle is half its intercepted arc} \\ &= 90^\circ && \text{double the inscribed angle} \\ \text{central angle} &= 90^\circ && \text{a central angle equals its intercepted arc} \end{aligned}$$

Check. Halving the $90°$ arc gives back the $45°$ inscribed angle, and the central angle ($90°$) is double the inscribed angle ($45°$), as expected.

Answer: arc $= 90°$, central angle $= 90°$.

Approach Read · Plan · Solve · Check

Read. Is the angle central (vertex at the center) or inscribed (vertex on the circle)? Which arc does it intercept?

Plan. Central angle = its arc. Inscribed angle = half its arc = half a central angle on the same arc. Angle inscribed in a semicircle = 90 degrees.

Solve. Apply the matching relationship between the angle and its intercepted arc.

Check. Did I halve for an inscribed angle (and not for a central one)? Did I use the SAME intercepted arc? Did I recognize a semicircle as giving 90 degrees?

Common traps — every wrong answer is one of these

inscribed/central factor error.
Treats an inscribed angle as equal to the central angle (or the full arc) instead of half, or applies the factor of two in the wrong direction.

arc-measure vs central-angle confusion.
Does not equate a central angle with its intercepted arc (halves or doubles it), or confuses arc measure in degrees with arc length.

semicircle error.
Fails to recognize that an angle inscribed in a semicircle is 90 degrees, using the 180-degree arc directly or another value.

Practice

1. *A central angle of a circle measures 80 degrees.*

What is the measure of its intercepted arc?

(A) 100 degrees
(B) 160 degrees
(C) 80 degrees
(D) 40 degrees

2. *An inscribed angle subtends the same arc as a central angle of 100 degrees.*

What is the measure of the inscribed angle?

(A) 25 degrees
(B) 100 degrees
(C) 50 degrees
(D) 200 degrees

3. *An inscribed angle intercepts an arc of 60 degrees.*

What is the measure of the inscribed angle?

(A) 15 degrees
(B) 120 degrees
(C) 60 degrees
(D) 30 degrees

4. *An angle is inscribed in a semicircle, with its rays passing through the two endpoints of a diameter.*

What is the measure of the angle?

(A) 90 degrees

(B) 180 degrees

(C) It cannot be determined

(D) 45 degrees

5. *A central angle of a circle measures 120 degrees.*

What are the measure of its intercepted arc and the measure of an inscribed angle on that same arc?

(A) Arc 60 degrees; inscribed angle 120 degrees

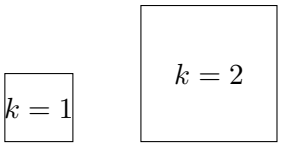
(B) Arc 120 degrees; inscribed angle 60 degrees

(C) Arc 120 degrees; inscribed angle 120 degrees

(D) Arc 240 degrees; inscribed angle 60 degrees

Scale factor: length, area & volume

The skill. If two similar figures have a linear scale factor k (the ratio of corresponding lengths), then their areas are in the ratio k^2 and their volumes are in the ratio k^3. So doubling every length ($k = 2$) multiplies area by 4 and volume by 8. Reversing this, a known area ratio gives the linear ratio by a square root, and a known volume ratio gives it by a cube root. Lengths use k, areas use k squared, volumes use k cubed.



Linear factor k: area scales by k^2, volume by k^3.

Worked example

Two similar solids have surface areas in the ratio $9 : 16$. Find the ratio of their volumes.

Read. Given: an AREA ratio $9 : 16$ for two similar solids. Asked: the VOLUME ratio.

Plan. Areas scale as k^2, so recover the linear factor k by a square root; then volumes scale as k^3.

Solve.

$$k = \sqrt{\frac{9}{16}} \qquad \text{areas scale as } k^2\text{, so take the square root}$$

$$k = \frac{3}{4} \qquad \text{simplify the linear factor}$$

$$k^3 = \left(\frac{3}{4}\right)^3 \qquad \text{volumes scale as } k^3$$

$$k^3 = \frac{27}{64} \qquad \text{cube the ratio, giving } 27 : 64$$

Check. Squaring the linear ratio $3 : 4$ gives $9 : 16$ (the given areas), and cubing it gives $27 : 64$; exponents match each dimension.

Answer: $27 : 64$.

Approach Read · Plan · Solve · Check

Read. Am I going from a length ratio to an area or volume ratio, or backward from an area/volume ratio to a length ratio?

Plan. Length to area: square the factor. Length to volume: cube it. Area to length: square root. Volume to length: cube root. Area to volume or vice versa: pass through the linear factor.

Solve. Apply the correct power or root of the scale factor.

Check. Did I use k^2 for area and k^3 for volume (not k itself)? Did I match the exponent to the dimension? For a reverse problem, did I take the right root?

Common traps — every wrong answer is one of these

linear-factor-to-area/volume error.
Applies the linear scale factor k directly to area or volume (e.g. doubling the sides doubles the area), instead of using k^2 or k^3.

area/volume exponent swap.
Uses the square where the cube is needed or vice versa: k^2 for volume or k^3 for area.

reverse scale error.
Given an area or volume ratio, fails to take the square or cube root to recover the linear ratio (or stops at the linear ratio when an area/volume answer is wanted).

Practice

1. *Two similar rectangles have corresponding sides in the ratio 1 : 3.*

What is the ratio of their areas?

(A) $1 : 9$

(B) 1 : 6

(C) 1 : 27

(D) 1 : 3

2. *Two similar solids have corresponding edges in the ratio 1 : 2.*

What is the ratio of their volumes?

(A) 1 : 6

(B) 1 : 2

(C) 1 : 4

(D) 1 : 8

3. *Every dimension of a cube is doubled.*

Its volume becomes how many times as large?

(A) 4 times

(B) 6 times

(C) 8 times

(D) 2 times

4. *Two similar figures have areas in the ratio 25 : 1.*

What is the ratio of their corresponding sides?

(A) 5 : 1

(B) 25 : 1

(C) 625 : 1

(D) 12.5 : 1

5. *Two similar solids have volumes in the ratio 27 : 1.*

What is the ratio of their surface areas?

(A) 27 : 1

(B) 9 : 1

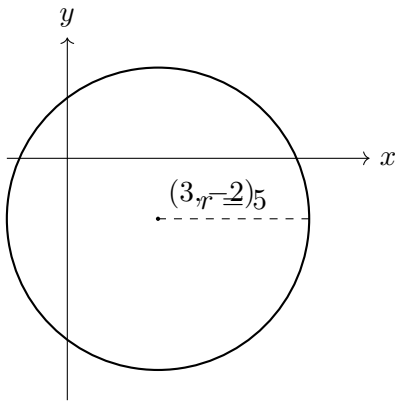
(C) 3 : 1

(D) 729 : 1

Circle equations: center & radius by completing the square

The skill. A circle given in general form $x^2 + y^2 + Dx + Ey + F = 0$ is converted to standard form $(x - h)^2 + (y - k)^2 = r^2$ by completing the square separately in x and in y. Group the x-terms

and y-terms, add the square of half each linear coefficient to complete each square, and add the same amounts to the other side to stay balanced. The center (h, k) reads with signs OPPOSITE to the parentheses, and the radius is the SQUARE ROOT of the right-hand side, not the right-hand side itself.



Standard form $(x-h)^2+(y-k)^2=r^2$: center (h,k), radius r.

Worked example

Convert $x^2+y^2-6x+4y-12=0$ to standard form, and state the circle's center and radius.

Read. Given: a circle in general form. Asked: standard form, plus center and radius.

Plan. Group x-terms and y-terms, complete each square by adding (half the linear coefficient)^2, and add the same amounts to the other side.

Solve.

$$(x^2-6x)+(y^2+4y)=12 \qquad \text{group the terms and move the constant right}$$

$$(x^2-6x+9)+(y^2+4y+4)=12+9+4 \qquad \text{add the square of half each coefficient to both sides}$$

$$(x-3)^2+(y+2)^2=25 \qquad \text{write each group as a perfect square}$$

$$r=5 \qquad \text{radius is } \sqrt{25}\text{; center reads } (3,-2)$$

Check. Expanding $(x-3)^2+(y+2)^2=25$ gives $x^2+y^2-6x+4y+13=25$, i.e. $-12=0$ shifted correctly; center signs are flipped and the radius is the square root of 25.

Answer: $(x-3)^2+(y+2)^2=25$; center $(3,-2)$, radius 5.

Approach Read · Plan · Solve · Check

Read. Is the equation already in standard form (read center and radius) or in general form (complete the square first)?

Plan. Group x-terms and y-terms. Complete each square by adding (half the linear coefficient) squared, adding the same to the other side. Rewrite as $(x-h)^2+(y-k)^2=r^2$; read center with opposite signs and radius as the square root.

Solve. Complete both squares, balance, and extract center and radius.

Check. Did I HALVE before squaring each coefficient? Did I add the completing constants to BOTH sides? Did I flip the signs for the center and square-root the right side for the radius?

Common traps — every wrong answer is one of these

half-then-square error.
Adds the wrong constant when completing a square in the circle equation, e.g. the square of the whole coefficient instead of the square of half of it.

balance error.
Adds the completing constants to one side only, or mishandles the constant term F, so the right-hand side is wrong.

center-sign/radius error.
Reads the center with the wrong signs, or treats the right-hand side as the radius instead of its square root.

Practice

1. *You are completing the square on the x-terms $x^2 + 6x$ of a circle's equation.*

What constant should be added?

(A) 36
(B) 6
(C) 9
(D) 3

2. *A circle has equation $(x - 2)^2 + (y + 3)^2 = 16$.*

What is its center?

(A) (-2, 3)
(B) (2, 3)
(C) (2, -3)
(D) (-2, -3)

3. *A circle has equation $(x - 1)^2 + (y - 4)^2 = 25$.*

What is its radius?

(A) 12.5
(B) 25

(C) 625

(D) 5

4. *Convert $x^2 + y^2 - 4x + 6y + 9 = 0$ to standard form.*

Which is the result?

(A) $(x - 2)^2 + (y + 3)^2 = 4$

(B) $(x - 2)^2 + (y + 3)^2 = 13$

(C) $(x - 2)^2 + (y + 3)^2 = 22$

(D) $(x - 2)^2 + (y + 3)^2 = -9$

5. *Consider the circle $x^2 + y^2 + 8x - 2y + 8 = 0$.*

What are its center and radius?

(A) Center (-4, 1), radius 3

(B) Center (-8, 2), radius 3

(C) Center (-4, 1), radius 9

(D) Center (4, -1), radius 3

DOMAIN 5

Student-Produced Response

The points you lose without missing the math

"Take care of the sense, and the sounds will take care of themselves."
— Lewis Carroll

Cross-cutting test mechanics. About one question in four is a grid-in (no choices): the answer-entry rules that quietly cost points.

SPR answer entry: format & symbols

The skill. Student-produced response answers must be entered in an accepted form regardless of correct math: up to 5 characters (6 with a negative sign); no symbols ($, comma, %); mixed numbers as an improper fraction or decimal (3 1/2 -> 7/2 or 3.5, never the literal '3 1/2', which is read as 31/2); a fraction that does not fit as its decimal; a decimal that does not fit truncated or rounded at the FOURTH digit (2/3 -> .6666 or .6667, with or without a leading zero, not .66); a percent as its decimal equivalent (45% -> .45). A correct value entered in the wrong form earns no credit.

3	·	7	5

The value 15/4 entered as the decimal 3.75 — one character per cell, no symbols, never the mixed form "3 3/4".

Worked example

A ribbon 15 feet long is cut into 4 equal pieces. What is the length, in feet, of each piece?

Read. Given a 15-ft ribbon split into 4 equal parts; asked for one piece's length in feet.

Plan. Divide the total length by the number of pieces, then choose an accepted grid-in form.

Solve. Each piece is $15 \div 4 = \frac{15}{4} = 3.75$ feet.

Check. $4 \times 3.75 = 15$, correct. The value is $3\frac{3}{4}$, but a mixed number is illegal in the grid; both

15/4 and 3.75 fit within 5 characters and are accepted.

Answer: $\frac{15}{4}$ (valid grid entry: 15/4 or 3.75, never 3 3/4).

Approach Read · Plan · Solve · Check

Read. After computing, decide the form: integer, fraction, or decimal, and whether it fits the entry field.

Plan. Convert mixed numbers to improper fractions or decimals; drop all symbols; if a fraction does not fit, use its decimal; if a decimal does not fit, keep four digits by truncating or rounding; convert a percent to its decimal.

Solve. Write the entry in accepted characters only and check the length limit.

Check. No symbols? Mixed number converted? Four digits of a non-fitting decimal? Percent expressed as a decimal? Length within 5 (or 6 with a sign)?

Common traps — every wrong answer is one of these

mixed-number entry error.
Enters a mixed number literally (e.g. '3 1/2' or '31/2') instead of the improper fraction 7/2 or the decimal 3.5; the field reads '3 1/2' as the fraction 31/2.

symbol inclusion error.
Includes a disallowed symbol such as a dollar sign, comma, or percent sign (e.g. $2.53, 68,132).

decimal precision error.
When a value does not fit and is entered as a decimal, supplies too few digits instead of truncating or rounding at the fourth digit (e.g. .66 or .67 for 2/3 rather than .6666 / .6667).

percent-not-converted error.
Enters a percent answer as the percent itself rather than its decimal equivalent (45% or 45 instead of .45), or shifts the place value wrongly (4.5).

Practice

1. *You solve a problem and get the value 3 and one half.*

Which is an accepted way to enter it?

(A) 31/2
(B) 3-1/2
(C) 3 1/2
(D) 7/2

2. *A problem's answer is the number 68,132.*

Which is an accepted way to enter it?

(A) 68132
(B) 68.132
(C) 68,132
(D) 68 132

3. *Your answer is 2/3 and you choose to enter it as a decimal.*

Which is an accepted entry?

(A) .66
(B) .67
(C) .6667
(D) 0.7

4. *A problem's answer is 45%.*

Which is an accepted way to enter it?

(A) 45%
(B) 45
(C) .45
(D) 4.5

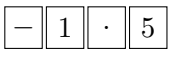
5. *Your answer is 7/4.*

Which is an accepted entry?

(A) 1 3/4
(B) 7/4
(C) 1.7
(D) 13/4

Entering negatives & decimals

The skill. Student-produced responses accept a leading minus sign and a decimal point. Enter a negative decimal as -a.b — keep the minus sign, use a decimal point (not a space), and include the decimal part.



The value −1.5 entered with a leading minus and a decimal point — the sign is kept and a point, not a space, separates the whole and fractional parts.

Worked example

Solve for x: $4x + 9 = 3$. Enter the value of x.

Read. Given a linear equation; asked for x, which will turn out negative.

Plan. Isolate x, then write the entry keeping the sign and a decimal point.

Solve. $4x = 3 - 9 = -6$, so $x = -\frac{6}{4} = -1.5$.

Check. $4(-1.5) + 9 = -6 + 9 = 3$, correct. Entry uses a leading minus and a decimal point, not a space; -1.5 (or $-3/2$) fits the field. Never write "-1 1/2".

Answer: -1.5 (valid grid entry: -1.5 or -3/2).

Approach Read · Plan · Solve · Check

Read. Is the value negative and/or a decimal?

Plan. Keep the minus; use a decimal point.

Solve. -a.b.

Check. Minus sign kept? Decimal point (not space)? Full value?

Common traps — every wrong answer is one of these

dropped the sign.
Omits the negative sign.

space not decimal.
Uses a space instead of a decimal point.

dropped the decimal part.
Rounds off the fractional part.

Practice

1. *Your grid-in answer is the value negative 5 point 2 (that is, -5.2). Which is an accepted SPR entry?*

   Which is correct?

   (A) -5 2

   (B) -5

   (C) -5.2

   (D) 5.2

2. *Your grid-in answer is the value negative 7 point 2 (that is, -7.2). Which is an accepted SPR entry?*

Which is correct?

(A) -7 2

(B) -7.2

(C) -7

(D) 7.2

3. *Your grid-in answer is the value negative 5 point 8 (that is, -5.8). Which is an accepted SPR entry?*

Which is correct?

(A) 5.8

(B) -5 8

(C) -5.8

(D) -5

4. *Your grid-in answer is the value negative 7 point 8 (that is, -7.8). Which is an accepted SPR entry?*

Which is correct?

(A) -7.8

(B) -7

(C) 7.8

(D) -7 8

5. *Your grid-in answer is -3.75. Which is an accepted SPR entry?*

Which is correct?

(A) -3 75

(B) 3.75

(C) -3.75

(D) -3

Fractions vs decimals: rounding & truncation

The skill. For a repeating decimal like n/3, enter the exact fraction (always accepted) or a decimal that FILLS the grid. Truncating to a couple of places (0.33) under-fills the grid and is not accepted; ratio colons and inverted fractions are wrong.

1	·	3	3	3

The repeating value $4/3 = 1.333\ldots$ entered as 1.333 — the decimal fills every cell rather than stopping short at the under-filled 1.33.

Worked example

A 4-foot board is shared equally among 3 people. How many feet of board does each person receive?

Read. Given 4 feet split evenly among 3 people; asked for feet per person.

Plan. Divide 4 by 3; recognize the repeating decimal and enter the exact fraction or a grid-filling decimal.

Solve. Each share is $\frac{4}{3} = 1.3333\ldots$ feet.

Check. $3 \times \frac{4}{3} = 4$, correct. The bar goes over 4/3 (not 3/4, not 4:3). As a decimal, fill the grid: 1.333, not the under-filled 1.33.

Answer: $\frac{4}{3}$ (valid grid entry: 4/3 or 1.333).

Approach Read · Plan · Solve · Check

Read. Is the decimal repeating?

Plan. Enter the fraction, or fill the grid.

Solve. n/3.

Check. Fraction bar (not colon)? Right way up? Not truncated?

Common traps — every wrong answer is one of these

truncated too short.
Rounds to two places, under-filling the grid.

ratio notation.
Uses a colon instead of a fraction bar.

inverted.
Flips the fraction.

Practice

1. *Your answer is 2/3 (a repeating decimal). Which is an accepted SPR entry?*

   Which is correct?

   (A) 2:3
   (B) 3/2
   (C) 0.66
   (D) 2/3

2. *Your answer is 1/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 1/3
(B) 3/1
(C) 0.33
(D) 1:3

3. *Your answer is 1/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 1:3
(B) 1/3
(C) 3/1
(D) 0.33

4. *Your answer is 5/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 5:3
(B) 5/3
(C) 3/5
(D) 1.66

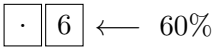
5. *Your answer is 2/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 3/2
(B) 2:3
(C) 0.6
(D) 2/3

Converting percents & mixed numbers

The skill. A percent answer must be entered as a decimal or fraction, not with a % sign. p% = p/100 = 0.p. Drop the percent sign and divide by 100 (not 10).



The percent 60% *entered as the decimal* .6 *— the* % *sign is dropped and the number is divided by* 100*, not* 10*.*

Worked example

On a quiz a student answers 9 of 15 questions correctly. Enter the fraction correct as a grid-in value.

Read. Given 9 correct out of 15; asked for the proportion, entered without a % sign.

Plan. Form the ratio, simplify, and convert the percent to its decimal by dividing by 100.

Solve. $\frac{9}{15} = \frac{3}{5} = 60\% = \frac{60}{100} = 0.6$.

Check. $0.6 \times 15 = 9$, correct. The % sign is dropped and the division is by 100 (not 10, which would give 6); both 0.6 and 3/5 are accepted entries.

Answer: 0.6 (valid grid entry: .6 or 3/5, not 60 or 60%).

Approach Read · Plan · Solve · Check

Read. Is the answer a percent?

Plan. Convert: p% = p/100.

Solve. 0.p.

Check. Removed the % sign? Divided by 100 (not 10)?

Common traps — every wrong answer is one of these

no conversion.
Enters the number without converting.

kept the % sign.
Leaves the percent sign in.

divided by 10.
Uses /10 instead of /100.

Practice

1. *A problem's answer is 25% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 0.25
(B) 25
(C) 25%
(D) 25/10

2. *A problem's answer is 50% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 50/10
(B) 50%
(C) 0.50
(D) 50

3. *A problem's answer is 75% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 75
(B) 0.75
(C) 75%
(D) 75/10

4. *A problem's answer is 20% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 20%
(B) 0.20
(C) 20
(D) 20/10

5. *A problem's answer is 40% (as a proportion). Which is an accepted SPR entry?*

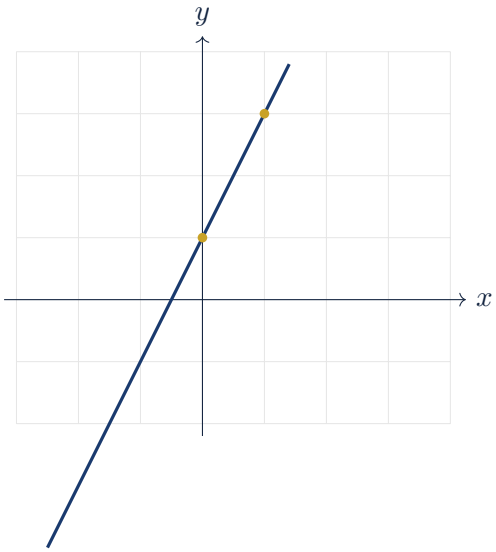
Which is correct?

(A) 40%
(B) 4
(C) 0.4
(D) 40

Visual Diagnostic Set

These questions require you to read the figure before calculating. For each miss, name the visual trap: *scale, intercept, vertex, slope, area,* or *trend.*

1. Read a line, not just an equation



The line shown passes through $(0, 1)$ and $(1, 3)$. Which equation represents it?

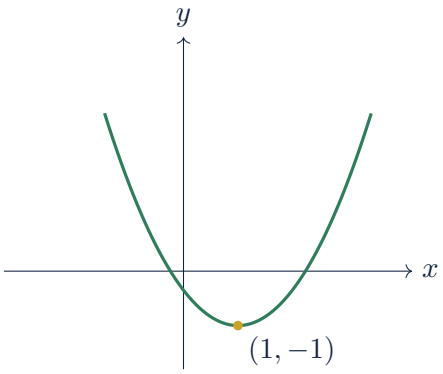
(A) $y = \frac{1}{2}x + 1$

(B) $y = 2x + 1$

(C) $y = 2x - 1$

(D) $y = x + 2$

2. Read the vertex and direction



Which statement must be true?

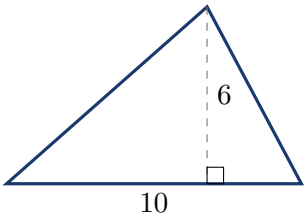
(A) The maximum value is -1.

(B) The axis of symmetry is $x = -1$.

(C) The minimum value is -1.

(D) The y-intercept is -1.

3. Use the labeled height



What is the area of the triangle?

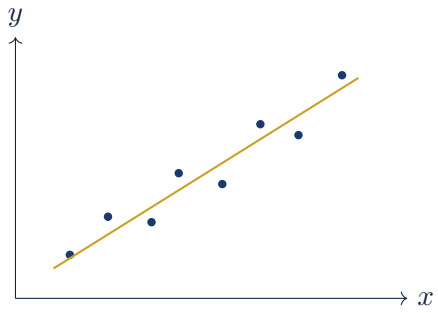
(A) 16

(B) 30

(C) 60

(D) 80

4. Read trend, not individual noise



Which description best matches the association?

(A) Strong negative linear association

(B) Positive linear association

(C) No association

(D) Positive exponential association

Worked diagnoses

Answers and named traps. 1: **B** (slope is rise/run, not run/rise). 2: **C** (the vertex is a minimum because the parabola opens upward). 3: **B** (use $\frac{1}{2}bh$; the slanted side is not the height). 4: **B** (describe the overall direction, not each point-to-point move).

Formula Reference

The PSAT/NMSQT provides the reference below on *every* math question. Your job is to **select** the right formula, not to recall it — so practice recognizing which one a problem calls for. Two things are **not** provided and must be memorized: **surface-area** formulas and **trigonometry** (SOH-CAH-TOA).

Area & circumference	**Volume**
Circle: $A = \pi r^2,\ C = 2\pi r$	Box: $V = \ell w h$
Rectangle: $A = \ell w$	Cylinder: $V = \pi r^2 h$
Triangle: $A = \frac{1}{2}bh$	Sphere: $V = \frac{4}{3}\pi r^3$
Pythagorean: $a^2 + b^2 = c^2$	Cone: $V = \frac{1}{3}\pi r^2 h$
	Pyramid: $V = \frac{1}{3}\ell w h$
Special right triangles	**Angle measure**
30–60–90: sides $x,\ x\sqrt{3},\ 2x$	Circle: $360^\circ = 2\pi$ radians
45–45–90: sides $s,\ s,\ s\sqrt{2}$	Triangle: angles sum to 180°

Memorize (not on the sheet): surface area (e.g. box $= 2(\ell w + \ell h + wh)$, cylinder $= 2\pi r^2 + 2\pi r h$, sphere $= 4\pi r^2$); and right-triangle trig: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$.

Part III

Reading and Writing

DOMAIN 6

Information and Ideas

Read for what the text claims—and what supports it

> *"A careful reader is not a faster reader; a careful reader is one who can say where an idea lives on the page."*
>
> — Quantum Tutoring

Roughly 26% of the Reading and Writing section. Central ideas and details, command of textual evidence, command of quantitative evidence, and inferences.

Central ideas and main purpose

The skill. Every passage is built around one controlling idea, and every sentence in it either states that idea, develops it, or qualifies it. Your job is to name that idea in your own words before you look at the choices, then find the choice that matches your summary in *scope*—not too broad, not too narrow. "Main purpose" questions add one twist: they ask what the author is *doing* (describing, contrasting, correcting a misconception), not just what the text is *about*.

Worked example

For most of the nineteenth century, doctors treated the common cold with bleeding and harsh purges, sure that illness had to be driven out of the body by force. Only later did physicians accept that a cold runs its course no matter what they do, and that rest and fluids serve a patient better than violent remedies. The story of the cold, in a sense, is the story of doctors learning when to do less.

What is the main purpose of the text?

Read. The last sentence names the point directly—"learning when to do less"—and the middle sentence marks a shift from force to rest. It asks for the author's purpose, not just the topic.

Plan. Start with a verb: to trace how the medical approach to colds changed toward gentler, less aggressive treatment.

Solve. The choice that captures the whole passage describes a shift in thinking about treatment—from forcing illness out to letting the cold pass.

Check. "To describe a bleeding procedure" is Too Narrow (one detail). "To prove modern medicine cures every illness" is Too Broad and unsupported. "To argue doctors of the past were foolish" is the Wrong Action—the author explains a change, it does not scold.

Answer: To trace how doctors' approach to treating the cold shifted toward doing less.

Approach Read · Plan · Solve · Check

Read. The first and last sentences carry the most weight; note any pivot word ("but," "however," "yet") that signals where the real point lands.

Plan. Say the central idea aloud in one plain sentence before reading the choices. If it is a purpose question, start your sentence with a verb.

Solve. Match your summary to the choice that covers the *whole* passage, not just one line.

Check. Cross out any choice that is true but minor, or that stretches beyond what the text supports.

Common traps — every wrong answer is one of these

Too Narrow.
A true statement about one detail or one sentence, mistaken for the main point of the whole passage.

Too Broad.
A grand generalization the passage never fully commits to; it sounds important but covers more ground than the text.

Half-Right.
The first part matches the passage, but the second half adds a claim the text never makes.

Wrong Action.
For purpose questions: the topic is right, but the verb is wrong (says "argues" when the author only "describes").

Practice

1. *For decades, biologists assumed that a tree in a forest competes with its neighbors for light and water, each one a solitary contestant. Recent studies of underground fungal networks, however, show that trees routinely trade sugars and warning signals through their roots. The forest, it turns out, behaves less like a crowd of rivals than like a single cooperating community.*

   What is the main idea of the text?

(A) Trees compete fiercely for sunlight in every forest.

(B) New evidence about root networks suggests forests are more cooperative than biologists once believed.

(C) Fungi that connect tree roots can carry sugars from one tree to another.

(D) All living systems in nature ultimately depend on cooperation rather than competition.

2. *The novelist described her hometown in loving, exact detail: the bakery's cracked front step, the river that turned brown after storms, the whistle of the six o'clock train. She rarely explained why these things mattered. She simply set them down, one after another, and trusted the reader to feel their weight.*

What is the main purpose of the text?

(A) To praise the novelist as the greatest writer of her generation.

(B) To characterize a writer's method of conveying meaning through concrete detail rather than explanation.

(C) To argue that the best novels are always set in small towns.

(D) To describe the specific sound of a six o'clock train.

3. *Public libraries were once quiet buildings devoted almost entirely to lending books. Today many of them lend tools, host job-training classes, and offer free internet to people who have none at home. The mission has not been abandoned so much as widened: the modern library still exists to give people access to what they cannot afford alone.*

What is the central idea of the text?

(A) Libraries have expanded their services while keeping their core purpose of providing shared access.

(B) Some libraries now lend tools in addition to books.

(C) Libraries have completely abandoned their original focus on books.

(D) Every public institution must modernize or risk becoming useless.

4. *Early maps of the ocean floor showed a smooth, featureless plain. When sonar surveys arrived in the twentieth century, they revealed towering ridges, deep trenches, and vast mountain chains hidden beneath the water. The old maps had not been wrong out of carelessness; mapmakers had simply drawn what their instruments could reach.*

What is the main purpose of the text?

(A) To claim that sonar is the most important scientific tool ever invented.

(B) To explain that improved instruments revealed an ocean floor far more rugged than earlier maps suggested.

(C) To criticize early mapmakers for their careless and inaccurate work.

(D) To describe the specific depth of ocean trenches measured by sonar.

5. *A city council debated whether to replace an aging bus line with a new one. Supporters pointed to faster travel times; opponents worried about riders on the old route who might be stranded. In the end the council did neither immediately, choosing instead to run both lines for a year and study which one people actually used.*

What is the main idea of the text?

(A) The new bus line offered faster travel times than the old one.

(B) Every transit decision should be made by studying rider behavior for exactly one year.

(C) The council refused to make any decision at all about the bus lines.

(D) Facing competing concerns, the council chose to gather evidence before making a permanent decision.

Supporting details (locating a specific detail)

The skill. Detail questions ask what the text actually says—not what is generally true, and not what you already know about the topic. The correct answer is almost always a paraphrase of one specific sentence. Your task is to locate that sentence, confirm it answers the exact question asked, and resist any choice that merely sounds plausible.

Worked example

When the river flooded in 1931, the town lost its wooden bridge, two riverside warehouses, and the small record office at the end of Main Street. The bridge was rebuilt within a year, this time in steel. The warehouses were never replaced, and the destroyed records left gaps in the town's history that researchers still puzzle over today.

According to the text, which structure was rebuilt after the 1931 flood?

Read. The question asks specifically what was rebuilt—not what was lost, and not what was left unreplaced.

Plan. The answer sits in the second sentence, which singles out one structure and says it was rebuilt in steel within a year.

Solve. Only the bridge is described as rebuilt; the text says so word for word.

Check. "The warehouses" is a Detail Swap—real words, wrong structure, since they were never replaced. "The record office" is Close but Off (lost, not rebuilt). "Everything the flood destroyed" is Extreme Wording the text contradicts.

Answer: The bridge, which was rebuilt in steel.

Approach Read · Plan · Solve · Check

Read. Pin down exactly what the question is asking for: a reason, a result, a description, a date.

Plan. Predict roughly where in the passage the answer sits, then reread that spot word for word.

Solve. Pick the choice that restates what the text literally says about that point.

Check. Point to the exact words in the passage that prove your choice; if you cannot, it is probably wrong.

Common traps — every wrong answer is one of these

Outside Knowledge.
A statement that is true in the real world but is never actually stated in the passage.

Detail Swap.
Uses real words from the text but attaches them to the wrong subject, cause, or time.

Extreme Wording.
Takes a modest claim in the text and inflates it with "always," "never," "all," or "only."

Close but Off.
Answers a slightly different question than the one asked—right topic, wrong point.

Practice

1. *The Arctic tern makes the longest migration of any known animal, flying from the Arctic to the Antarctic and back each year. It does not fly in a straight line; instead it follows a winding path that traces the winds, adding thousands of miles to the trip. This zigzag route, though longer, lets the bird glide on favorable currents and save energy.*

According to the text, why does the Arctic tern follow a winding rather than a straight route?

(A) Because terns cannot navigate in a straight line.
(B) To avoid predators along the coastlines.
(C) To take advantage of favorable winds and conserve energy.
(D) Because it is the fastest animal migration on record.

2. *Marie planted her first garden in a narrow strip of soil behind the apartment. The tomatoes failed and the beans were eaten by rabbits, but the sunflowers thrived, growing taller than the fence by August. She kept only the sunflowers the next year, and the year after that she added a second row.*

According to the text, which plant succeeded in Marie's first garden?

(A) The tomatoes.
(B) The beans.
(C) All of the plants she grew.

(D) The sunflowers.

3. *The invention of the mechanical clock changed how towns organized their days. Before clocks, work often began at sunrise and ended at dusk, so the length of a workday shifted with the seasons. Once public clocks appeared in town squares, hours became fixed and equal, and a "day's work" could be measured the same way in winter as in summer.*

According to the text, how did work hours change after public clocks appeared?

(A) They grew longer in every town.

(B) They began earlier, at sunrise, for the first time.

(C) They were abolished in favor of seasonal schedules.

(D) They became fixed and equal regardless of season.

4. *Honeybees communicate the location of food through a movement scientists call the waggle dance. The direction of the dance points toward the food relative to the sun, and the length of the waggling run signals how far away it is. Longer waggles mean a longer flight; shorter ones mean the food is close.*

According to the text, what does the length of the waggling run indicate?

(A) The distance to the food source.

(B) The direction of the food relative to the sun.

(C) The number of bees available to gather food.

(D) The quality of the food that was found.

5. *The old lighthouse on the point was automated in 1969, and its last keeper moved ashore. For years the building stood empty, its windows salted over. Then a local historical society raised money to restore it, and today volunteers open it to visitors on summer weekends.*

According to the text, who currently opens the lighthouse to visitors?

(A) The last lighthouse keeper.

(B) A government automation crew.

(C) Paid employees hired in 1969.

(D) Volunteers from a historical society.

Command of textual evidence

The skill. These questions give you a claim, a hypothesis, or a conclusion, and ask which quotation from the passage would most directly support it. The right answer is not the most interesting or most quoted line; it is the one that logically *backs up the specific claim*. Restate the claim in plain terms, then test each quotation against it: does this sentence actually make that claim more believable?

Worked example

A paleontologist claimed that a newly discovered dinosaur had been a fast runner. She recorded several observations. The animal's leg bones were unusually long and slender—a build linked in living animals to running speed. Its fossil lay beside the mud of an ancient lakebed. Its teeth were small and sharply pointed.

Which observation, if true, most directly supports the claim that the dinosaur was a fast runner?

Read. The exact claim is about *speed*—how fast the animal could run—not what it ate or where it died.

Plan. Turn it into a question: "Which observation makes fast running more believable?" The answer must connect to movement.

Solve. Long, slender leg bones tied to speed in living animals directly back the claim about running.

Check. The lakebed location is a Neutral Fact about where it was found. The small pointed teeth are Off-Claim—evidence about diet, not speed. Neither strengthens the running claim.

Answer: The observation that its leg bones were long and slender, a build linked to speed.

Approach Read · Plan · Solve · Check

Read. Nail down the exact claim you must support—who or what, and in which direction.

Plan. Turn the claim into a question: "Which quote proves *this*?" Ignore quotes that are merely true or vivid.

Solve. Choose the quotation whose meaning directly matches and strengthens the claim.

Check. Reject quotes that support a different claim, contradict the claim, or are just background.

Common traps — every wrong answer is one of these

Off-Claim.
A real quotation that supports some other point in the passage, not the specific claim asked about.

Backwards.
A quotation that actually weakens or contradicts the claim it is supposed to support.

Neutral Fact.
A true, on-topic quotation that neither supports nor undercuts the claim—it just sits there.

Overreach.
A quotation that would support a much stronger claim than the one you were given, so it doesn't match.

Practice

1. *A researcher studying urban crows claimed that the birds can recognize and remember individual human faces. Over several months she recorded the following observations: crows that had been captured once by a person wearing a particular mask later scolded anyone wearing that same mask, even people who had never handled them. Crows ignored the same individuals when they wore different masks. The birds also passed their reaction to new crows that had never been captured at all.*

Which finding, if true, most directly supports the researcher's claim that crows recognize individual faces?

(A) Crows passed their reaction to new crows that had never been captured.

(B) The birds were studied over several months in an urban setting.

(C) Crows ignored people who wore a different mask than the capture mask.

(D) Crows scolded anyone wearing the specific mask worn during their capture, even strangers.

2. *An economist argued that raising a city's minimum wage did not cause the job losses many had predicted. She examined restaurant employment before and after the increase. Employment stayed nearly flat in the year following the raise. A neighboring city that did not raise its wage saw employment fall slightly over the same period. Prices at the restaurants rose by a few cents per item.*

Which finding most directly supports the economist's argument that the wage increase did not cause job losses?

(A) Prices at the restaurants rose by a few cents per item.

(B) Many people had predicted that the increase would cause job losses.

(C) Employment in the neighboring city fell slightly during the same period.

(D) Restaurant employment stayed nearly flat in the year after the wage increase.

3. *A literary critic proposed that the poet's later work grew more hopeful than her early work. In her twenties the poet wrote almost entirely about loss and endings. Critics at the time praised her "unflinching gloom." In her fifties she published a collection whose poems close, again and again, on images of dawn, sprouting seeds, and reunion.*

Which detail most directly supports the critic's claim that the poet's later work grew more hopeful?

(A) Critics praised her early work for its "unflinching gloom."

(B) She continued to publish poetry into her fifties.

(C) Her later poems repeatedly close on images of dawn, seeds, and reunion.

(D) In her twenties she wrote almost entirely about loss and endings.

4. *A geologist hypothesized that a certain desert valley had once held a large lake. She noted several observations. Bands of fine, layered clay—the kind that settles only in still, deep water—ran along the valley walls. Fossilized shells of freshwater snails were scattered across the valley floor. The surrounding hills were made of hard volcanic rock.*

Which observation most directly supports the hypothesis that the valley once held a lake?

(A) The valley is located in a desert region today.

(B) The clay bands ran along the upper valley walls.

(C) Fossilized freshwater snail shells were scattered across the valley floor.

(D) The surrounding hills were made of hard volcanic rock.

5. *A teacher believed that students learned vocabulary better by using new words in sentences than by copying definitions. She split her class into two groups for two weeks. One group wrote original sentences with each new word; the other copied each word's dictionary definition three times. On a test given a month later, the sentence-writing group remembered noticeably more words. Both groups spent the same amount of time studying.*

Which result most directly supports the teacher's belief?

(A) The definition-copying group wrote each word's meaning three times.

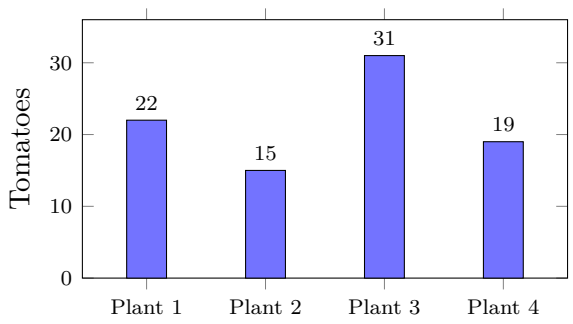
(B) A month later, the sentence-writing group remembered noticeably more words.

(C) Both groups spent the same amount of time studying.

(D) The class was divided into two groups for the experiment.

Command of quantitative evidence

The skill. Here a short passage is paired with data—a table or graph. Your job is to read the data accurately and pick the choice the numbers actually support, or the choice that correctly completes the passage using the data. The trap is that most wrong answers misread the figure: they flip a comparison, cite a number that isn't there, or draw a conclusion the data can't reach. Read the units and labels first; the numbers mean nothing without them. *(Practice items describe their table or graph in words—treat the description as the figure; the worked example below shows one drawn out.)*



The gardener's tally: read the exact counts and compare direction before choosing—Plant 3 is highest at 31.

Worked example

A gardener compared how many tomatoes four plants produced over one summer. Her tally listed Plant 1 with 22 tomatoes, Plant 2 with 15, Plant 3 with 31, and Plant 4 with 19. She concluded that the plant in the sunniest corner of the yard—Plant 3—had produced the most.

Which choice best uses the data to support the gardener's conclusion?

Read. The figure counts tomatoes per plant. The conclusion is that Plant 3, in the sunniest spot, produced the most.

Plan. To fit the claim, the data must show Plant 3's count above every other plant's count.

Solve. Plant 3's 31 tomatoes is the highest of the four totals, so the choice citing 31 as the largest supports the conclusion.

Check. "Plant 2 produced 40" is a Number Not There. "Plant 1 produced more than Plant 3" is a Flipped Comparison (22 is less than 31). "The extra sun caused the higher yield" is an Unsupported Leap—the tally shows counts, not cause.

Answer: Plant 3 produced the most tomatoes (31), more than any other plant.

Approach Read · Plan · Solve · Check

Read. Read the caption, axis labels, and units before any number; know what each figure measures.

Plan. Restate the claim the passage is building, then decide what the data would need to show to fit it.

Solve. Find the exact values the question needs and compare them in the right direction.

Check. Confirm your choice matches both the data and the passage's point; reject anything the numbers don't state.

Common traps — every wrong answer is one of these

Number Not There.
Cites a figure or category that does not appear in the described data at all.

Flipped Comparison.
Reverses the direction—says one value is greater when the data show it is smaller.

Right Data, Wrong Point.
Reads the numbers correctly but draws a conclusion the passage was not making.

Unsupported Leap.
Goes beyond the data, adding a cause, a prediction, or a trend the figure never shows.

Practice

1. *A student measured how quickly water evaporated from open dishes left in different rooms of her house over one day. Her table listed four rooms and the milliliters of water lost from each: the sunroom lost 40 mL, the kitchen lost 25 mL, the bedroom lost 18 mL, and the basement lost 9 mL. She concluded that warmer, sunnier rooms lost more water.*

Which choice best uses the data to support the student's conclusion?

(A) The garage lost 30 mL of water during the experiment.

(B) The sunroom, the warmest room, lost the most water (40 mL), while the cool basement lost the least (9 mL).

(C) The kitchen lost more water than the sunroom.

(D) Every room lost exactly the same amount of water over the day.

2. *A town tracked the number of bicycles counted crossing its main bridge each morning before and after it added a protected bike lane. Before the lane, an average of 120 bicycles crossed each morning. In the first month after, the count rose to 210; by the sixth month it reached 340. Pedestrian counts stayed roughly the same throughout.*

Which choice most accurately reflects the data?

(A) Pedestrian crossings more than doubled after the lane opened.

(B) Exactly 500 bicycles crossed the bridge in the sixth month.

(C) Bicycle crossings rose from 120 before the lane to 340 by the sixth month after.

(D) Bicycle crossings fell after the protected lane was added.

3. *A biologist recorded the average wingspan of four butterfly species in a preserve. Species A averaged 5 cm, Species B averaged 7 cm, Species C averaged 9 cm, and Species D averaged 6 cm. She noted that Species C had the widest wings of the four.*

Which choice is best supported by the data?

(A) Species A had the widest wingspan of the four species.

(B) Species D averaged 12 cm, the largest in the study.

(C) Species C's wide wings allow it to fly faster than the others.

(D) Species C, at 9 cm, had a wider average wingspan than Species B, at 7 cm.

4. *A cafeteria surveyed students about their preferred lunch drink. Of 200 students, 90 chose water, 60 chose milk, 30 chose juice, and 20 chose a sports drink. The manager wanted to stock the most popular option most heavily.*

Which choice is best supported by the survey data?

(A) A majority of students—over 100—chose milk.

(B) Sports drinks were the second most popular choice.

(C) Water was the most popular drink, chosen by 90 of the 200 students.

(D) Juice was chosen by more students than water.

5. *Researchers compared how long two battery brands powered the same flashlight. In three trials, Brand X lasted 8, 9, and 7 hours, while Brand Y lasted 5, 6, and 4 hours. The team wanted to recommend the longer-lasting brand for emergency kits.*

Which choice best completes the recommendation using the data?

(A) Brand X lasted 12 hours in its best trial, the longest of the study.

(B) Brand X should be recommended, since it lasted longer than Brand Y in every trial.

(C) Brand Y should be recommended, since it lasted longer in every trial.

(D) The two brands lasted exactly the same amount of time in each trial.

Inferences

The skill. An inference is a conclusion the text does not state outright but strongly implies. The correct answer follows *necessarily* from the passage—if the text is true, the answer must also be true. You are not guessing or adding your own opinions; you are finishing the thought the author has led you to. Choose the choice that stays closest to what the text guarantees, not the boldest or most surprising claim.

Worked example

The library charges a late fee for every book that is returned after its due date. This month Marcus borrowed six books, and he returned all six. When he checked his account, he found that he owed no late fees at all.

Which choice most logically completes the text?

Read. Two facts are fixed: a late fee applies to any book returned late, and Marcus owed no fees on his six returned books.

Plan. Ask what *must* follow. If every late book brings a fee and Marcus owed none, then none of his books can have been late.

Solve. It must be true that Marcus returned all six books on or before their due dates.

Check. "Marcus will never owe a fee again" is Overstated. "The library dropped its late-fee policy" Needs More—the passage says the policy still applies. "Marcus returned his books late" is a Contradiction of the no-fee fact.

Answer: Marcus returned all six books on or before their due dates.

Approach Read · Plan · Solve · Check

Read. Track the logical relationships—especially contrasts, causes, and conditions the author sets up.

Plan. Ask what *must* be true given these sentences, then predict the missing conclusion before reading choices.

Solve. Select the choice that the passage forces you to accept, using only its own information.

Check. Reject any choice that needs facts the passage never gave you, or that overstates its certainty.

Common traps — every wrong answer is one of these

Overstated.
A conclusion in the right direction but pushed too far—more certain or extreme than the text allows.

Needs More.
A plausible idea that requires an extra fact the passage never provides.

Reversed Logic.
Flips a cause and effect or a condition, drawing the opposite of what follows.

Contradiction.
A conclusion that actually clashes with something the passage states.

Practice

1. *The desert plant stores water in its thick stem and opens the tiny pores in its skin only at night. During the day, when the air is hot and dry, those pores stay sealed. This timing means the plant loses far less water than plants that open their pores under the midday sun.*

   Which choice most logically completes the text?

   (A) Daytime air in the desert is usually cool and humid.

   (B) Plants that open their pores during the day tend to lose more water than this desert plant does.

   (C) This desert plant never loses any water at all.

   (D) The desert plant opens its pores during the hottest part of the day.

2. *Every member of the hiking club who finished the ridge trail last summer had trained for at least three months beforehand. Sam is a member of the club, but he joined only two weeks ago and has not yet trained.*

   Which choice most logically completes the text?

   (A) Sam did not finish the ridge trail last summer.

   (B) Sam will never be able to finish the ridge trail.

   (C) Sam trained for more than three months before joining.

(D) Only club members are allowed on the ridge trail.

3. *The museum's new exhibit can hold no more than fifty visitors at a time for safety reasons. On opening day, more than four hundred people arrived within the first hour, and each wanted to see the exhibit.*

Which choice most logically completes the text?

(A) All four hundred visitors saw the exhibit at the same time.

(B) The exhibit was closed for the entire opening day.

(C) The safety limit was raised to four hundred visitors that morning.

(D) Many of the visitors had to wait before they could enter the exhibit.

4. *Researchers found that a particular songbird sings more loudly in cities than in quiet forests. In the study, the louder a bird's usual surroundings were, the louder that bird sang. The birds seemed to adjust their volume to be heard over the noise around them.*

Which choice most logically completes the text?

(A) Birds in quiet forests sing more loudly than birds in cities.

(B) The noise of a city permanently damages the birds' hearing.

(C) A songbird living near a noisy highway would likely sing more loudly than one in a silent meadow.

(D) Songbirds are unable to change the volume of their songs.

5. *The bakery sells out of its sourdough loaves by mid-morning almost every day. The owner has said she could sell twice as many if she had the oven space, but her single oven can bake only so many loaves before closing time.*

Which choice most logically completes the text?

(A) Customers do not want to buy the bakery's sourdough loaves.

(B) The bakery has several ovens running all day.

(C) The owner plans to stop selling sourdough entirely.

(D) The bakery's sourdough sales are limited more by its oven capacity than by customer demand.

DOMAIN 7

Craft and Structure

How a text is built, word by word

> *"The difference between the almost right word and the right word is really a large matter—'tis the difference between the lightning-bug and the lightning."*
>
> — Mark Twain

Roughly 28% of the Reading and Writing section. Words in context, text structure and purpose, and cross-text connections.

Words in context

The skill. A Words-in-Context item hands you a sentence with a blank (or an underlined word) and asks you to supply the most *logical* academic word. The passage always plants a clue—a definition, a contrast, or an example—that pins down exactly one meaning. Your job is to read the clue first and predict the meaning *before* you look at the choices, so the answer options cannot bend your thinking.

Worked example

Early maps of the coast were far from reliable; because no surveyor had measured the inlets directly, cartographers filled the gaps with _____ shapes, best guesses drawn from sailors' rough reports.

Which choice completes the text with the most logical and precise word?

Read. The whole sentence, not just the blank: the maps were "far from reliable," and the shapes were "best guesses" from "rough reports."

Plan. The clue is the appositive "best guesses"—it defines the blank. Predict a plain fill: *uncertain* or *guessed-at*.

Solve. *Conjectural* means based on guesswork rather than direct evidence, which matches "best guesses" exactly.

Check. *Precise* points the wrong way (Near-opposite)—the maps were unreliable. *Coastal* is Right neighborhood, wrong nuance: it fits the topic but names no guesswork. *Elaborate* Sounds smart but ignores the clue about guessing.

Answer: conjectural.

Approach Read · Plan · Solve · Check

Read. Read the whole sentence, not just the words hugging the blank.

Plan. Underline the clue—definition, contrast word, or example—and predict a plain-English fill.

Solve. Match each choice to your prediction; keep only the one that fits the clue's direction.

Check. Reread the sentence with your pick; it must be true and idiomatic, not just possible.

Common traps — every wrong answer is one of these

Near-opposite.
A word that points the wrong direction—often the answer if you missed a contrast signal like *but* or *although*.

Right neighborhood, wrong nuance.
A word in the same topic but with an intensity or shade the clue does not support.

Wrong sense.
A word whose *other* common meaning fits, but not the meaning the sentence needs.

Sounds smart.
A hard, impressive word that has nothing to do with the planted clue.

Clue-blind.
A choice that ignores the definition the sentence hands you and forces its own meaning.

Practice

1. *The committee praised the report for being ____: it stated each finding in a few plain words, with nothing padded or repeated.*

Which choice completes the text with the most logical and precise word?

(A) concise

(B) verbose

(C) tentative

(D) ornate

2. *Unlike her flashy rivals, the architect favored a ____ style, stripping each building down to clean lines and bare surfaces.*

Which choice completes the text with the most logical and precise word?

(A) modest
(B) unstable
(C) austere
(D) lavish

3. *The evidence was far from settled; every claim in the draft remained ____, offered only as a possibility to be tested later.*

Which choice completes the text with the most logical and precise word?

(A) provisional
(B) definitive
(C) cautious
(D) invisible

4. *Because the two accounts of the storm agreed on every detail, the historian treated their close ____ as strong support for the story.*

Which choice completes the text with the most logical and precise word?

(A) resemblance
(B) correspondent
(C) correspondence
(D) contradiction

5. *Rather than settling the dispute, the ruling only ____ it, adding new grievances that pushed the two towns further apart.*

Which choice completes the text with the most logical and precise word?

(A) resolved
(B) complicated
(C) aggregated
(D) aggravated

Words in context: precise connotation

The skill. Sometimes two choices are near-synonyms that would both make grammatical sense—yet only one carries the *tone* the sentence needs. Connotation items reward you for hearing the difference

between *thrifty* and *stingy*, or *curious* and *nosy*. Ask whether the sentence's attitude is positive, neutral, or negative, and how strong it is; then pick the word whose flavor matches.

Worked example

Colleagues admired how the editor asked about everyone's projects and remembered small details months later; her interest, they agreed, came from real warmth, never from wanting to pry.

Which choice best describes the editor, as her colleagues saw her?

Read. The attitude is approving—"admired," "real warmth"—and the sentence explicitly rules out prying.

Plan. Two near-synonyms will circle "interested in others." Name the shade: positive and warm, not intrusive.

Solve. *Inquisitive* carries an approving, genuinely curious tone that matches "real warmth."

Check. *Nosy* has the Wrong charge—it means prying, which the sentence denies. *Indifferent* is Too weak (really the opposite). *Obsessive* is Too strong, overshooting friendly curiosity.

Answer: inquisitive.

Approach Read · Plan · Solve · Check

Read. Note the sentence's overall attitude—approving, neutral, or critical.

Plan. Set the two near-synonyms side by side; name the shade that separates them.

Solve. Keep the word whose connotation and intensity match the sentence's tone.

Check. Swap in your pick; the emotional color should feel exactly right, not close.

Common traps — every wrong answer is one of these

Wrong charge.
A synonym with the opposite emotional sign—praise where the sentence blames, or the reverse.

Too strong.
A word that overshoots the sentence's mildness and sounds extreme.

Too weak.
A word that undershoots, draining the force the sentence clearly wants.

Denotation-only.
A choice that means roughly the right thing but ignores the tone entirely.

Register mismatch.
A word whose formality clashes with the passage's voice.

Practice

1. *Reviewers admired the novel's hero, whose refusal to abandon the search—long after others quit—struck them as noble rather than foolish.*

Which choice best describes the hero's refusal, as the reviewers saw it?

(A) stubborn
(B) pushy
(C) relentless
(D) persistent

2. *The mayor's opponents complained that his careful budgeting had crossed a line: he now refused to spend on even basic repairs the town badly needed.*

Which choice best captures the opponents' view of the mayor's habits?

(A) generous
(B) economical
(C) miserly
(D) frugal

3. *The teacher's comments were honest but kind; she pointed out each flaw gently, never letting a criticism sting.*

Which choice best describes the teacher's comments?

(A) harsh
(B) evasive
(C) candid
(D) blunt

4. *Fans loved how the young star kept her fame from changing her; she still helped clean up after shows and remembered every crew member's name.*

Which choice best describes the star, as her fans saw her?

(A) indifferent
(B) unassuming
(C) timid
(D) arrogant

5. *Investigators described the accountant's records as almost unnaturally exact—every figure checked, every receipt filed, nothing left to chance.*

Which choice best describes the accountant's records?

(A) adequate

(B) careless

(C) meticulous

(D) fussy

Text structure and purpose

The skill. Structure questions ask what a sentence *does* within a passage, or how the whole passage is organized. The right answer names a *function*—to introduce a problem, to qualify a claim, to give an example—rather than restating the content. Track the logical moves: where a claim appears, where a counter-move arrives, and where the writer lands. Transition words are your map.

Worked example

For decades, doctors told patients to rest a sprained ankle completely until the pain faded. More recent studies, though, show that gentle movement within a day or two speeds healing. Many clinics have since revised their advice.

What is the main function of the second sentence?

Read. Track the moves: sentence one gives the old rule; the transition "though" signals a turn; sentence three reports the result.

Plan. Label the target sentence's job as a verb phrase: "it presents newer findings that overturn the old rule."

Solve. The choice that names that function—introducing evidence that contradicts the long-standing advice—is correct.

Check. "It restates the first sentence" is Content, not function and misses the reversal. "It states the clinics' decision" is Wrong location (that is sentence three). "It proves rest is always harmful" is Overreach beyond a modest finding.

Answer: It introduces newer evidence that challenges the long-standing advice in the first sentence.

Approach Read · Plan · Solve · Check

Read. Read for the moves, not the topic: claim, example, objection, conclusion.

Plan. Label the target sentence's job in a verb phrase ("it qualifies. . . ").

Solve. Choose the option that names that job; reject options that only summarize content.

Check. Confirm the choice fits the sentence's place in the flow, before and after.

Common traps — every wrong answer is one of these

Content, not function.
A choice that accurately restates *what* the sentence says but not *why* it is there.

Wrong location.
A function that belongs to a different sentence in the passage.

Overreach.
A choice that inflates a modest sentence into the passage's main claim.

Reversed move.
Calling a concession a rebuttal, or an example a counterexample.

Half-right.
A choice correct about the topic but wrong about the direction of the argument.

Practice

1. *Many gardeners assume that richer soil always yields bigger harvests. In fact, some native wildflowers bloom best in thin, rocky ground, where fewer competitors crowd them out. Their roots, adapted over centuries, actually suffer in soil packed with nutrients.*

What is the main function of the second sentence?

(A) It concedes that richer soil usually produces bigger harvests.

(B) It offers evidence that challenges the assumption stated in the first sentence.

(C) It restates the claim of the first sentence in different words.

(D) It introduces the passage's central argument about root systems.

2. *The new bridge design promised to cut travel time in half. Engineers, however, warned that the lightweight cables might sway dangerously in high wind. The city council delayed the vote until further testing could be done.*

What is the main function of the second sentence?

(A) It confirms that the design will cut travel time in half.

(B) It raises a concern that complicates the promise made in the first sentence.

(C) It summarizes the benefits described in the first sentence.

(D) It states the council's final decision about the bridge.

3. *Historians once credited a single inventor with the printing press. Recent work tells a messier story: dozens of workshops across several cities refined the tools at once. No lone genius, this account suggests, deserves the whole prize.*

Which choice best describes the overall structure of the passage?

(A) It compares two cities that each claimed the invention.

(B) It presents a traditional view, then replaces it with a more complex account.

(C) It lists the steps involved in building a printing press.

(D) It argues that a single inventor deserves full credit after all.

4. *Coral reefs shelter a quarter of all ocean species. When water warms even slightly, the coral expels the algae that feed it and turns bone white. Scientists call this bleaching, and a reef may not recover from it for decades.*

What is the main function of the first sentence?

(A) It explains why warm water expels algae from coral.

(B) It predicts how long a damaged reef will take to recover.

(C) It establishes the reefs' importance before the passage describes a threat to them.

(D) It defines the term "bleaching" used later in the passage.

5. *At first, the volunteers sorted the archive by hand, a slow and error-prone task. Then a student wrote a short program to scan the labels automatically. Within a week, the backlog that had taken months to build was gone.*

Which choice best describes the overall structure of the passage?

(A) It traces a shift from a laborious method to a faster solution and its result.

(B) It weighs the costs and benefits of writing custom software.

(C) It criticizes the volunteers for working too slowly.

(D) It lists several unrelated tasks the archive required.

Purpose of a detail

The skill. These items point to one concrete detail—a statistic, an anecdote, a quotation—and ask *why* the author put it there. A detail always serves the sentences around it: it supports a claim, illustrates an idea, undercuts an expectation, or sets up what comes next. Find the claim the detail is doing work for, then describe the job it does.

Worked example

The museum claimed its new lighting would protect the oldest paintings. Within a year, conservators measuring the canvases found that fading had slowed to a third of its former rate.

Why does the author include the conservators' measurement?

Read. The detail is the measured drop in fading; the nearest claim is that the lighting "would protect the oldest paintings."

Plan. Ask what the detail does for that claim: it proves, illustrates, contrasts, or sets up? Here it delivers hard evidence that the claim held.

Solve. The detail backs the museum's claim with a concrete measurement, so the correct choice names that supporting job.

Check. "It reports when the paintings were made" is True but idle. "It shows the lighting failed" ties to the Wrong claim—fading slowed. "The museum is the world's best" is Too broad for one measurement.

Answer: To give concrete evidence that the new lighting worked as the museum claimed.

Approach Read · Plan · Solve · Check

Read. Locate the detail and the claim nearest to it.

Plan. Ask: does this detail prove, illustrate, contrast, or set up that claim?

Solve. Pick the option that names that relationship in the author's own logic.

Check. Remove the detail in your mind; the choice should name what would be lost.

Common traps — every wrong answer is one of these

True but idle.
A statement true about the detail that names no purpose it serves.

Wrong claim.
A purpose tied to a point the detail does not actually support.

Too broad.
A choice that treats one small detail as the passage's whole point.

Off-tone.
A purpose that misreads the author's attitude toward the detail.

Literal echo.
A choice that just repeats the detail instead of explaining its job.

Practice

1. *The library's new hours have not helped the students who need it most. A recent survey found that most part-time workers finish their shifts after ten at night—an hour after the doors now close.*

   Why does the author include the survey finding about shift times?

   (A) To praise the library for extending its evening hours.

   (B) To report exactly when the library's doors close each night.

   (C) To show that the new hours fail to serve the very students mentioned.

   (D) To prove that most students at the school work part-time jobs.

2. *Critics doubted that the small town could host the festival. Yet last summer it drew forty thousand visitors, filled every inn for miles, and left the organizers scrambling to add parking.*

What is the main purpose of the details about visitors and inns?

(A) To provide evidence that the critics' doubts were misplaced.

(B) To explain why the town needed more parking in general.

(C) To argue that small towns should never host large events.

(D) To describe the region's most popular inns for travelers.

3. *Dr. Ellis wanted her readers to feel how vast the migration was. She wrote not of "many birds" but of a river of wings three miles wide, blotting the noon sun for a full hour as it passed.*

Why does the author describe "a river of wings three miles wide"?

(A) To specify the exact number of birds in the migration.

(B) To suggest that the birds blocked sunlight by accident.

(C) To criticize Dr. Ellis for exaggerating the event.

(D) To give readers a vivid sense of the migration's enormous scale.

4. *The startup insisted its batteries were built to last. When a reviewer ran one through five hundred charge cycles and found it still held ninety percent of its capacity, that boast suddenly seemed earned.*

What is the main purpose of the detail about five hundred charge cycles?

(A) To explain how a lithium battery stores its charge.

(B) To recommend that readers buy the startup's product.

(C) To lend concrete support to the company's durability claim.

(D) To reveal that the battery lost most of its capacity.

5. *Not everyone welcomed the railway. One farmer, whose family had crossed the valley by wagon for generations, refused to sell his land and kept a rusting plow at his gate as a kind of protest.*

Why does the author mention the rusting plow at the farmer's gate?

(A) To illustrate the farmer's stubborn resistance to the railway's arrival.

(B) To show that the farmer could no longer afford new equipment.

(C) To prove that wagons were faster than trains in the valley.

(D) To explain how the family had crossed the valley for generations.

Cross-text connections

The skill. Here you get two short passages on one topic and must reason about how they relate—usually how the author of one would *respond* to a claim in the other. First nail down each

author's position and how confident each is. Then predict the response: agreement, disagreement, a qualification, or a fresh angle. Answer from what each author actually argues, not from your own view of the topic.

Worked example

Passage 1: A conservationist argues that reintroducing wolves to the park will restore its balance, since the wolves will thin the overgrown deer herds that strip young trees bare.

Passage 2: A wildlife biologist agrees the deer must be controlled but notes that in similar parks the returning wolves preyed mostly on elk, leaving the deer population—and the damaged trees—largely untouched.

How would the author of Passage 2 most likely respond to the claim in Passage 1?

Read. Passage 1: wolves will thin the deer and save the trees. Passage 2: wolves elsewhere hunted elk instead, so the deer stayed.

Plan. Mark overlap and divergence: both want the deer controlled (overlap), but they diverge on whether wolves will actually do it.

Solve. Author 2 would grant the goal yet doubt the mechanism, citing cases where wolves ignored deer—a qualified challenge.

Check. "Wolves should never be reintroduced" is an Outside view neither author holds. Attributing the deer worry to Passage 1 alone risks the Flipped speaker error. Calling it flat rejection is an Overstated clash—Author 2 shares the goal.

Answer: By agreeing the deer are a problem but doubting that wolves will curb them, since elsewhere the wolves hunted elk instead.

Approach Read · Plan · Solve · Check

Read. Summarize each passage's main claim in one short sentence.

Plan. Mark where the two overlap and where they diverge.

Solve. Predict Author 2's likely reply to Author 1 (or vice versa), then match a choice.

Check. The response must stay inside what that author wrote—no imported opinions.

Common traps — every wrong answer is one of these

One-passage answer.
A choice supported by only one passage, ignoring the relationship asked about.

Flipped speaker.
Attributing Passage 1's view to Passage 2's author, or the reverse.

Overstated clash.
Turning a mild qualification into flat disagreement (or the reverse).

Outside view.
A response that sounds reasonable but neither author actually holds.

Topic-only match.
A choice on the right subject but silent on how the authors connect.

Practice

1. *Passage 1: City planners argue that adding bike lanes reduces traffic, since every cyclist is one fewer car on the road. Wider adoption, they say, would ease the daily jams downtown.*

Passage 2: A transit researcher notes that new bike lanes fill mainly with people who used to walk or ride the bus, not with former drivers. The number of cars, her data show, barely moves.

How would the author of Passage 2 most likely respond to the claim in Passage 1?

(A) By recommending that buses be removed to make room for cyclists.
(B) By arguing that bike lanes rarely replace car trips, so traffic changes little.
(C) By agreeing that bike lanes are the fastest way to clear downtown jams.
(D) By insisting that cities should stop building bike lanes altogether.

2. *Passage 1: A literary scholar contends that a novel's first draft reveals the author's true intentions, before editors smooth away the rough, honest edges.*

Passage 2: Another scholar replies that early drafts are full of dead ends the author later rejected; the polished final version is the one the author chose to stand behind.

What is the main point of disagreement between the two authors?

(A) Whether novels are more honest than other kinds of writing.
(B) Whether readers prefer rough drafts to polished books.
(C) Whether a first draft or the final version better reflects the author's real intentions.
(D) Whether editors should be allowed to revise a novelist's work.

3. *Passage 1: An economist claims that raising the minimum wage always costs jobs, because businesses facing higher payroll must hire fewer workers.*

Passage 2: A labor analyst points to towns that raised wages and saw employment hold steady, as better-paid workers spent more locally and kept shops busy.

How would the author of Passage 2 most likely respond to the claim in Passage 1?

(A) By arguing that wages should never be set by any government.
(B) By claiming that local shops are unaffected by their customers' pay.
(C) By offering real cases in which higher wages did not reduce employment.

(D) By agreeing that higher wages always force businesses to cut jobs.

4. *Passage 1: One historian argues that the canal transformed the region by itself, opening trade routes that no road could match.*

Passage 2: A second historian agrees the canal mattered but insists it succeeded only because new railways carried goods the final miles to inland towns.

How does the view in Passage 2 relate to the view in Passage 1?

(A) It accepts the canal's importance but qualifies it by adding a second cause.

(B) It flatly denies that the canal had any effect on the region.

(C) It claims that roads, not canals, carried most of the region's trade.

(D) It repeats Passage 1's argument without adding anything new.

5. *Passage 1: A psychologist maintains that praising children for being "smart" builds their confidence and pushes them to attempt harder tasks.*

Passage 2: In her experiments, a researcher found that children praised for being smart later avoided hard problems, fearing that failure would prove them not smart after all.

How would the author of Passage 2 most likely respond to the claim in Passage 1?

(A) By concluding that confidence has no effect on which tasks children attempt.

(B) By presenting evidence that such praise can make children avoid challenges.

(C) By agreeing that praising intelligence reliably builds lasting confidence.

(D) By arguing that children should never be praised for anything at all.

DOMAIN 8

Expression of Ideas

Say it clearly, and connect the pieces

"Good prose is a windowpane; the reader should see the thought, not the glass."
— Adapted from a writer's maxim

Roughly 20% of the Reading and Writing section. Transitions and rhetorical synthesis—using a set of notes to accomplish a specific writing goal.

Transitions: choosing the logical connector

The skill. A transition question hands you two sentences and a blank between them; your job is to name the relationship the second sentence has to the first, then pick the word that labels exactly that relationship. The sentences already contain the logic—you are not adding meaning, only announcing it. There are five families worth memorizing: *cause/effect* (therefore, thus, as a result, consequently), *contrast* (however, nevertheless, on the other hand, still), *addition* (moreover, furthermore, in addition, also), *example* (for instance, for example, notably), and *sequence/time* (first, then, meanwhile, eventually, finally). Decide the family before you look at the choices, because the choices are engineered so that one word from each rival family sounds plausible if you skip that step.

Worked example

Solar panels were once too costly for most homeowners. ____, a decade of falling manufacturing costs has put them within reach of millions.

Which choice completes the text with the most logical transition?

Read. Sentence one: panels used to be too expensive. Sentence two: now they are affordable. The second sentence reverses the situation the first describes.

Plan. The blank must signal contrast—"that was the case before, but this is the case now."

Solve. A contrast connector fits: *However* announces the reversal from "too costly" to "within reach."

Check. *Therefore* treats the new affordability as a result of the old high price (fake causation); *Moreover* pretends the second fact merely piles onto the first instead of overturning it (redundant addition); *For example* offers no instance of anything.

Answer: However.

Approach Read · Plan · Solve · Check

Read. Read both sentences fully—especially the second one, which carries the relationship.

Plan. Say the relationship aloud in plain words: "this is the result of that," "this contradicts that," "this is one more of the same."

Solve. Match your plain-words label to the transition family; choose the word from that family.

Check. Reread the pair with your choice inserted—does the logic click, or merely sound formal?

Common traps — every wrong answer is one of these

Wrong family.
The choice is a real, common transition, but it signals a relationship (contrast, say) that the sentences do not actually have.

False contrast.
A "however" or "nevertheless" is offered where the second sentence agrees with or extends the first—no reversal is present.

Fake causation.
"Therefore" or "as a result" is dangled after two facts that are merely related, not cause and effect.

Redundant addition.
"Moreover" or "in addition" is used to bolt on a sentence that actually explains or exemplifies the first rather than adding a new, parallel point.

Formal but empty.
A weighty-sounding connector ("consequently," "notwithstanding") is picked because it feels academic, not because the logic requires it.

Practice

1. *The desert receives less than five centimeters of rain each year.* _____, *the plants that survive there store water in thick, waxy stems.*

   Which choice completes the text with the most logical transition?

(A) As a result,

(B) For example,

(C) In contrast,

(D) However,

2. *Maya practiced the violin for two hours every evening. _____, she reviewed her music theory notes each morning before class.*

Which choice completes the text with the most logical transition?

(A) Nevertheless,

(B) Therefore,

(C) In addition,

(D) For instance,

3. *The committee expected the new policy to reduce complaints. _____, the number of complaints actually rose in the following quarter.*

Which choice completes the text with the most logical transition?

(A) Consequently,

(B) Meanwhile,

(C) Instead,

(D) Likewise,

4. *Several everyday materials conduct heat far better than others. _____, a copper spoon left in hot soup grows warm long before a wooden one does.*

Which choice completes the text with the most logical transition?

(A) Therefore,

(B) Furthermore,

(C) For example,

(D) On the other hand,

5. *The engineers first drained the reservoir to inspect the cracked wall. _____, they sealed each fracture with a fast-curing resin.*

Which choice completes the text with the most logical transition?

(A) In particular

(B) By contrast

(C) Then

(D) Nonetheless

Transitions: diagnosing the relationship

The skill. The hard transition items are not the ones where you don't know the words—they are the ones where two families both feel possible. The cure is diagnosis: instead of asking "which word sounds right?" ask "what is the second sentence *doing* to the first?" A sentence can only be doing one primary job—echoing, reversing, extending, illustrating, or advancing in time. Test a suspected transition by trying to break it: if you insert "however" but can find no idea being contradicted, "however" is disqualified no matter how natural it reads. Watch especially for sentences that seem to contrast but are really giving a consequence, and for sentences that seem additive but are really a restatement. Naming the job first makes the wrong word visibly wrong.

Worked example

Downtown parking fees climbed sharply last year. ____, ridership on the new light-rail line has reached record highs.

Which choice completes the text with the most logical transition?

Read. Claim one: parking became much more expensive. Claim two: more people ride the train. Ask what sentence two is *doing*—it reports the outcome that the higher fees produced.

Plan. The job is consequence, so the connector must mean "and so"; reject any word whose relationship you cannot point to.

Solve. *Consequently* names exactly that cause-and-effect link between costlier parking and fuller trains.

Check. *However* fails the falsification test—nothing is contradicted (overreading contrast); *Similarly* treats two linked facts as parallel echoes (surface-word matching); *Meanwhile* forces a mere time overlap where real causation exists (time-word overreach).

Answer: Consequently.

Approach Read · Plan · Solve · Check

Read. Isolate the claim in sentence one and the claim in sentence two.

Plan. Ask: does sentence two reverse, repeat, extend, illustrate, or follow in time?

Solve. Pick the connector whose whole purpose is that one job; reject any that require a relationship you can't point to.

Check. Try to falsify your pick—if "therefore" has no cause, or "however" has no contrast, discard it.

Common traps — every wrong answer is one of these

Surface-word matching.
A shared keyword ("problem," "instead") lures you toward contrast when the sentences are actually cooperative.

Overreading contrast.
Two different examples of the same idea are misread as opposing, so a reversal word is chosen.

Consequence mistaken for addition.
A result is treated as "one more fact," so "also" replaces the needed "therefore."

Restatement mistaken for example.
The second sentence rephrases the first, but "for instance" is chosen as if a new specific case were introduced.

Time-word overreach.
"Meanwhile" or "then" is chosen for logical relationships that have nothing to do with sequence.

Practice

1. *Nocturnal animals face a problem daytime hunters never confront: near-total darkness.* _____, *many have evolved eyes with an extra reflective layer that doubles the light each cell receives.*

Which choice completes the text with the most logical transition? (The second sentence names a response to the problem, not a contrast.)

(A) Nevertheless,
(B) However,
(C) In response,
(D) On the contrary,

2. *The novelist set her early stories in crowded cities. Her later work unfolds in silent, half-empty towns;* _____, *the loneliness of her characters only deepened over her career.*

Which choice completes the text with the most logical transition? (The second clause continues a trend, not reverses it.)

(A) by contrast
(B) however
(C) instead
(D) in fact

3. *The two bridges look nearly identical from the riverbank.* _____, *one rests on stone piers sunk into bedrock while the other hangs entirely from steel cables.*

Which choice completes the text with the most logical transition? (Sentence two reveals a genuine difference beneath the similarity.)

(A) Therefore,
(B) Similarly,

(C) In other words,

(D) In reality,

4. *Volunteers cleared the invasive weeds from the marsh in a single weekend.* _____, *native reeds that had not bloomed in years reappeared the following spring.*

Which choice completes the text with the most logical transition? (Sentence two is an outcome of the action, not a parallel fact.)

(A) As a result,

(B) For example,

(C) Even so,

(D) Additionally,

5. *The report argued that the factory was the town's largest employer.* _____, *it stated plainly that nearly one in three local jobs depended on the plant.*

Which choice completes the text with the most logical transition? (Sentence two restates the claim in sharper terms.)

(A) In contrast,

(B) Put another way,

(C) Conversely,

(D) Meanwhile,

Rhetorical synthesis: state a main point from notes

The skill. In a synthesis question you are handed a short list of research notes and a stated goal, and you must choose the sentence that fulfills that goal using the facts given. When the goal is "state the main point" or "present the central conclusion," the winning sentence pulls together the notes into a single claim—not a stray detail, and not a fact copied from one bullet. The best answer usually names the subject and asserts something general that the notes collectively support. Beware choices that are perfectly true yet trivial: a lone statistic from one note is accurate but is not a *main* point. Ask what the whole set of notes is about, then find the choice that says that.

Worked example

While researching an invention, a student has taken these notes:

- The printing press let identical copies of a text be made quickly.
- Books, once rare and costly, became cheap enough for ordinary readers.
- Literacy rates rose across Europe within a few generations.
- New ideas spread between distant cities faster than ever before.

The student wants to state the main point of the notes. Which choice most effectively accomplishes this goal?

Read. The goal is the main point; every note circles one idea—the press transformed how widely knowledge could travel.

Plan. A correct choice must gather the notes into one general claim about the press's broad impact, not simply repeat a single bullet.

Solve. "By making books cheap and plentiful, the printing press spread literacy and new ideas across Europe" pulls all four notes under one claim.

Check. "The press made identical copies quickly" reports a lone bullet (one-note answer); "Literacy rose within a few generations" is a true but partial detail (too narrow); "The printing press is a fascinating invention" states no real point (vague filler).

Answer: By making books cheap and plentiful, the printing press spread literacy and new ideas across Europe.

Approach Read · Plan · Solve · Check

Read. Read the goal first, then the notes; underline the idea the notes share.

Plan. Phrase the main point in your own words before reading the choices.

Solve. Choose the sentence that matches your phrasing and draws on the notes as a whole.

Check. Confirm the choice is supported by the notes and actually answers the stated goal.

Common traps — every wrong answer is one of these

One-note answer.
The choice reports a single bullet accurately but ignores the broader point the notes build toward.

Unsupported addition.
The sentence sounds like a strong conclusion but asserts something no note establishes.

Wrong goal.
The choice emphasizes a contrast or gives an example when the goal asked for the central point.

Too narrow.
A true detail is dressed up as a thesis but is far smaller than what the notes cover.

Vague filler.
The sentence is so general ("This topic is interesting") that it states no real point at all.

Practice

1. While researching urban gardens, a student has taken these notes:

- Community gardens turn vacant lots into shared growing space.
- Residents report knowing more of their neighbors after joining.
- Gardens supply fresh produce in areas with few grocery stores.
- Studies link the plots to lower neighborhood stress levels.

The student wants to state the main point of the notes. Which choice most effectively accomplishes this goal?

(A) Community gardens are built on lots that were previously vacant.
(B) Some residents say they now know more of their neighbors.
(C) Community gardens are becoming popular in cities around the world.
(D) Community gardens offer neighborhoods social, nutritional, and psychological benefits at once.

2. While studying a migratory bird, a student has taken these notes:

- The Arctic tern breeds in the far north each summer.
- It winters near Antarctica, at the opposite end of the planet.
- Its yearly round trip can exceed 70,000 kilometers.
- Over a lifetime it may travel far enough to reach the moon and back.

The student wants to state the main point of the notes. Which choice most effectively accomplishes this goal?

(A) The Arctic tern winters somewhere near Antarctica.
(B) The Arctic tern undertakes one of the longest annual migrations of any animal on Earth.
(C) The Arctic tern breeds in the far north during the summer months.
(D) Many birds travel long distances between seasons.

3. While researching a material, a student has taken these notes:

- Bamboo can grow nearly a meter in a single day.
- Its fibers rival some steels in tensile strength for their weight.
- It regrows from the same root system without replanting.
- Builders use it for scaffolding, flooring, and even bridges.

The student wants to state the main point of the notes. Which choice most effectively accomplishes this goal?

(A) Bamboo's rapid growth, strength, and renewability make it a remarkably versatile building material.
(B) Bamboo can grow nearly a meter in one day.
(C) Bamboo is used in some places to build scaffolding.

(D) Bamboo is a plant that has attracted the interest of builders.

4. While studying a preservation project, a student has taken these notes:

- A team is digitizing fragile 200-year-old town records.
- High-resolution scans capture ink that is fading from the paper.
- The originals are then sealed in climate-controlled storage.
- Anyone can now read the documents online for free.

The student wants to state the main point of the notes. Which choice most effectively accomplishes this goal?

(A) The project uses climate-controlled storage for the documents.

(B) Digitizing the town records both protects the fragile originals and opens them to the public.

(C) The scans are made at high resolution.

(D) Old records can be difficult to read when the ink has faded.

Rhetorical synthesis: emphasize a similarity or difference

The skill. Here the goal narrows: the notes describe two things, and you must write a sentence that highlights either how they are *alike* or how they *differ*. The correct answer must (1) mention both items and (2) foreground the specific relationship the goal names. A sentence about only one item cannot emphasize a comparison, and a sentence that lists both but draws no relationship merely reports. If the goal says "difference," the answer's grammar should pivot—*whereas*, *but*, *unlike*, *by contrast*; if it says "similarity," the answer should join—*both*, *like*, *share*, *alike*. Match the connective spine of the sentence to the goal.

Worked example

While comparing two telescopes, a student has taken these notes:

- The Halden telescope gathers light with a large curved mirror.
- The Prewitt telescope gathers light with a large glass lens.
- Both are used mainly to photograph distant galaxies.
- Both sit atop the same mountain observatory.

The student wants to emphasize a difference between the two telescopes. Which choice most effectively accomplishes this goal?

Read. Two items, the Halden and Prewitt telescopes; the notes that separate them concern how each gathers light—a mirror versus a lens.

Plan. Because the goal is difference, the sentence must name both telescopes and pivot on a contrast word (*whereas*, *while*) on the trait that actually differs.

Solve. "Whereas the Halden telescope gathers light with a curved mirror, the Prewitt uses a large glass lens" names both and pivots on the differing trait.

Check. "Both photograph distant galaxies" stresses likeness when a difference was asked for (opposite relationship); "The Halden uses a curved mirror" covers only one telescope (single-item answer); "Both sit on the same mountain" compares them on a trait unrelated to the goal (off-axis comparison).

Answer: Whereas the Halden telescope gathers light with a curved mirror, the Prewitt telescope uses a large glass lens.

Approach Read · Plan · Solve · Check

Read. Identify the two items and the notes attached to each.

Plan. Decide whether the goal wants likeness or contrast; pick the fact from each item that shows it.

Solve. Choose the sentence naming both items and pivoting or joining them as the goal requires.

Check. Verify both items appear and that the stated relationship—not the opposite—is emphasized.

Common traps — every wrong answer is one of these

Opposite relationship.
The goal asks for a similarity but the choice stresses a difference, or vice versa.

Single-item answer.
The sentence describes only one of the two things, so no comparison is made.

Lists without linking.
Both items appear, but the sentence just states facts side by side without asserting how they relate.

Off-axis comparison.
The choice compares the items on a trait the notes don't tie to the goal, missing the intended point.

Unsupported claim.
The comparison asserts a likeness or gap the notes never establish.

Practice

1. While comparing two composers, a student has taken these notes:

- Composer A wrote mainly for small string quartets.
- Composer B wrote mainly for large symphony orchestras.
- Both drew heavily on the folk melodies of their home regions.
- Both composed in the early twentieth century.

The student wants to emphasize a similarity between the two composers. Which choice most effectively accomplishes this goal?

(A) Both composers worked in the early twentieth century, though their styles diverged sharply.
(B) Although they wrote for very different ensembles, both composers built their music on regional folk melodies.
(C) Composer A wrote for small string quartets, while Composer B wrote for large orchestras.
(D) Composer A drew on the folk melodies of his home region.

2. While comparing two lakes, a student has taken these notes:

- Lake Verin is fed by melting glaciers and stays very cold year-round.
- Lake Osha is fed by warm springs and rarely drops below room temperature.
- Both lakes lie within the same national park.
- Both are popular with summer visitors.

The student wants to emphasize a difference between the two lakes. Which choice most effectively accomplishes this goal?

(A) Lake Osha is fed by warm springs and rarely turns cold.
(B) Lake Verin and Lake Osha are both popular with summer visitors.
(C) Whereas glacier-fed Lake Verin stays frigid all year, spring-fed Lake Osha remains warm.
(D) Both Lake Verin and Lake Osha lie within the same national park.

3. While comparing two printing methods, a student has taken these notes:

- Letterpress presses ink into the paper, leaving a slight indentation.
- Digital printing lays ink flat on the surface with no indentation.
- Both can reproduce fine text clearly.
- Both are used by small stationery studios today.

The student wants to emphasize a difference between the two methods. Which choice most effectively accomplishes this goal?

(A) Both methods are used today by small stationery studios.

(B) Unlike digital printing, which leaves ink flat on the page, letterpress presses each character into the paper.

(C) Both letterpress and digital printing can reproduce fine text clearly.

(D) Letterpress printing leaves a slight indentation in the paper.

4. While comparing two athletes, a student has taken these notes:

- Sprinter X trains almost entirely with short, explosive bursts.
- Marathoner Y trains almost entirely with long, steady runs.
- Both follow strict, carefully measured diets.
- Both credit their success to years of daily discipline.

The student wants to emphasize a similarity between the two athletes. Which choice most effectively accomplishes this goal?

(A) Marathoner Y trains almost entirely with long, steady runs.

(B) Sprinter X and Marathoner Y differ sharply in how they train.

(C) Despite their opposite training styles, both athletes attribute their success to years of daily discipline.

(D) Sprinter X trains with short, explosive bursts, whereas Marathoner Y runs long and steady.

Rhetorical synthesis: introduce a topic to a specific audience

The skill. The last synthesis type asks for a sentence tuned to a stated audience and purpose: "introduce the study to an audience unfamiliar with astronomy," "present the finding to readers who already know the basics." Two things decide the answer: the sentence must accomplish the specific job named (introduce, define, motivate), and its assumed knowledge must match the named audience. For an unfamiliar audience, the winning sentence defines terms and starts from the ground; for an expert audience, it can skip the basics and get to the new claim. A sentence that is accurate but pitched at the wrong level—too technical for novices, too basic for experts—loses even when it's true.

Worked example

While preparing an article, a student has taken these notes:

- An aquifer is an underground layer of rock that stores and carries water.
- Wells draw drinking water from aquifers in many regions.
- Rain slowly refills them over years or even centuries.
- Pumping water out faster than it refills can drain an aquifer.

The student wants to introduce the topic to an audience unfamiliar with aquifers. Which choice most effectively accomplishes this goal?

Read. The audience is new to the subject and the job is to introduce it, so the sentence must name the topic and define the key term from the ground up.

Plan. A correct choice defines "aquifer" in plain words and frames what the piece is about, assuming no prior knowledge.

Solve. "An aquifer is an underground layer of rock that stores and carries water, supplying wells in many parts of the world" names and defines the term for a newcomer.

Check. "Over-pumping can drain an aquifer faster than rain refills it" offers a detail with no context (detail without frame) and assumes the reader already knows the word (wrong pitch); "Water is one of our most precious resources" never names the actual subject (off-topic opener).

Answer: An aquifer is an underground layer of rock that stores and carries water, supplying wells in many parts of the world.

Approach Read · Plan · Solve · Check

Read. Note the audience and the job (introduce, motivate, define) in the goal.

Plan. Ask how much this audience already knows and what the sentence must do.

Solve. Choose the sentence that does that job at the right level of assumed knowledge.

Check. Reread as that audience—is anything unexplained, or anything condescendingly obvious?

Common traps — every wrong answer is one of these

Wrong pitch.
The sentence assumes expertise a general audience lacks, or over-explains for an expert one.

Wrong job.
The choice draws a conclusion or gives an example when the goal was merely to introduce the topic.

Detail without frame.
A specific fact is offered with no context, so a newcomer has nothing to anchor it to.

Unsupported claim.
The introduction smuggles in an assertion the notes never support.

Off-topic opener.
The sentence is broadly related but never actually names the subject the passage will cover.

Practice

1. While preparing an article, a student has taken these notes:

- Bioluminescence is the production of light by living organisms.
- Certain deep-sea fish use it to lure prey in total darkness.
- Fireflies use it to signal potential mates.
- The light comes from a chemical reaction inside the organism.

The student wants to introduce the topic to an audience unfamiliar with bioluminescence. Which choice most effectively accomplishes this goal?

(A) The chemical reaction responsible relies on the enzyme that oxidizes luciferin.

(B) Many organisms have surprising ways of surviving in the dark.

(C) Bioluminescence, the production of light by living organisms, appears in creatures as different as deep-sea fish and fireflies.

(D) Deep-sea anglerfish dangle a glowing lure to draw prey toward their jaws.

2. While preparing a report, a student has taken these notes:

- A "keystone species" has an outsized effect on its ecosystem.
- Sea otters eat urchins that would otherwise devour kelp forests.
- Remove the otters, and the kelp forests collapse.
- The term was coined by ecologist Robert Paine in the 1960s.

The student wants to introduce the concept to readers who have never heard of a keystone species. Which choice most effectively accomplishes this goal?

(A) Robert Paine coined the term in the 1960s while studying tidal pools.

(B) Without sea otters, urchin populations explode and strip the kelp bare.

(C) Ecosystems are held together by relationships we are only beginning to map.

(D) A keystone species is one whose presence has an outsized effect on the ecosystem around it.

3. While preparing a piece for experienced hikers, a student has taken these notes:

- Switchbacks are zigzagging trail segments cut into steep slopes.
- They trade a longer path for a gentler, safer grade.
- Cutting straight across them erodes the hillside.
- A new trail on Mount Reyes added a mile of switchbacks last year.

The student wants to introduce the new trail to an audience of experienced hikers already familiar with switchbacks. Which choice most effectively accomplishes this goal?

(A) Switchbacks are zigzagging segments that trail crews cut into steep slopes.

(B) Hikers should never cut across a switchback, because doing so erodes the hill.

(C) Mountains offer countless rewards to those willing to climb them.

(D) Mount Reyes's rebuilt trail adds a mile of switchbacks to ease its steepest climb.

4. While preparing an explainer, a student has taken these notes:

- A "tessellation" is a pattern of shapes that fit together with no gaps.
- Bathroom tiles and honeycombs are everyday examples.
- Only certain shapes, like triangles and hexagons, tile a plane alone.
- The artist M. C. Escher built famous prints from tessellations.

The student wants to introduce the topic to readers unfamiliar with the term. Which choice most effectively accomplishes this goal?

(A) Patterns are everywhere once you learn to look for them.

(B) A tessellation is a pattern of shapes that fit together perfectly, with no gaps or overlaps.

(C) Escher's prints famously exploit the strangeness of interlocking shapes.

(D) Regular hexagons tile a plane, while regular pentagons cannot.

DOMAIN 9

Standard English Conventions

The rules that keep meaning intact

> *"A sentence is a small machine for carrying thought; punctuation is the set of gears that keeps it from grinding."*
>
> — Anonymous, on the craft of clear writing

Roughly 26% of the Reading and Writing section. Sentence boundaries and punctuation; subject–verb and pronoun agreement; verb forms; and modifiers and parallel structure.

Sentence Boundaries

The skill. A complete sentence needs at least one independent clause—a subject and a predicate that can stand alone. When two independent clauses meet, they must be joined legally: end the first with a period; link them with a semicolon; or link them with a comma plus a coordinating conjunction (for, and, nor, but, or, yet, so—the FANBOYS). Joining two independent clauses with only a comma produces a comma splice; joining them with no punctuation at all produces a run-on (a fused sentence). A comma alone is legal only when at least one side is *not* an independent clause.

Worked example

The lighthouse had been dark for a decade ____ the old sailors still steered by its silhouette against the stars.

Which choice conforms to the conventions of Standard English?

Read. Test each side of the blank. "The lighthouse had been dark for a decade" stands alone; "the old sailors still steered by its silhouette against the stars" stands alone. Two independent clauses meet here.

Plan. Two independent clauses must be joined legally: a period, a semicolon, or a comma plus a FANBOYS conjunction. A comma alone will not do.

Solve. A semicolon links the two closely related clauses without adding a conjunction, so "…for a decade; the old sailors…" is correct.

Check. A bare comma would be a comma splice; no punctuation at all would fuse the clauses into a run-on; a period after "dark" alone would illegally chop the sentence before its clause is complete.

Answer: a semicolon.

Approach Read · Plan · Solve · Check

Read. Identify the material on each side of the boundary mark.

Plan. Test each side: can it stand alone as a sentence? If both can, you have two independent clauses.

Solve. Join two independent clauses with a period, a semicolon, or a comma plus FANBOYS; use a comma alone only when one side is dependent.

Check. Reread. No comma splice, no fused clauses, and no full stop chopping a single clause in two.

Common traps — every wrong answer is one of these

Comma splice.
Two independent clauses joined by a comma with no conjunction.

Fused sentence (run-on).
Two independent clauses jammed together with no punctuation.

Missing conjunction.
A comma is present but the required FANBOYS word is absent.

Illegal full stop.
A period (or semicolon) placed inside a single clause, severing a subject from its verb or a modifier from its clause.

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The greenhouse kept its own weather ___ inside, the ferns unfurled while frost webbed the outer glass.

(A) ,
(B) (no punctuation)
(C) , while
(D) ;

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Marisol rewired the sensor twice ___ it still refused to register the faintest change in temperature.

(A) (no punctuation)
(B) , yet
(C) , it
(D) ; yet,

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The archive had no catalog ___ every visiting scholar simply memorized the shelves.

(A) (no punctuation)
(B) , therefore
(C) ;
(D) ,

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Because the tide had turned ___ the fishermen hauled their nets in before dusk.

(A) . The
(B) , and
(C) ,
(D) ;

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The comet would not return for a lifetime ___ so the villagers stayed up to watch it cross.

(A) . So
(B) ,
(C) ;
(D) , and

Punctuation Within a Sentence

The skill. Two jobs dominate here. First, *introducing*: a colon or a dash may introduce a list, an explanation, or an amplification—but only when the words before the mark form a complete independent clause. Do not place a colon between a verb and its object or after a preposition. Second, *setting off nonessential elements*: information that could be lifted out without changing the core meaning is bracketed by a matched pair of commas, dashes, or parentheses. The pair must match—a comma cannot close what a dash opened—and essential (restrictive) information takes no punctuation at all.

Worked example

The chef built the entire tasting menu around one ingredient ____ a saffron she pressed and dried by hand each autumn.

Which choice conforms to the conventions of Standard English?

Read. The words before the blank, "The chef built the entire tasting menu around one ingredient," form a complete independent clause, and what follows amplifies "one ingredient."

Plan. A colon may introduce an explanation or amplification, but only when a full independent clause precedes it. Here one does.

Solve. Place a colon after "ingredient" to introduce the amplifying phrase: "...around one ingredient: a saffron she pressed..."

Check. A comma would leave the amplification only weakly attached; a dash could work but a semicolon is wrong because "a saffron she pressed and dried by hand each autumn" is not an independent clause, and "such as" before a colon would double the introducer.

Answer: a colon.

Approach Read · Plan · Solve · Check

Read. Locate the mark and ask what it is doing: introducing, or setting material off?

Plan. For a colon or dash that introduces, confirm a full clause precedes it. For a set-off element, confirm both ends are marked and matched.

Solve. Keep introducing marks after complete clauses only; enclose nonessential material in a matched pair; leave essential material unpunctuated.

Check. Remove the enclosed phrase—the sentence should still stand. Confirm the opening and closing marks are the same type.

Common traps — every wrong answer is one of these

Colon after a fragment.
A colon placed where the preceding words are not a complete clause (e.g., after "such as" or a bare verb).

Mismatched pair.
A nonessential phrase opened with a dash but closed with a comma, or opened with a comma but closed with a dash.

Unmatched single comma.
Only one of the two required commas around a nonessential element appears.

Restrictive over-punctuation.
Commas placed around essential (restrictive) information that the sentence needs to identify its subject.

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The expedition carried three instruments that had never failed in the field ____ a barometer, a sextant, and a hand-cranked radio.

(A) ,
(B) ; namely,
(C) :
(D) , such as:

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The violinist ____ who had learned the piece overnight ____ played it without a single hesitation.

(A) , . . . ,
(B) , . . . —
(C) — . . . ,
(D) (. . . ,

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

My oldest cousin ____ a cartographer by training ____ can sketch any coastline from memory.

(A) , . . . (none)
(B) : . . . :
(C) , . . . ,
(D) (no punctuation either side)

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The botanist wanted only one thing from the greenhouse ____ a cutting of the vine that flowered at midnight.

(A) :
(B) , such as
(C) ;
(D) ,

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The novel that won the regional prize ____ was set entirely in a single railway car.

(A) ,
(B) —
(C) :
(D) (no punctuation)

Subject–Verb Agreement

The skill. A verb must agree in number with its true subject—singular subject, singular verb; plural subject, plural verb. The difficulty is finding the true subject. Ignore intervening phrases: material between subject and verb (a prepositional phrase, a relative clause, an appositive) does not change the subject's number. Collective nouns (team, committee, flock) are usually singular when acting as a unit. Watch inverted order, where the subject follows the verb (as after "there is / there are" or a fronted phrase), and watch correlative subjects joined by "or / nor," where the verb agrees with the nearer element.

Worked example

The tall stack of weathered survey maps in the corner of the reading room ____ older than the library itself.

Which choice conforms to the conventions of Standard English?

Read. Find the verb slot, then ask who or what is older. The true subject is "stack," a singular noun.

Plan. Strike out the intervening phrases "of weathered survey maps" and "in the corner of the reading room"; they do not change the subject's number. A singular subject takes a singular verb.

Solve. Match "stack" with the singular "is": "The tall stack . . . is older than the library itself."

Check. Choosing "are" would be intervening-phrase attraction, agreeing with the nearby plural "maps" or "survey maps" instead of with the real subject "stack."

Answer: is.

Approach Read · Plan · Solve · Check

Read. Find the verb, then ask "who or what" performs it.

Plan. Strike out every phrase between the subject and the verb; note collective nouns, inversions, and or/nor pairs.

Solve. Match the verb to the bare subject's number, not to the nearest noun.

Check. Reunite subject and verb with the middle removed—do they sound right together?

Common traps — every wrong answer is one of these

Intervening-phrase attraction.
The verb is made to agree with a noun in an intervening phrase rather than with the subject.

Collective miscount.
A collective noun acting as one unit is given a plural verb (or vice versa).

Inverted-order slip.
After "there" or a fronted phrase, the verb agrees with something other than the delayed subject.

Or/nor mismatch.
With subjects joined by or/nor, the verb fails to agree with the nearer subject.

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The crate of antique brass telescopes ___ heavier than the movers had expected.

(A) was
(B) were
(C) have been
(D) are

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Along the ridge ___ a single line of wind-bent pines, marking the old property boundary.

(A) stand
(B) are standing
(C) were standing
(D) stands

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The jury, after eleven hours of argument, ___ its verdict without further comment.

(A) deliver
(B) are delivering
(C) have delivered
(D) delivers

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Neither the conductor nor the musicians ___ willing to shorten the encore.

(A) were
(B) was
(C) has been
(D) is

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Each of the returning astronauts ____ a private logbook that the agency never publishes.

(A) have kept

(B) keeps

(C) keep

(D) are keeping

Pronoun Agreement and Clarity

The skill. A pronoun must agree with its antecedent in number and must point to that antecedent unmistakably. Singular antecedents—including the indefinite pronouns each, either, neither, everyone, anybody, and a singular collective noun treated as a unit—take singular pronouns. A pronoun that could refer to two different nouns, or that refers to no stated noun at all, is faulty; the fix is either to make the reference single and obvious or to replace the pronoun with the noun itself. Clarity outranks brevity: if a reader must pause to ask "which one?" the sentence needs the noun.

Worked example

Each of the volunteers at the shelter must record ____ hours in the logbook at the end of a shift.

Which choice conforms to the conventions of Standard English?

Read. The pronoun in the blank stands for the subject "Each," not for the plural "volunteers" inside the intervening phrase.

Plan. Indefinite pronouns such as "each," "either," and "everyone" are singular and take a singular pronoun in Standard English.

Solve. Because a person's hours are meant, the singular "his or her" matches the singular antecedent "each": "Each . . . must record his or her hours."

Check. "Their" is the classic number-disagreement error, pulled toward the nearby plural "volunteers"; "its" wrongly treats a person as a thing.

Answer: his or her.

Approach Read · Plan · Solve · Check

Read. Circle each pronoun and name the exact noun it stands for.

Plan. Check number agreement, then check that only one noun can be the antecedent.

Solve. Match number; if reference is ambiguous or absent, restate the noun.

Check. Reread swapping the noun back in—does the meaning stay the same and stay singular in intent?

Common traps — every wrong answer is one of these

Number disagreement.
A plural pronoun (they, their) paired with a singular antecedent such as “each” or “a student.”

Ambiguous reference.
A pronoun that could point to either of two preceding nouns.

Missing antecedent.
A pronoun (often “it” or “this”) that refers to no stated noun at all.

Collective disagreement.
A collective noun treated as singular in the verb but plural in its pronoun, or the reverse.

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Every applicant to the fellowship must submit ____ portfolio before the equinox.

(A) their
(B) its
(C) them
(D) his or her

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

When the historian met the archivist, ____ handed over the missing ledger.

(A) the archivist
(B) she
(C) they
(D) it

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The migrating flock changed direction the moment ____ leader caught the updraft.

(A) them
(B) its
(C) their
(D) it’s

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Neither of the two composers would admit that ____ own score had grown too long.

(A) its

(B) them

(C) his

(D) their

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The lecture ran past midnight and the projector kept failing, but ____ never dulled the audience's enthusiasm.

(A) it

(B) this

(C) they

(D) the late hour

Verb Forms and Tense Consistency

The skill. Choose the verb form the sentence's own logic demands, then hold the tense steady unless the meaning requires a shift. Past actions completed before another past action take the past perfect (had done); actions continuing to the present take the present perfect (has done). Do not slide between past and present narration without cause. Watch irregular past participles (had *gone*, not had *went*) and the tense signaled by time markers such as "by the time," "since," and "already." A change of tense should track a real change in time, not a lapse of attention.

Worked example

By the time the curtain finally rose, the understudy ____ every line that the exhausted lead had forgotten.

Which choice conforms to the conventions of Standard English?

Read. The baseline time frame is past: "the curtain finally rose." The understudy's learning happened before that moment.

Plan. An action completed before another past action takes the past perfect (had done). The marker "by the time" confirms an earlier-than-past sequence.

Solve. Use the past perfect: "…the understudy had memorized every line…"

Check. Simple past "memorized" would flatten the sequence the marker demands; present "memorizes" would be an unmotivated tense shift; "has memorized" wrongly reaches to the present instead of to a point before the past.

Answer: had memorized.

Approach Read · Plan · Solve · Check

Read. Identify the sentence's baseline time frame from its other verbs and time markers.

Plan. Decide whether the target action is simultaneous with, before, or after that frame.

Solve. Select the matching form—simple, perfect, or progressive—with the correct participle.

Check. Read the verbs in sequence; the timeline should advance logically with no unmotivated jump.

Common traps — every wrong answer is one of these

Unmotivated tense shift.
The verb jumps from past to present (or back) with no change in the actual time described.

Wrong perfect.
Simple past used where an earlier-than-past action requires the past perfect (or vice versa).

Faulty participle.
A nonstandard past participle, such as "had wrote" or "has drank."

Sequence mismatch.
The chosen tense contradicts a time marker like "by the time" or "already."

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

By the time the rescue boat arrived, the stranded surveyors ___ their last flare an hour earlier.

(A) have fired
(B) had fired
(C) fired
(D) fire

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The curator described how the fresco cracked over centuries and then ___ how restorers had stabilized it.

(A) will explain
(B) explained
(C) explains
(D) is explaining

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Since the observatory opened in 1911, its dome ___ open on all but a handful of clouded nights.

(A) creaked
(B) creaks
(C) had creaked
(D) has creaked

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The apprentice realized that she ___ the wrong alloy into the mold before the foreman noticed.

(A) pours
(B) had poured
(C) poured
(D) has poured

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The choir rehearsed for weeks, and on opening night it ___ the anthem flawlessly.

(A) has sung
(B) had sang
(C) sang
(D) sings

Modifiers and Parallel Structure

The skill. A modifier must sit next to the word it describes. An opening participial or descriptive phrase attaches to the subject that immediately follows the comma; if that subject cannot logically perform the described action, the modifier dangles or is misplaced. The cure is to name the right subject or to rewrite the phrase as a clause. Parallel structure is the partner rule: items in a series, and elements joined by paired conjunctions (not only… but also, either… or), must share the same grammatical form—all nouns, all gerunds, or all matching clauses.

Worked example

Circling the flooded harbor at dusk, ___

Which choice conforms to the conventions of Standard English?

Read. The opening phrase "Circling the flooded harbor at dusk" describes whoever is doing the circling; that doer must be the subject right after the comma.

Plan. An opening participial phrase attaches to the subject that immediately follows it. If the stated subject cannot logically circle the harbor, the modifier dangles.

Solve. Only a person can circle the harbor, so the subject must name that person: "… the harbor pilot spotted the overturned skiff."

Check. "…the overturned skiff was easy to see" or "…there was an overturned skiff" leaves the circling attached to a skiff or to nothing at all—the classic dangling modifier.

Answer: the harbor pilot spotted the overturned skiff.

Approach Read · Plan · Solve · Check

Read. Find any opening or embedded modifier and any list or paired-conjunction structure.

Plan. Ask what the modifier is meant to describe, and confirm every series item shares one grammatical form.

Solve. Place the modifier beside its true target; recast list items so their forms match.

Check. Read the modifier with the subject that follows it, and read the series items in isolation—each should fit.

Common traps — every wrong answer is one of these

Dangling modifier.
An opening phrase whose logical subject never appears in the sentence.

Misplaced modifier.
A modifier positioned so it seems to describe the wrong noun.

Faulty parallelism.
Items in a series that mix grammatical forms (e.g., two gerunds and an infinitive).

Correlative mismatch.
The elements after "not only" and "but also" (or "either"/"or") are not grammatically matched.

Practice

1. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Rummaging through the attic at dawn, ____

(A) there was a diary bound in oilcloth.
(B) the diary bound in oilcloth was discovered.
(C) the archivist found a diary bound in oilcloth.
(D) a diary bound in oilcloth turned up.

2. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The training program rewarded recruits for their stamina, their patience, and ____

(A) how precise they were.
(B) to be precise under pressure.

(C) their precision under pressure.

(D) being precise under pressure.

3. *Which choice completes the text so that it conforms to the conventions of Standard English?*

Painted in the last week before the flood, ____

(A) the ceiling still shows the muralist's hurried brushwork.

(B) there was hurried brushwork on the ceiling.

(C) exhaustion was visible in the muralist's ceiling.

(D) the muralist finished the ceiling in exhausted silence.

4. *Which choice completes the text so that it conforms to the conventions of Standard English?*

The engineer was praised not only for redesigning the bridge but also ____

(A) to train the crew that built it.

(B) the crew that built it was hers to train.

(C) for training the crew that built it.

(D) she trained the crew that built it.

5. *Which choice completes the text so that it conforms to the conventions of Standard English?*

A good field notebook records what the observer sees, sketches the terrain, and ____

(A) noting the exact time of each event.

(B) to note the exact time of each event.

(C) notes the exact time of each event.

(D) the exact time of each event is noted.

Answer Keys

Algebra

Solve linear equations in one variable — Q1
Correct: **(C)**. *Traps:* **(A)** coefficient/division error; **(B)** transposition sign error; **(D)** coefficient/-division error.

Solve linear equations in one variable — Q2
Correct: **(B)**. *Traps:* **(A)** coefficient/division error; **(C)** both-sides imbalance; **(D)** transposition sign error.

Solve linear equations in one variable — Q3
Correct: **(C)**. *Traps:* **(A)** coefficient/division error; **(B)** transposition sign error; **(D)** distribution error.

Solve linear equations in one variable — Q4
Correct: **(D)**. *Traps:* **(A)** coefficient/division error; **(B)** transposition sign error; **(C)** distribution error.

Solve linear equations in one variable — Q5
Correct: **(D)**. *Traps:* **(A)** special-case blindness; **(B)** special-case blindness; **(C)** special-case blindness.

Slope as ratio (rise/run, geometric) — Q1
Correct: **(D)**. *Traps:* **(A)** sign/direction; **(B)** single-point ratio / coordinate-as-slope; **(C)** run/rise inversion (dx/dy).

Slope as ratio (rise/run, geometric) — Q2
Correct: **(C)**. *Traps:* **(A)** sign/direction; **(B)** run/rise inversion (dx/dy); **(D)** single-point ratio / coordinate-as-slope.

Slope as ratio (rise/run, geometric) — Q3
Correct: **(D)**. *Traps:* **(A)** run/rise inversion (dx/dy); **(B)** horizontal/vertical swap; **(C)** single-point ratio / coordinate-as-slope.

Slope as ratio (rise/run, geometric) — Q4
Correct: **(C)**. *Traps:* **(A)** run/rise inversion (dx/dy); **(B)** horizontal/vertical swap; **(D)** single-point ratio / coordinate-as-slope.

Slope as ratio (rise/run, geometric) — Q5
Correct: **(B)**. *Traps:* **(A)** single-point ratio / coordinate-as-slope; **(C)** pair-dependence belief; **(D)** run/rise inversion (dx/dy).

Slope as rate of change (contextual) — Q1
Correct: **(D)**. *Traps:* **(A)** step-size error; **(B)** units inverted/dropped; **(C)** sign/direction.

Slope as rate of change (contextual) — Q2
Correct: **(A)**. *Traps:* **(B)** non-constancy belief; **(C)** role swap; **(D)** non-constancy belief.

Slope as rate of change (contextual) — Q3
Correct: **(C)**. *Traps:* **(A)** total-for-rate; **(B)** total-for-rate; **(D)** units inverted/dropped.

Slope as rate of change (contextual) — Q4
Correct: **(B)**. *Traps:* **(A)** total-for-rate; **(C)** role swap; **(D)** point-as-intercept.

Slope as rate of change (contextual) — Q5
Correct: **(C)**. *Traps:* **(A)** non-constancy belief; **(B)** non-constancy belief; **(D)** non-constancy belief.

Forms of a line (slope-intercept, point-slope, standard) — Q1
Correct: **(A)**. *Traps:* **(B)** intercept confusion; **(C)** intercept-as-slope swap; **(D)** intercept confusion.

Forms of a line (slope-intercept, point-slope, standard) — Q2
Correct: **(B)**. *Traps:* **(A)** standard-form rearrangement error; **(C)** coefficient-as-slope ignoring form; **(D)** standard-form rearrangement error.

Forms of a line (slope-intercept, point-slope, standard) — Q3
Correct: **(B)**. *Traps:* **(A)** coefficient-as-slope ignoring form; **(C)** intercept confusion; **(D)** intercept confusion.

Forms of a line (slope-intercept, point-slope, standard) — Q4
Correct: **(D)**. *Traps:* **(A)** point-slope sign error; **(B)** point-slope sign error; **(C)** point-slope sign error.

Forms of a line (slope-intercept, point-slope, standard) — Q5
Correct: **(A)**. *Traps:* **(B)** standard-form rearrangement error; **(C)** intercept confusion; **(D)** standard-form rearrangement error.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q1
Correct: **(C)**. *Traps:* **(A)** expansion/solve sign error; **(B)** y-value-as-intercept; **(D)** y-value-as-intercept.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q2
Correct: **(D)**. *Traps:* **(A)** y-value-as-intercept; **(B)** run/rise inversion (dx/dy); **(C)** expansion/solve sign error.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q3
Correct: **(A)**. *Traps:* **(B)** expansion/solve sign error; **(C)** y-value-as-intercept; **(D)** expansion/solve sign error.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q4
Correct: **(A)**. *Traps:* **(B)** parallel/perpendicular slope error; **(C)** parallel/perpendicular slope error; **(D)** parallel/perpendicular slope error.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q5
Correct: **(C)**. *Traps:* **(A)** y-value-as-intercept; **(B)** y-value-as-intercept; **(D)** sign/direction.

Interpret slope & intercept of a linear model in context — Q1
Correct: **(A)**. *Traps:* **(B)** rate-as-total; **(C)** coefficient/variable conflation; **(D)** role swap.

Interpret slope & intercept of a linear model in context — Q2
Correct: **(A)**. *Traps:* **(B)** rate-as-total; **(C)** role swap; **(D)** rate-as-total.

Interpret slope & intercept of a linear model in context — Q3
Correct: **(B)**. *Traps:* **(A)** sign/direction; **(C)** role swap; **(D)** units drop.

Interpret slope & intercept of a linear model in context — Q4
Correct: **(D)**. *Traps:* **(A)** units drop; **(B)** units drop; **(C)** role swap.

Interpret slope & intercept of a linear model in context — Q5
Correct: **(A)**. *Traps:* **(B)** coefficient/variable conflation; **(C)** rate-as-total; **(D)** role swap.

Solve linear systems (substitution / elimination) — Q1
Correct: **(B)**. *Traps:* **(A)** incomplete / back-substitution error; **(C)** coefficient/division error; **(D)** substitution scope error.

Solve linear systems (substitution / elimination) — Q2
Correct: **(D)**. *Traps:* **(A)** incomplete / back-substitution error; **(B)** incomplete / back-substitution error; **(C)** elimination sign/scaling error.

Solve linear systems (substitution / elimination) — Q3
Correct: **(A)**. *Traps:* **(B)** incomplete / back-substitution error; **(C)** elimination sign/scaling error; **(D)** incomplete / back-substitution error.

Solve linear systems (substitution / elimination) — Q4
Correct: **(A)**. *Traps:* **(B)** transposition sign error; **(C)** coefficient/division error; **(D)** incomplete / back-substitution error.

Solve linear systems (substitution / elimination) — Q5
Correct: **(B)**. *Traps:* **(A)** solution-count misread; **(C)** solution-count misread; **(D)** solution-count misread.

Systems in context & number of solutions — Q1
Correct: **(B)**. *Traps:* **(A)** translation/setup error; **(C)** translation/setup error; **(D)** translation/setup error.

Systems in context & number of solutions — Q2
Correct: **(B)**. *Traps:* **(A)** contextual answer-interpretation; **(C)** contextual answer-interpretation; **(D)** coefficient/division error.

Systems in context & number of solutions — Q3
Correct: **(C)**. *Traps:* **(A)** solution-count from structure; **(B)** solution-count from structure; **(D)** solution-count from structure.

Systems in context & number of solutions — Q4
Correct: **(B)**. *Traps:* **(A)** translation/setup error; **(C)** transposition sign error; **(D)** contextual answer-interpretation.

Systems in context & number of solutions — Q5
Correct: **(A)**. *Traps:* **(B)** solution-count from structure; **(C)** solution-count from structure; **(D)** solution-count from structure.

Linear inequalities & systems of inequalities — Q1
Correct: **(B)**. *Traps:* **(A)** sign-flip omission; **(C)** transposition sign error; **(D)** coefficient/division error.

Linear inequalities & systems of inequalities — Q2
Correct: **(A)**. *Traps:* **(B)** boundary type error; **(C)** wrong-side shading; **(D)** boundary type error.

Linear inequalities & systems of inequalities — Q3
Correct: **(C)**. *Traps:* **(A)** boundary type error; **(B)** wrong-side shading; **(D)** wrong-side shading.

Linear inequalities & systems of inequalities — Q4
Correct: **(A)**. *Traps:* **(B)** system region error; **(C)** system region error; **(D)** system region error.

Linear inequalities & systems of inequalities — Q5
Correct: **(A)**. *Traps:* **(B)** boundary type error; **(C)** boundary type error; **(D)** boundary type error.

Literal equations & rearranging formulas — Q1
Correct: **(D)**. *Traps:* **(A)** incomplete division; **(B)** inverse/order error; **(C)** inverse/order error.

Literal equations & rearranging formulas — Q2
Correct: **(C)**. *Traps:* **(A)** inverse/order error; **(B)** inverse/order error; **(D)** incomplete division.

Literal equations & rearranging formulas — Q3
Correct: **(B)**. *Traps:* **(A)** fails to factor a repeated variable; **(C)** fails to factor a repeated variable; **(D)** inverse/order error.

Literal equations & rearranging formulas — Q4
Correct: **(D)**. *Traps:* **(A)** incomplete division; **(B)** inverse/order error; **(C)** inverse/order error.

Literal equations & rearranging formulas — Q5
Correct: **(D)**. *Traps:* **(A)** inverse/order error; **(B)** inverse/order error; **(C)** incomplete division.

Build & interpret linear models in context — Q1
Correct: **(D)**. *Traps:* **(A)** rate sign/operation error; **(B)** rate/initial role swap; **(C)** rate sign/operation error.

Build & interpret linear models in context — Q2
Correct: **(A)**. *Traps:* **(B)** rate sign/operation error; **(C)** rate/initial role swap; **(D)** rate/initial role swap.

Build & interpret linear models in context — Q3
Correct: **(A)**. *Traps:* **(B)** inequality direction/type error; **(C)** rate/initial role swap; **(D)** inequality direction/type error.

Build & interpret linear models in context — Q4
Correct: **(D)**. *Traps:* **(A)** rate sign/operation error; **(B)** rate/initial role swap; **(C)** rate sign/operation error.

Build & interpret linear models in context — Q5
Correct: **(D)**. *Traps:* **(A)** rate/initial role swap; **(B)** inequality direction/type error; **(C)** inequality direction/type error.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q1
Correct: **(A)**. *Traps:* **(B)** degenerate-coefficient error; **(C)** degenerate-coefficient error; **(D)** degenerate-coefficient error.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q2
Correct: **(D)**. *Traps:* **(A)** proportionality setup/sign error; **(B)** proportionality setup/sign error; **(C)** proportionality setup/sign error.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q3
Correct: **(D)**. *Traps:* **(A)** proportionality setup/sign error; **(B)** proportionality setup/sign error; **(C)** count-to-condition mapping error.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q4
Correct: **(B)**. *Traps:* **(A)** count-to-condition mapping error; **(C)** count-to-condition mapping error; **(D)** count-to-condition mapping error.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q5
Correct: **(B)**. *Traps:* **(A)** degenerate-coefficient error; **(C)** count-to-condition mapping error; **(D)** count-to-condition mapping error.

Solve a linear system graphically (intersection point) — Q1
Correct: **(A)**. *Traps:* **(B)** coordinate order flip; **(C)** single-line read; **(D)** single-line read.

Solve a linear system graphically (intersection point) — Q2
Correct: **(C)**. *Traps:* **(A)** slope/intercept mismatch; **(B)** parallel-as-solution; **(D)** parallel-as-solution.

Solve a linear system graphically (intersection point) — Q3
Correct: **(D)**. *Traps:* **(A)** coordinate order flip; **(B)** single-line read; **(C)** single-line read.

Solve a linear system graphically (intersection point) — Q4
Correct: **(B)**. *Traps:* **(A)** parallel-as-solution; **(C)** slope/intercept mismatch; **(D)** parallel-as-solution.

Solve a linear system graphically (intersection point) — Q5
Correct: **(D)**. *Traps:* **(A)** slope/intercept mismatch; **(B)** parallel-as-solution; **(C)** slope/intercept mismatch.

Equivalent linear expressions (distribute and combine like terms) — Q1
Correct: **(C)**. *Traps:* **(A)** combines unlike terms; **(B)** coefficient arithmetic slip; **(D)** partial distribution.

Equivalent linear expressions (distribute and combine like terms) — Q2
Correct: **(B)**. *Traps:* **(A)** coefficient arithmetic slip; **(C)** negative-sign drop; **(D)** combines unlike terms.

Equivalent linear expressions (distribute and combine like terms) — Q3
Correct: **(A)**. *Traps:* **(B)** negative-sign drop; **(C)** coefficient arithmetic slip; **(D)** partial distribution.

Equivalent linear expressions (distribute and combine like terms) — Q4
Correct: **(C)**. *Traps:* **(A)** combines unlike terms; **(B)** partial distribution; **(D)** coefficient arithmetic slip.

Equivalent linear expressions (distribute and combine like terms) — Q5
Correct: **(D)**. *Traps:* **(A)** negative-sign drop; **(B)** coefficient arithmetic slip; **(C)** coefficient arithmetic slip.

Equivalent linear expressions by factoring a common factor — Q1
Correct: **(A)**. *Traps:* **(B)** incomplete factoring; **(C)** factor doesn't divide all terms; **(D)** quotient arithmetic slip.

Equivalent linear expressions by factoring a common factor — Q2
Correct: **(D)**. *Traps:* **(A)** quotient arithmetic slip; **(B)** incomplete factoring; **(C)** incomplete factoring.

Equivalent linear expressions by factoring a common factor — Q3
Correct: **(A)**. *Traps:* **(B)** quotient arithmetic slip; **(C)** sign error factoring a negative; **(D)** sign error factoring a negative.

Equivalent linear expressions by factoring a common factor — Q4
Correct: **(A)**. *Traps:* **(B)** incomplete factoring; **(C)** quotient arithmetic slip; **(D)** sign error factoring a negative.

Equivalent linear expressions by factoring a common factor — Q5
Correct: **(C)**. *Traps:* **(A)** incomplete factoring; **(B)** factor doesn't divide all terms; **(D)** quotient arithmetic slip.

Absolute-value equations & functions — Q1
Correct: **(D)**. *Traps:* **(A)** dropped-negative-case error; **(B)** dropped-negative-case error; **(C)** dropped-negative-case error.

Absolute-value equations & functions — Q2
Correct: **(D)**. *Traps:* **(A)** dropped-negative-case error; **(B)** dropped-negative-case error; **(C)** dropped-negative-case error.

Absolute-value equations & functions — Q3
Correct: **(A)**. *Traps:* **(B)** negative-RHS blindness; **(C)** negative-RHS blindness; **(D)** negative-RHS blindness.

Absolute-value equations & functions — Q4
Correct: **(A)**. *Traps:* **(B)** vertex/graph sign error; **(C)** vertex/graph sign error; **(D)** vertex/graph sign error.

Absolute-value equations & functions — Q5
Correct: **(D)**. *Traps:* **(A)** inequality and/or error; **(B)** inequality and/or error; **(C)** inequality and/or error.

Absolute-value equations with a coefficient — Q1
Correct: **(A)**. *Traps:* **(B)** dropped the negative case; **(C)** sign-flipped both; **(D)** ignored the coefficient.

Absolute-value equations with a coefficient — Q2
Correct: **(D)**. *Traps:* **(A)** dropped the negative case; **(B)** sign-flipped both; **(C)** ignored the coefficient.

Absolute-value equations with a coefficient — Q3
Correct: **(A)**. *Traps:* **(B)** ignored the coefficient; **(C)** sign-flipped both; **(D)** dropped the negative case.

Absolute-value equations with a coefficient — Q4
Correct: **(A)**. *Traps:* **(B)** sign-flipped both; **(C)** dropped the negative case; **(D)** ignored the coefficient.

Absolute-value equations with a coefficient — Q5
Correct: **(B)**. *Traps:* **(A)** dropped the negative case; **(C)** ignored the coefficient; **(D)** sign-flipped both.

Absolute-value inequalities: less-than (between) — Q1
Correct: **(C)**. *Traps:* **(A)** one-sided; **(B)** used outside form; **(D)** ignored the shift.

Absolute-value inequalities: less-than (between) — Q2
Correct: **(D)**. *Traps:* **(A)** used outside form; **(B)** ignored the shift; **(C)** one-sided.

Absolute-value inequalities: less-than (between) — Q3
Correct: **(D)**. *Traps:* **(A)** one-sided; **(B)** ignored the shift; **(C)** used outside form.

Absolute-value inequalities: less-than (between) — Q4
Correct: **(B)**. *Traps:* **(A)** ignored the shift; **(C)** one-sided; **(D)** used outside form.

Absolute-value inequalities: less-than (between) — Q5
Correct: **(A)**. *Traps:* **(B)** one-sided; **(C)** ignored the shift; **(D)** used outside form.

Absolute-value inequalities: greater-than (outside) — Q1
Correct: **(A)**. *Traps:* **(B)** ignored the shift; **(C)** used between form; **(D)** one piece only.

Absolute-value inequalities: greater-than (outside) — Q2
Correct: **(D)**. *Traps:* **(A)** one piece only; **(B)** used between form; **(C)** ignored the shift.

Absolute-value inequalities: greater-than (outside) — Q3
Correct: **(C)**. *Traps:* **(A)** one piece only; **(B)** used between form; **(D)** ignored the shift.

Absolute-value inequalities: greater-than (outside) — Q4
Correct: **(B)**. *Traps:* **(A)** one piece only; **(C)** used between form; **(D)** ignored the shift.

Absolute-value inequalities: greater-than (outside) — Q5
Correct: **(C)**. *Traps:* **(A)** used between form; **(B)** one piece only; **(D)** ignored the shift.

Graphs of absolute-value functions (vertex form) — Q1
Correct: **(C)**. *Traps:* **(A)** x-sign error; **(B)** both signs wrong; **(D)** y-sign error.

Graphs of absolute-value functions (vertex form) — Q2
Correct: **(A)**. *Traps:* **(B)** both signs wrong; **(C)** x-sign error; **(D)** y-sign error.

Graphs of absolute-value functions (vertex form) — Q3
Correct: **(B)**. *Traps:* **(A)** both signs wrong; **(C)** x-sign error; **(D)** y-sign error.

Graphs of absolute-value functions (vertex form) — Q4
Correct: **(C)**. *Traps:* **(A)** x-sign error; **(B)** y-sign error; **(D)** both signs wrong.

Graphs of absolute-value functions (vertex form) — Q5
Correct: **(B)**. *Traps:* **(A)** both signs wrong; **(C)** y-sign error; **(D)** x-sign error.

Advanced Math

Evaluate & interpret function notation f(x) — Q1
Correct: **(B)**. *Traps:* **(A)** notation-as-multiplication; **(C)** notation-as-multiplication; **(D)** input/output swap.

Evaluate & interpret function notation f(x) — Q2
Correct: **(C)**. *Traps:* **(A)** transposition sign error; **(B)** notation-as-multiplication; **(D)** transposition sign error.

Evaluate & interpret function notation f(x) — Q3
Correct: **(A)**. *Traps:* **(B)** input/output swap; **(C)** input/output swap; **(D)** input/output swap.

Evaluate & interpret function notation f(x) — Q4
Correct: **(A)**. *Traps:* **(B)** transformation-vs-evaluation; **(C)** notation-as-multiplication; **(D)** distribution error.

Evaluate & interpret function notation f(x) — Q5
Correct: **(A)**. *Traps:* **(B)** input/output swap; **(C)** notation-as-multiplication; **(D)** input/output swap.

Expand binomials / FOIL — Q1
Correct: **(A)**. *Traps:* **(B)** sum/product confusion; **(C)** sum/product confusion; **(D)** missing cross terms (incomplete FOIL).

Expand binomials / FOIL — Q2
Correct: **(B)**. *Traps:* **(A)** missing cross terms (incomplete FOIL); **(C)** sign error in products; **(D)** sign error in products.

Expand binomials / FOIL — Q3
Correct: **(D)**. *Traps:* **(A)** missing cross terms (incomplete FOIL); **(B)** perfect-square doubling error; **(C)** sum/product confusion.

Expand binomials / FOIL — Q4
Correct: **(A)**. *Traps:* **(B)** sign error in products; **(C)** sign error in products; **(D)** distribution error.

Expand binomials / FOIL — Q5
Correct: **(C)**. *Traps:* **(A)** perfect-square doubling error; **(B)** sign error in products; **(D)** sign error in products.

Factor quadratics — Q1
Correct: **(D)**. *Traps:* **(A)** sign error in factors; **(B)** sum/product factor-pair error; **(C)** sum/product factor-pair error.

Factor quadratics — Q2
Correct: **(B)**. *Traps:* **(A)** sum/product factor-pair error; **(C)** sum/product factor-pair error; **(D)** sign error in factors.

Factor quadratics — Q3
Correct: **(D)**. *Traps:* **(A)** leading-coefficient/common-factor mishandling; **(B)** leading-coefficient/common-factor mishandling; **(C)** sign error in factors.

Factor quadratics — Q4
Correct: **(D)**. *Traps:* **(A)** special-form error; **(B)** sum/product factor-pair error; **(C)** special-form error.

Factor quadratics — Q5
Correct: **(C)**. *Traps:* **(A)** leading-coefficient/common-factor mishandling; **(B)** special-form error; **(D)** special-form error.

Solve quadratics by factoring & the zero-product property — Q1
Correct: **(C)**. *Traps:* **(A)** transposition sign error; **(B)** sum/product factor-pair error; **(D)** dropped or miscounted root.

Solve quadratics by factoring & the zero-product property — Q2
Correct: **(D)**. *Traps:* **(A)** not-set-to-zero; **(B)** not-set-to-zero; **(C)** transposition sign error.

Solve quadratics by factoring & the zero-product property — Q3
Correct: **(D)**. *Traps:* **(A)** divide-by-variable; **(B)** dropped or miscounted root; **(C)** transposition sign error.

Solve quadratics by factoring & the zero-product property — Q4
Correct: **(C)**. *Traps:* **(A)** transposition sign error; **(B)** dropped or miscounted root; **(D)** coefficient/division error.

Solve quadratics by factoring & the zero-product property — Q5
Correct: **(A)**. *Traps:* **(B)** sum/product factor-pair error; **(C)** transposition sign error; **(D)** dropped or miscounted root.

Quadratic formula & discriminant (number of real solutions) — Q1
Correct: **(B)**. *Traps:* **(A)** formula structure error; **(C)** formula substitution sign error; **(D)** formula structure error.

Quadratic formula & discriminant (number of real solutions) — Q2
Correct: **(B)**. *Traps:* **(A)** discriminant solution-count misread; **(C)** discriminant solution-count misread; **(D)** discriminant solution-count misread.

Quadratic formula & discriminant (number of real solutions) — Q3
Correct: **(C)**. *Traps:* **(A)** discriminant solution-count misread; **(B)** discriminant solution-count misread; **(D)** discriminant solution-count misread.

Quadratic formula & discriminant (number of real solutions) — Q4
Correct: **(C)**. *Traps:* **(A)** formula structure error; **(B)** formula substitution sign error; **(D)** formula substitution sign error.

Quadratic formula & discriminant (number of real solutions) — Q5
Correct: **(B)**. *Traps:* **(A)** discriminant solution-count misread; **(C)** formula structure error; **(D)** coefficient/division error.

Forms of a quadratic & the parabola — Q1
Correct: **(D)**. *Traps:* **(A)** vertex-form h sign; **(B)** feature-from-form confusion; **(C)** feature-from-form confusion.

Forms of a quadratic & the parabola — Q2
Correct: **(B)**. *Traps:* **(A)** feature-from-form confusion; **(C)** axis from standard form; **(D)** axis from standard form.

Forms of a quadratic & the parabola — Q3
Correct: **(B)**. *Traps:* **(A)** direction/extremum confusion; **(C)** direction/extremum confusion; **(D)** feature-from-form confusion.

Forms of a quadratic & the parabola — Q4
Correct: **(A)**. *Traps:* **(B)** dropped or miscounted root; **(C)** transposition sign error; **(D)** feature-from-form confusion.

Forms of a quadratic & the parabola — Q5
Correct: **(B)**. *Traps:* **(A)** feature-from-form confusion; **(C)** discriminant solution-count misread; **(D)** discriminant solution-count misread.

Interpret a quadratic model in context — Q1
Correct: **(C)**. *Traps:* **(A)** extremum interpretation; **(B)** answer-interpretation/units; **(D)** extremum interpretation.

Interpret a quadratic model in context — Q2
Correct: **(A)**. *Traps:* **(B)** answer-interpretation/units; **(C)** extremum interpretation; **(D)** zero-in-context meaning.

Interpret a quadratic model in context — Q3
Correct: **(A)**. *Traps:* **(B)** zero-in-context meaning; **(C)** zero-in-context meaning; **(D)** zero-in-context meaning.

Interpret a quadratic model in context — Q4
Correct: **(C)**. *Traps:* **(A)** feature-from-form confusion; **(B)** extremum interpretation; **(D)** answer-interpretation/units.

Interpret a quadratic model in context — Q5
Correct: **(A)**. *Traps:* **(B)** extremum interpretation; **(C)** extremum interpretation; **(D)** zero-in-context meaning.

Completing the square — Q1
Correct: **(A)**. *Traps:* **(B)** half-then-square error; **(C)** half-then-square error; **(D)** half-then-square error.

Completing the square — Q2
Correct: **(A)**. *Traps:* **(B)** balance error; **(C)** vertex-sign/form error; **(D)** balance error.

Completing the square — Q3
Correct: **(C)**. *Traps:* **(A)** vertex-sign/form error; **(B)** vertex-sign/form error; **(D)** vertex-sign/form error.

Completing the square — Q4
Correct: **(D)**. *Traps:* **(A)** leading-coefficient error; **(B)** leading-coefficient error; **(C)** leading-coefficient error.

Completing the square — Q5
Correct: **(C)**. *Traps:* **(A)** half-then-square error; **(B)** balance error; **(D)** vertex-sign/form error.

Composite functions & evaluating compositions — Q1
Correct: **(C)**. *Traps:* **(A)** table/graph lookup error; **(B)** table/graph lookup error; **(D)** order/inside-out error.

Composite functions & evaluating compositions — Q2
Correct: **(C)**. *Traps:* **(A)** composition-as-multiplication error; **(B)** composition-as-multiplication error; **(D)** order/inside-out error.

Composite functions & evaluating compositions — Q3
Correct: **(C)**. *Traps:* **(A)** table/graph lookup error; **(B)** table/graph lookup error; **(D)** table/graph lookup error.

Composite functions & evaluating compositions — Q4
Correct: **(B)**. *Traps:* **(A)** commutativity error; **(C)** commutativity error; **(D)** commutativity error.

Composite functions & evaluating compositions — Q5
Correct: **(A)**. *Traps:* **(B)** composition-as-multiplication error; **(C)** table/graph lookup error; **(D)** order/inside-out error.

Domain & range of functions — Q1
Correct: **(A)**. *Traps:* **(B)** restriction-source error; **(C)** restriction-source error; **(D)** restriction-source error.

Domain & range of functions — Q2
Correct: **(D)**. *Traps:* **(A)** domain/range swap; **(B)** range-from-vertex error; **(C)** domain/range swap.

Domain & range of functions — Q3
Correct: **(B)**. *Traps:* **(A)** range-from-vertex error; **(C)** range-from-vertex error; **(D)** range-from-vertex error.

Domain & range of functions — Q4
Correct: **(D)**. *Traps:* **(A)** restriction-source error; **(B)** restriction-source error; **(C)** restriction-source error.

Domain & range of functions — Q5
Correct: **(C)**. *Traps:* **(A)** graph-reading extent error; **(B)** domain/range swap; **(D)** domain/range swap.

Piecewise-defined functions — Q1
Correct: **(A)**. *Traps:* **(B)** evaluate-all-pieces error; **(C)** wrong-branch selection; **(D)** boundary/endpoint error.

Piecewise-defined functions — Q2
Correct: **(A)**. *Traps:* **(B)** wrong-branch selection; **(C)** wrong-branch selection; **(D)** evaluate-all-pieces error.

Piecewise-defined functions — Q3
Correct: **(B)**. *Traps:* **(A)** boundary/endpoint error; **(C)** boundary/endpoint error; **(D)** boundary/endpoint error.

Piecewise-defined functions — Q4
Correct: **(D)**. *Traps:* **(A)** interval-misread error; **(B)** evaluate-all-pieces error; **(C)** interval-misread error.

Piecewise-defined functions — Q5
Correct: **(C)**. *Traps:* **(A)** wrong-branch selection; **(B)** evaluate-all-pieces error; **(D)** wrong-branch selection.

Polynomial arithmetic & structure — Q1
Correct: **(C)**. *Traps:* **(A)** combining unlike terms; **(B)** combining unlike terms; **(D)** subtraction sign distribution.

Polynomial arithmetic & structure — Q2
Correct: **(D)**. *Traps:* **(A)** subtraction sign distribution; **(B)** subtraction sign distribution; **(C)** subtraction sign distribution.

Polynomial arithmetic & structure — Q3
Correct: **(D)**. *Traps:* **(A)** multiplication/exponent error; **(B)** multiplication/exponent error; **(C)** multiplication/exponent error.

Polynomial arithmetic & structure — Q4
Correct: **(B)**. *Traps:* **(A)** degree/leading-coefficient error; **(C)** degree/leading-coefficient error; **(D)** degree/leading-coefficient error.

Polynomial arithmetic & structure — Q5
Correct: **(A)**. *Traps:* **(B)** subtraction sign distribution; **(C)** multiplication/exponent error; **(D)** combining unlike terms.

Zeros, factors & the factor theorem — Q1
Correct: **(A)**. *Traps:* **(B)** factor/zero sign error; **(C)** factor/zero sign error; **(D)** factor/zero sign error.

Zeros, factors & the factor theorem — Q2
Correct: **(C)**. *Traps:* **(A)** factor-theorem meaning error; **(B)** factor/zero sign error; **(D)** factor-theorem meaning error.

Zeros, factors & the factor theorem — Q3
Correct: **(A)**. *Traps:* **(B)** factor-theorem meaning error; **(C)** factor/zero sign error; **(D)** factor-theorem meaning error.

Zeros, factors & the factor theorem — Q4
Correct: **(D)**. *Traps:* **(A)** number-of-zeros error; **(B)** number-of-zeros error; **(C)** number-of-zeros error.

Zeros, factors & the factor theorem — Q5
Correct: **(D)**. *Traps:* **(A)** factor/zero sign error; **(B)** number-of-zeros error; **(C)** factor/zero sign error.

Polynomial graphs: end behavior, zeros, multiplicity — Q1
Correct: **(D)**. *Traps:* **(A)** end-behavior error; **(B)** end-behavior error; **(C)** end-behavior error.

Polynomial graphs: end behavior, zeros, multiplicity — Q2
Correct: **(C)**. *Traps:* **(A)** zeros-as-x-intercepts error; **(B)** zeros-as-x-intercepts error; **(D)** zeros-as-x-intercepts error.

Polynomial graphs: end behavior, zeros, multiplicity — Q3
Correct: **(A)**. *Traps:* **(B)** multiplicity error; **(C)** multiplicity error; **(D)** multiplicity error.

Polynomial graphs: end behavior, zeros, multiplicity — Q4
Correct: **(A)**. *Traps:* **(B)** end-behavior error; **(C)** end-behavior error; **(D)** end-behavior error.

Polynomial graphs: end behavior, zeros, multiplicity — Q5
Correct: **(B)**. *Traps:* **(A)** degree-vs-zero-count error; **(C)** degree-vs-zero-count error; **(D)** degree-vs-zero-count error.

Polynomial division & the remainder theorem — Q1
Correct: **(D)**. *Traps:* **(A)** remainder-theorem sign error; **(B)** remainder-theorem sign error; **(C)** remainder-theorem sign error.

Polynomial division & the remainder theorem — Q2
Correct: **(A)**. *Traps:* **(B)** remainder-vs-factor direction error; **(C)** remainder-vs-factor direction error; **(D)** remainder-vs-factor direction error.

Polynomial division & the remainder theorem — Q3
Correct: **(C)**. *Traps:* **(A)** division mechanics error; **(B)** division mechanics error; **(D)** division mechanics error.

Polynomial division & the remainder theorem — Q4
Correct: **(B)**. *Traps:* **(A)** nonzero-remainder interpretation error; **(C)** nonzero-remainder interpretation error; **(D)** nonzero-remainder interpretation error.

Polynomial division & the remainder theorem — Q5
Correct: **(D)**. *Traps:* **(A)** remainder-theorem sign error; **(B)** remainder-vs-factor direction error; **(C)** remainder-vs-factor direction error.

Synthetic division & the quotient — Q1
Correct: **(B)**. *Traps:* **(A)** used the divisor; **(C)** used the coefficient; **(D)** sign error.

Synthetic division & the quotient — Q2
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the divisor; **(D)** used the coefficient.

Synthetic division & the quotient — Q3
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the coefficient; **(D)** used the divisor.

Synthetic division & the quotient — Q4
Correct: **(C)**. *Traps:* **(A)** sign error; **(B)** used the divisor; **(D)** used the coefficient.

Synthetic division & the quotient — Q5
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the divisor; **(D)** used the coefficient.

Binomial expansion (perfect squares) — Q1
Correct: **(D)**. *Traps:* **(A)** did not square the constant; **(B)** dropped the middle term; **(C)** did not double.

Binomial expansion (perfect squares) — Q2
Correct: **(D)**. *Traps:* **(A)** dropped the middle term; **(B)** did not square the constant; **(C)** did not double.

Binomial expansion (perfect squares) — Q3
Correct: **(D)**. *Traps:* **(A)** did not double; **(B)** did not square the constant; **(C)** dropped the middle term.

Binomial expansion (perfect squares) — Q4
Correct: **(D)**. *Traps:* **(A)** did not square the constant; **(B)** did not double; **(C)** dropped the middle term.

Binomial expansion (perfect squares) — Q5
Correct: **(C)**. *Traps:* **(A)** did not double; **(B)** dropped the middle term; **(D)** did not square the constant.

Special products & factoring patterns — Q1
Correct: **(C)**. *Traps:* **(A)** kept a middle term; **(B)** did not square k; **(D)** wrong sign.

Special products & factoring patterns — Q2
Correct: **(B)**. *Traps:* **(A)** wrong sign; **(C)** did not square k; **(D)** kept a middle term.

Special products & factoring patterns — Q3
Correct: **(B)**. *Traps:* **(A)** wrong sign; **(C)** kept a middle term; **(D)** did not square k.

Special products & factoring patterns — Q4
Correct: **(D)**. *Traps:* **(A)** did not square k; **(B)** wrong sign; **(C)** kept a middle term.

Special products & factoring patterns — Q5
Correct: **(B)**. *Traps:* **(A)** did not square k; **(C)** kept a middle term; **(D)** wrong sign.

Sum & product of roots — Q1
Correct: **(A)**. *Traps:* **(B)** swapped sum & product; **(C)** product sign error; **(D)** sum sign error.

Sum & product of roots — Q2
Correct: **(D)**. *Traps:* **(A)** swapped sum & product; **(B)** product sign error; **(C)** sum sign error.

Sum & product of roots — Q3
Correct: **(D)**. *Traps:* **(A)** swapped sum & product; **(B)** product sign error; **(C)** sum sign error.

Sum & product of roots — Q4
Correct: **(A)**. *Traps:* **(B)** swapped sum & product; **(C)** sum sign error; **(D)** product sign error.

Sum & product of roots — Q5
Correct: **(B)**. *Traps:* **(A)** sum sign error; **(C)** product sign error; **(D)** swapped sum & product.

Exponent rules & properties — Q1
Correct: **(D)**. *Traps:* **(A)** add/multiply exponent confusion; **(B)** coefficient/distribution error; **(C)** add/multiply exponent confusion.

Exponent rules & properties — Q2
Correct: **(B)**. *Traps:* **(A)** add/multiply exponent confusion; **(C)** add/multiply exponent confusion; **(D)** coefficient/distribution error.

Exponent rules & properties — Q3
Correct: **(C)**. *Traps:* **(A)** negative-exponent error; **(B)** negative-exponent error; **(D)** negative-exponent error.

Exponent rules & properties — Q4
Correct: **(B)**. *Traps:* **(A)** zero/fractional-exponent error; **(C)** zero/fractional-exponent error; **(D)** zero/fractional-exponent error.

Exponent rules & properties — Q5
Correct: **(A)**. *Traps:* **(B)** add/multiply exponent confusion; **(C)** coefficient/distribution error; **(D)** coefficient/distribution error.

Scientific notation — Q1
Correct: **(B)**. *Traps:* **(A)** decimal place/count error; **(C)** exponent-sign error; **(D)** decimal place/count error.

Scientific notation — Q2
Correct: **(A)**. *Traps:* **(B)** exponent-sign error; **(C)** normalization error; **(D)** decimal place/count error.

Scientific notation — Q3
Correct: **(D)**. *Traps:* **(A)** operation exponent error; **(B)** operation exponent error; **(C)** operation exponent error.

Scientific notation — Q4
Correct: **(A)**. *Traps:* **(B)** normalization error; **(C)** normalization error; **(D)** normalization error.

Scientific notation — Q5
Correct: **(C)**. *Traps:* **(A)** decimal place/count error; **(B)** operation exponent error; **(D)** operation exponent error.

Exponential functions ($y = a*b^x$) — Q1
Correct: **(A)**. *Traps:* **(B)** base/coefficient role confusion; **(C)** evaluation error; **(D)** linear/exponential confusion.

Exponential functions ($y = a*b^x$) — Q2
Correct: **(D)**. *Traps:* **(A)** growth/decay direction error; **(B)** growth/decay direction error; **(C)** growth/decay direction error.

Exponential functions ($y = a*b^x$) — Q3
Correct: **(A)**. *Traps:* **(B)** evaluation error; **(C)** evaluation error; **(D)** linear/exponential confusion.

Exponential functions ($y = a*b^x$) — Q4
Correct: **(B)**. *Traps:* **(A)** growth/decay direction error; **(C)** linear/exponential confusion; **(D)** linear/exponential confusion.

Exponential functions ($y = a*b^x$) — Q5
Correct: **(D)**. *Traps:* **(A)** base/coefficient role confusion; **(B)** linear/exponential confusion; **(C)** base/coefficient role confusion.

Exponential growth & decay in context — Q1
Correct: **(B)**. *Traps:* **(A)** growth/decay direction error; **(C)** rate-to-base error; **(D)** rate-to-base error.

Exponential growth & decay in context — Q2
Correct: **(A)**. *Traps:* **(B)** simple vs compound error; **(C)** growth/decay direction error; **(D)** rate-to-base error.

Exponential growth & decay in context — Q3
Correct: **(B)**. *Traps:* **(A)** simple vs compound error; **(C)** rate-to-base error; **(D)** simple vs compound error.

Exponential growth & decay in context — Q4
Correct: **(D)**. *Traps:* **(A)** simple vs compound error; **(B)** simple vs compound error; **(C)** rate-to-base error.

Exponential growth & decay in context — Q5
Correct: **(A)**. *Traps:* **(B)** rate-to-base error; **(C)** simple vs compound error; **(D)** rate-to-base error.

Solving exponential equations by matching bases — Q1
Correct: **(A)**. *Traps:* **(B)** exponential-as-linear error; **(C)** exponential-as-linear error; **(D)** exponential-as-linear error.

Solving exponential equations by matching bases — Q2
Correct: **(C)**. *Traps:* **(A)** exponential-as-linear error; **(B)** rewriting error; **(D)** rewriting error.

Solving exponential equations by matching bases — Q3
Correct: **(B)**. *Traps:* **(A)** base-matching failure; **(C)** base-matching failure; **(D)** base-matching failure.

Solving exponential equations by matching bases — Q4
Correct: **(A)**. *Traps:* **(B)** negative/fractional-exponent error; **(C)** negative/fractional-exponent error; **(D)** negative/fractional-exponent error.

Solving exponential equations by matching bases — Q5
Correct: **(C)**. *Traps:* **(A)** base-matching failure; **(B)** base-matching failure; **(D)** base-matching failure.

Graphs of exponential functions — Q1
Correct: **(A)**. *Traps:* **(B)** used the base; **(C)** multiplied a·b; **(D)** assumed origin.

Graphs of exponential functions — Q2
Correct: **(B)**. *Traps:* **(A)** multiplied a·b; **(C)** assumed origin; **(D)** used the base.

Graphs of exponential functions — Q3
Correct: **(D)**. *Traps:* **(A)** multiplied a·b; **(B)** assumed origin; **(C)** used the base.

Graphs of exponential functions — Q4
Correct: **(A)**. *Traps:* **(B)** used the base; **(C)** multiplied a·b; **(D)** assumed origin.

Graphs of exponential functions — Q5
Correct: **(C)**. *Traps:* **(A)** assumed origin; **(B)** used the base; **(D)** multiplied a·b.

Compound interest — Q1
Correct: **(C)**. *Traps:* **(A)** percent not converted; **(B)** simple interest; **(D)** dropped the 1 +.

Compound interest — Q2
Correct: **(C)**. *Traps:* **(A)** percent not converted; **(B)** simple interest; **(D)** dropped the 1 +.

Compound interest — Q3
Correct: **(A)**. *Traps:* **(B)** simple interest; **(C)** dropped the 1 +; **(D)** percent not converted.

Compound interest — Q4
Correct: **(D)**. *Traps:* **(A)** simple interest; **(B)** percent not converted; **(C)** dropped the 1 +.

Compound interest — Q5
Correct: **(D)**. *Traps:* **(A)** dropped the 1 +; **(B)** percent not converted; **(C)** simple interest.

Half-life & doubling time — Q1
Correct: **(C)**. *Traps:* **(A)** multiplied in the exponent; **(B)** linear not exponential; **(D)** growth instead of decay.

Half-life & doubling time — Q2
Correct: **(B)**. *Traps:* **(A)** linear not exponential; **(C)** multiplied in the exponent; **(D)** growth instead of decay.

Half-life & doubling time — Q3
Correct: **(D)**. *Traps:* **(A)** multiplied in the exponent; **(B)** growth instead of decay; **(C)** linear not exponential.

Half-life & doubling time — Q4
Correct: **(A)**. *Traps:* **(B)** multiplied in the exponent; **(C)** growth instead of decay; **(D)** linear not exponential.

Half-life & doubling time — Q5
Correct: **(C)**. *Traps:* **(A)** growth instead of decay; **(B)** linear not exponential; **(D)** multiplied in the exponent.

Radicals & rational exponents (simplify & convert) — Q1
Correct: **(C)**. *Traps:* **(A)** rational-exponent conversion error; **(B)** rational-exponent conversion error; **(D)** rational-exponent conversion error.

Radicals & rational exponents (simplify & convert) — Q2
Correct: **(C)**. *Traps:* **(A)** radical-of-sum error; **(B)** radical-of-sum error; **(D)** simplification error.

Radicals & rational exponents (simplify & convert) — Q3
Correct: **(B)**. *Traps:* **(A)** simplification error; **(C)** simplification error; **(D)** simplification error.

Radicals & rational exponents (simplify & convert) — Q4
Correct: **(B)**. *Traps:* **(A)** rational-exponent conversion error; **(C)** negative rational exponent error; **(D)** negative rational exponent error.

Radicals & rational exponents (simplify & convert) — Q5
Correct: **(C)**. *Traps:* **(A)** negative rational exponent error; **(B)** rational-exponent conversion error; **(D)** rational-exponent conversion error.

Radical & rational equations (solving with extraneous checks) — Q1
Correct: **(B)**. *Traps:* **(A)** extraneous-root acceptance; **(C)** extraneous-root acceptance; **(D)** extraneous-root acceptance.

Radical & rational equations (solving with extraneous checks) — Q2
Correct: **(D)**. *Traps:* **(A)** squaring/expansion error; **(B)** squaring/expansion error; **(C)** squaring/-expansion error.

Radical & rational equations (solving with extraneous checks) — Q3
Correct: **(C)**. *Traps:* **(A)** excluded-value error; **(B)** excluded-value error; **(D)** excluded-value error.

Radical & rational equations (solving with extraneous checks) — Q4
Correct: **(D)**. *Traps:* **(A)** squaring/expansion error; **(B)** extraneous-root acceptance; **(C)** extraneous-root acceptance.

Radical & rational equations (solving with extraneous checks) — Q5
Correct: **(A)**. *Traps:* **(B)** excluded-value error; **(C)** squaring/expansion error; **(D)** extraneous-root acceptance.

Simplify radicals (perfect-square factors) — Q1
Correct: **(B)**. *Traps:* **(A)** radical mis-handling; **(C)** incomplete simplification; **(D)** radical mis-handling.

Simplify radicals (perfect-square factors) — Q2
Correct: **(C)**. *Traps:* **(A)** radical mis-handling; **(B)** incomplete simplification; **(D)** radical mis-handling.

Simplify radicals (perfect-square factors) — Q3
Correct: **(C)**. *Traps:* **(A)** incomplete simplification; **(B)** radical mis-handling; **(D)** radical mis-handling.

Simplify radicals (perfect-square factors) — Q4
Correct: **(D)**. *Traps:* **(A)** incomplete simplification; **(B)** radical mis-handling; **(C)** radical mis-handling.

Simplify radicals (perfect-square factors) — Q5
Correct: **(C)**. *Traps:* **(A)** incomplete simplification; **(B)** radical mis-handling; **(D)** left the square under the root.

Add & subtract like radicals — Q1
Correct: **(C)**. *Traps:* **(A)** false 'cannot combine'; **(B)** multiplied the coefficients; **(D)** added the radicands.

Add & subtract like radicals — Q2
Correct: **(C)**. *Traps:* **(A)** multiplied the coefficients; **(B)** false 'cannot combine'; **(D)** added the radicands.

Add & subtract like radicals — Q3
Correct: **(A)**. *Traps:* **(B)** false 'cannot combine'; **(C)** added the radicands; **(D)** multiplied the coefficients.

Add & subtract like radicals — Q4
Correct: **(C)**. *Traps:* **(A)** added the radicands; **(B)** false 'cannot combine'; **(D)** multiplied the coefficients.

Add & subtract like radicals — Q5
Correct: **(D)**. *Traps:* **(A)** added the radicands; **(B)** false 'cannot combine'; **(C)** multiplied the coefficients.

Multiply radicals & rationalize denominators — Q1
Correct: **(C)**. *Traps:* **(A)** radical lost or misplaced; **(B)** dropped the denominator; **(D)** radical lost or misplaced.

Multiply radicals & rationalize denominators — Q2
Correct: **(B)**. *Traps:* **(A)** radical lost or misplaced; **(C)** radical lost or misplaced; **(D)** dropped the denominator.

Multiply radicals & rationalize denominators — Q3
Correct: **(A)**. *Traps:* **(B)** radical lost or misplaced; **(C)** dropped the denominator; **(D)** radical lost or misplaced.

Multiply radicals & rationalize denominators — Q4
Correct: **(D)**. *Traps:* **(A)** radical lost or misplaced; **(B)** radical lost or misplaced; **(C)** dropped the denominator.

Multiply radicals & rationalize denominators — Q5
Correct: **(D)**. *Traps:* **(A)** left the radical in the denominator; **(B)** dropped the denominator; **(C)** radical lost or misplaced.

nth roots & cube roots — Q1
Correct: **(D)**. *Traps:* **(A)** wrong root/operation; **(B)** wrong root/operation; **(C)** multiplied by the index.

nth roots & cube roots — Q2
Correct: **(D)**. *Traps:* **(A)** multiplied by the index; **(B)** wrong root/operation; **(C)** wrong root/operation.

nth roots & cube roots — Q3
Correct: **(A)**. *Traps:* **(B)** wrong root/operation; **(C)** wrong root/operation; **(D)** multiplied by the index.

nth roots & cube roots — Q4
Correct: **(B)**. *Traps:* **(A)** multiplied by the index; **(C)** divided by the index; **(D)** wrong root/operation.

nth roots & cube roots — Q5
Correct: **(C)**. *Traps:* **(A)** multiplied by the index; **(B)** divided by the index; **(D)** wrong root/operation.

Operations with rational exponents — Q1
Correct: **(B)**. *Traps:* **(A)** skipped the power; **(C)** skipped the root; **(D)** skipped the root.

Operations with rational exponents — Q2
Correct: **(C)**. *Traps:* **(A)** skipped the root; **(B)** skipped the root; **(D)** skipped the power.

Operations with rational exponents — Q3
Correct: **(A)**. *Traps:* **(B)** skipped the root; **(C)** skipped the power; **(D)** skipped the root.

Operations with rational exponents — Q4
Correct: **(C)**. *Traps:* **(A)** skipped the root; **(B)** skipped the power; **(D)** skipped the root.

Operations with rational exponents — Q5
Correct: **(D)**. *Traps:* **(A)** skipped the root; **(B)** used the exponent as a plain multiplier; **(C)** skipped the power.

Radical functions: domain & graphs — Q1
Correct: **(B)**. *Traps:* **(A)** ignored the restriction; **(C)** sign error on a; **(D)** flipped the inequality.

Radical functions: domain & graphs — Q2
Correct: **(A)**. *Traps:* **(B)** ignored the restriction; **(C)** sign error on a; **(D)** flipped the inequality.

Radical functions: domain & graphs — Q3
Correct: **(C)**. *Traps:* **(A)** ignored the restriction; **(B)** flipped the inequality; **(D)** sign error on a.

Radical functions: domain & graphs — Q4
Correct: **(C)**. *Traps:* **(A)** sign error on a; **(B)** ignored the restriction; **(D)** flipped the inequality.

Radical functions: domain & graphs — Q5
Correct: **(B)**. *Traps:* **(A)** flipped the inequality; **(C)** ignored the restriction; **(D)** sign error on a.

Rational expressions: simplify & operate — Q1
Correct: **(B)**. *Traps:* **(A)** cancel-terms-not-factors error; **(C)** cancel-terms-not-factors error; **(D)** cancel-terms-not-factors error.

Rational expressions: simplify & operate — Q2
Correct: **(D)**. *Traps:* **(A)** no-common-denominator error; **(B)** no-common-denominator error; **(C)** no-common-denominator error.

Rational expressions: simplify & operate — Q3
Correct: **(C)**. *Traps:* **(A)** division/reciprocal error; **(B)** division/reciprocal error; **(D)** division/reciprocal error.

Rational expressions: simplify & operate — Q4
Correct: **(C)**. *Traps:* **(A)** domain/excluded-value error; **(B)** domain/excluded-value error; **(D)** domain/excluded-value error.

Rational expressions: simplify & operate — Q5
Correct: **(D)**. *Traps:* **(A)** domain/excluded-value error; **(B)** domain/excluded-value error; **(C)** cancel-terms-not-factors error.

Multiply & divide rational expressions — Q1
Correct: **(D)**. *Traps:* **(A)** cancelled then multiplied; **(B)** inverted the result; **(C)** multiplied without flipping.

Multiply & divide rational expressions — Q2
Correct: **(A)**. *Traps:* **(B)** inverted the result; **(C)** cancelled then multiplied; **(D)** multiplied without flipping.

Multiply & divide rational expressions — Q3
Correct: **(C)**. *Traps:* **(A)** multiplied without flipping; **(B)** inverted the result; **(D)** cancelled then multiplied.

Multiply & divide rational expressions — Q4
Correct: **(D)**. *Traps:* **(A)** multiplied without flipping; **(B)** cancelled then multiplied; **(C)** inverted the result.

Multiply & divide rational expressions — Q5
Correct: **(A)**. *Traps:* **(B)** multiplied without flipping; **(C)** inverted the result; **(D)** cancelled then multiplied.

Add & subtract rational expressions — Q1
Correct: **(C)**. *Traps:* **(A)** added the denominators; **(B)** multiplied the numerators; **(D)** multiplied the denominators.

Add & subtract rational expressions — Q2
Correct: **(A)**. *Traps:* **(B)** added the denominators; **(C)** multiplied the numerators; **(D)** multiplied the denominators.

Add & subtract rational expressions — Q3
Correct: **(B)**. *Traps:* **(A)** added the denominators; **(C)** multiplied the numerators; **(D)** multiplied the denominators.

Add & subtract rational expressions — Q4
Correct: **(B)**. *Traps:* **(A)** multiplied the denominators; **(C)** multiplied the numerators; **(D)** added the denominators.

Add & subtract rational expressions — Q5
Correct: **(D)**. *Traps:* **(A)** multiplied the denominators; **(B)** multiplied the numerators; **(C)** added the denominators.

Complex (compound) fractions — Q1
Correct: **(B)**. *Traps:* **(A)** inverted the result; **(C)** multiplied the denominators; **(D)** multiplied straight across.

Complex (compound) fractions — Q2
Correct: **(B)**. *Traps:* **(A)** inverted the result; **(C)** multiplied straight across; **(D)** multiplied the denominators.

Complex (compound) fractions — Q3
Correct: **(C)**. *Traps:* **(A)** multiplied straight across; **(B)** inverted the result; **(D)** multiplied the denominators.

Complex (compound) fractions — Q4
Correct: **(C)**. *Traps:* **(A)** multiplied straight across; **(B)** multiplied the denominators; **(D)** inverted the result.

Complex (compound) fractions — Q5
Correct: **(A)**. *Traps:* **(B)** inverted the result; **(C)** multiplied the denominators; **(D)** multiplied straight across.

Solve rational equations — Q1
Correct: **(B)**. *Traps:* **(A)** multiplied instead of dividing; **(C)** inverted; **(D)** ignored the coefficient.

Solve rational equations — Q2
Correct: **(C)**. *Traps:* **(A)** multiplied instead of dividing; **(B)** ignored the coefficient; **(D)** inverted.

Solve rational equations — Q3
Correct: **(D)**. *Traps:* **(A)** inverted; **(B)** multiplied instead of dividing; **(C)** ignored the coefficient.

Solve rational equations — Q4
Correct: **(C)**. *Traps:* **(A)** inverted; **(B)** ignored the coefficient; **(D)** multiplied instead of dividing.

Solve rational equations — Q5
Correct: **(D)**. *Traps:* **(A)** ignored the coefficient; **(B)** inverted; **(C)** multiplied instead of dividing.

Excluded values & domain — Q1
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** missed the exclusion; **(D)** used the numerator.

Excluded values & domain — Q2
Correct: **(A)**. *Traps:* **(B)** sign error; **(C)** used the numerator; **(D)** missed the exclusion.

Excluded values & domain — Q3
Correct: **(D)**. *Traps:* **(A)** used the numerator; **(B)** missed the exclusion; **(C)** sign error.

Excluded values & domain — Q4
Correct: **(A)**. *Traps:* **(B)** missed the exclusion; **(C)** used the numerator; **(D)** sign error.

Excluded values & domain — Q5
Correct: **(C)**. *Traps:* **(A)** used the numerator; **(B)** missed the exclusion; **(D)** sign error.

Rational functions: asymptotes & holes — Q1
Correct: **(B)**. *Traps:* **(A)** used the numerator; **(C)** sign error; **(D)** horizontal/vertical mix-up.

Rational functions: asymptotes & holes — Q2
Correct: **(A)**. *Traps:* **(B)** sign error; **(C)** used the numerator; **(D)** horizontal/vertical mix-up.

Rational functions: asymptotes & holes — Q3
Correct: **(C)**. *Traps:* **(A)** used the numerator; **(B)** sign error; **(D)** horizontal/vertical mix-up.

Rational functions: asymptotes & holes — Q4
Correct: **(C)**. *Traps:* **(A)** sign error; **(B)** used the numerator; **(D)** horizontal/vertical mix-up.

Rational functions: asymptotes & holes — Q5
Correct: **(D)**. *Traps:* **(A)** horizontal/vertical mix-up; **(B)** sign error; **(C)** used the numerator.

Direct & inverse variation — Q1
Correct: **(C)**. *Traps:* **(A)** reused the given y; **(B)** added instead of scaling; **(D)** scaled without finding k.

Direct & inverse variation — Q2
Correct: **(A)**. *Traps:* **(B)** added instead of scaling; **(C)** scaled without finding k; **(D)** reused the given y.

Direct & inverse variation — Q3
Correct: **(C)**. *Traps:* **(A)** reused the given y; **(B)** scaled without finding k; **(D)** added instead of scaling.

Direct & inverse variation — Q4
Correct: **(D)**. *Traps:* **(A)** scaled without finding k; **(B)** added instead of scaling; **(C)** reused the given y.

Direct & inverse variation — Q5
Correct: **(A)**. *Traps:* **(B)** scaled without finding k; **(C)** added instead of scaling; **(D)** reused the given y.

Nonlinear systems: line & parabola — Q1
Correct: **(C)**. *Traps:* **(A)** substitution/setup error; **(B)** substitution/setup error; **(D)** substitution/setup error.

Nonlinear systems: line & parabola — Q2
Correct: **(C)**. *Traps:* **(A)** x-only-not-point error; **(B)** x-only-not-point error; **(D)** x-only-not-point error.

Nonlinear systems: line & parabola — Q3
Correct: **(C)**. *Traps:* **(A)** intersection-count error; **(B)** intersection-count error; **(D)** intersection-count error.

Nonlinear systems: line & parabola — Q4
Correct: **(D)**. *Traps:* **(A)** dropped-second-solution error; **(B)** dropped-second-solution error; **(C)** dropped-second-solution error.

Nonlinear systems: line & parabola — Q5
Correct: **(B)**. *Traps:* **(A)** dropped-second-solution error; **(C)** intersection-count error; **(D)** x-only-not-point error.

Solve line & parabola by substitution — Q1
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** one solution only; **(D)** read off coefficients.

Solve line & parabola by substitution — Q2
Correct: **(C)**. *Traps:* **(A)** read off coefficients; **(B)** one solution only; **(D)** sign error.

Solve line & parabola by substitution — Q3
Correct: **(D)**. *Traps:* **(A)** read off coefficients; **(B)** one solution only; **(C)** sign error.

Solve line & parabola by substitution — Q4
Correct: **(A)**. *Traps:* **(B)** sign error; **(C)** one solution only; **(D)** read off coefficients.

Solve line & parabola by substitution — Q5
Correct: **(A)**. *Traps:* **(B)** sign error; **(C)** one solution only; **(D)** read off coefficients.

Line & circle systems — Q1
Correct: **(A)**. *Traps:* **(B)** used the line value; **(C)** used R^2; **(D)** one point only.

Line & circle systems — Q2
Correct: **(D)**. *Traps:* **(A)** one point only; **(B)** used R^2; **(C)** used the line value.

Line & circle systems — Q3
Correct: **(A)**. *Traps:* **(B)** used the line value; **(C)** one point only; **(D)** used R^2.

Line & circle systems — Q4
Correct: **(D)**. *Traps:* **(A)** used R^2; **(B)** used the line value; **(C)** one point only.

Line & circle systems — Q5
Correct: **(B)**. *Traps:* **(A)** used the line value; **(C)** used R^2; **(D)** one point only.

Number of intersections (discriminant) — Q1
Correct: **(B)**. *Traps:* **(A)** dropped the c; **(C)** added instead of subtracting; **(D)** dropped the 4.

Number of intersections (discriminant) — Q2
Correct: **(C)**. *Traps:* **(A)** dropped the c; **(B)** dropped the 4; **(D)** added instead of subtracting.

Number of intersections (discriminant) — Q3
Correct: **(D)**. *Traps:* **(A)** added instead of subtracting; **(B)** dropped the c; **(C)** dropped the 4.

Number of intersections (discriminant) — Q4
Correct: **(B)**. *Traps:* **(A)** added instead of subtracting; **(C)** dropped the 4; **(D)** dropped the c.

Number of intersections (discriminant) — Q5
Correct: **(C)**. *Traps:* **(A)** dropped the 4; **(B)** added instead of subtracting; **(D)** dropped the c.

Report intersection points (x and y) — Q1
Correct: **(B)**. *Traps:* **(A)** swapped coordinates; **(C)** used x as y; **(D)** gave only x.

Report intersection points (x and y) — Q2
Correct: **(B)**. *Traps:* **(A)** gave only x; **(C)** swapped coordinates; **(D)** used x as y.

Report intersection points (x and y) — Q3
Correct: **(A)**. *Traps:* **(B)** used x as y; **(C)** gave only x; **(D)** swapped coordinates.

Report intersection points (x and y) — Q4
Correct: **(D)**. *Traps:* **(A)** swapped coordinates; **(B)** used x as y; **(C)** gave only x.

Report intersection points (x and y) — Q5
Correct: **(A)**. *Traps:* **(B)** used x as y; **(C)** swapped coordinates; **(D)** gave only x.

Nonlinear systems in context — Q1
Correct: **(C)**. *Traps:* **(A)** swapped S and P; **(B)** sum sign; **(D)** product sign.

Nonlinear systems in context — Q2
Correct: **(B)**. *Traps:* **(A)** product sign; **(C)** swapped S and P; **(D)** sum sign.

Nonlinear systems in context — Q3
Correct: **(D)**. *Traps:* **(A)** sum sign; **(B)** product sign; **(C)** swapped S and P.

Nonlinear systems in context — Q4
Correct: **(B)**. *Traps:* **(A)** product sign; **(C)** sum sign; **(D)** swapped S and P.

Nonlinear systems in context — Q5
Correct: **(D)**. *Traps:* **(A)** sum sign; **(B)** product sign; **(C)** swapped S and P.

Function transformations — Q1
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Function transformations — Q2
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** horizontal-shift direction error; **(D)** inside/outside confusion.

Function transformations — Q3
Correct: **(B)**. *Traps:* **(A)** reflection-axis error; **(C)** reflection-axis error; **(D)** reflection-axis error.

Function transformations — Q4
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** stretch/compression direction error; **(D)** stretch/compression direction error.

Function transformations — Q5
Correct: **(A)**. *Traps:* **(B)** horizontal-shift direction error; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Horizontal shifts — Q1
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** horizontal-shift direction error.

Horizontal shifts — Q2
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** horizontal-shift direction error.

Horizontal shifts — Q3
Correct: **(B)**. *Traps:* **(A)** horizontal-shift direction error; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Horizontal shifts — Q4
Correct: **(C)**. *Traps:* **(A)** inside/outside confusion; **(B)** inside/outside confusion; **(D)** horizontal-shift direction error.

Horizontal shifts — Q5
Correct: **(C)**. *Traps:* **(A)** horizontal-shift direction error; **(B)** inside/outside confusion; **(D)** output-shift error.

Reflection over the x-axis — Q1
Correct: **(C)**. *Traps:* **(A)** inside/outside confusion; **(B)** reflection-axis error; **(D)** reflection-axis error.

Reflection over the x-axis — Q2
Correct: **(A)**. *Traps:* **(B)** reflection-axis error; **(C)** inside/outside confusion; **(D)** reflection-axis error.

Reflection over the x-axis — Q3
Correct: **(B)**. *Traps:* **(A)** reflection-axis error; **(C)** reflection-axis error; **(D)** inside/outside confusion.

Reflection over the x-axis — Q4
Correct: **(C)**. *Traps:* **(A)** inside/outside confusion; **(B)** reflection-axis error; **(D)** reflection-axis error.

Reflection over the x-axis — Q5
Correct: **(D)**. *Traps:* **(A)** inside/outside confusion; **(B)** no reflection applied; **(C)** reflection-axis error.

Reflection over the y-axis — Q1
Correct: **(D)**. *Traps:* **(A)** inside/outside confusion; **(B)** reflection-axis error; **(C)** reflection-axis error.

Reflection over the y-axis — Q2
Correct: **(B)**. *Traps:* **(A)** reflection-axis error; **(C)** inside/outside confusion; **(D)** reflection-axis error.

Reflection over the y-axis — Q3
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** reflection-axis error; **(D)** reflection-axis error.

Reflection over the y-axis — Q4
Correct: **(D)**. *Traps:* **(A)** inside/outside confusion; **(B)** reflection-axis error; **(C)** reflection-axis error.

Reflection over the y-axis — Q5
Correct: **(C)**. *Traps:* **(A)** inside/outside confusion; **(B)** output-negation error; **(D)** reflection-axis error.

Vertical stretch & compression — Q1
Correct: **(B)**. *Traps:* **(A)** stretch/compression direction error; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Vertical stretch & compression — Q2
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** stretch/compression direction error; **(D)** inside/outside confusion.

Vertical stretch & compression — Q3
Correct: **(A)**. *Traps:* **(B)** stretch/compression direction error; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Vertical stretch & compression — Q4
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** moved instead of scaled; **(D)** stretch/compression direction error.

Vertical stretch & compression — Q5
Correct: **(D)**. *Traps:* **(A)** stretch/compression direction error; **(B)** inside/outside confusion; **(C)** moved instead of scaled.

Horizontal stretch & compression — Q1
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** stretch/compression direction error; **(D)** inside/outside confusion.

Horizontal stretch & compression — Q2
Correct: **(D)**. *Traps:* **(A)** inside/outside confusion; **(B)** stretch/compression direction error; **(C)** inside/outside confusion.

Horizontal stretch & compression — Q3
Correct: **(A)**. *Traps:* **(B)** stretch/compression direction error; **(C)** inside/outside confusion; **(D)** inside/outside confusion.

Horizontal stretch & compression — Q4
Correct: **(D)**. *Traps:* **(A)** inside/outside confusion; **(B)** confused stretch with shift; **(C)** stretch/-compression direction error.

Horizontal stretch & compression — Q5
Correct: **(D)**. *Traps:* **(A)** stretch/compression direction error; **(B)** inside/outside confusion; **(C)** confused stretch with shift.

Combined transformations — Q1
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** horizontal-shift direction error.

Combined transformations — Q2
Correct: **(A)**. *Traps:* **(B)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** horizontal-shift direction error.

Combined transformations — Q3
Correct: **(B)**. *Traps:* **(A)** inside/outside confusion; **(C)** inside/outside confusion; **(D)** horizontal-shift direction error.

Combined transformations — Q4
Correct: **(C)**. *Traps:* **(A)** horizontal-shift direction error; **(B)** inside/outside confusion; **(D)** inside/outside confusion.

Combined transformations — Q5
Correct: **(B)**. *Traps:* **(A)** horizontal-shift direction error; **(C)** inside/outside confusion; **(D)** vertical-shift direction error.

Evaluating transformed functions — Q1
Correct: **(B)**. *Traps:* **(A)** operation/sign error; **(C)** dropped-transformation error; **(D)** operation/sign error.

Evaluating transformed functions — Q2
Correct: **(D)**. *Traps:* **(A)** dropped-transformation error; **(B)** operation/sign error; **(C)** operation/sign error.

Evaluating transformed functions — Q3
Correct: **(C)**. *Traps:* **(A)** operation/sign error; **(B)** operation/sign error; **(D)** dropped-transformation error.

Evaluating transformed functions — Q4
Correct: **(A)**. *Traps:* **(B)** operation/sign error; **(C)** dropped-transformation error; **(D)** operation/sign error.

Evaluating transformed functions — Q5
Correct: **(A)**. *Traps:* **(B)** used the input, not the output; **(C)** operation/sign error; **(D)** dropped-transformation error.

Complex numbers: arithmetic with i — Q1
Correct: **(C)**. *Traps:* **(A)** i-squared sign error; **(B)** i-squared sign error; **(D)** i-squared sign error.

Complex numbers: arithmetic with i — Q2
Correct: **(C)**. *Traps:* **(A)** combine real/imaginary error; **(B)** combine real/imaginary error; **(D)** combine real/imaginary error.

Complex numbers: arithmetic with i — Q3
Correct: **(D)**. *Traps:* **(A)** multiplication/FOIL error; **(B)** i-squared sign error; **(C)** i-squared sign error.

Complex numbers: arithmetic with i — Q4
Correct: **(C)**. *Traps:* **(A)** square-root-of-negative error; **(B)** square-root-of-negative error; **(D)** square-root-of-negative error.

Complex numbers: arithmetic with i — Q5
Correct: **(D)**. *Traps:* **(A)** i-squared sign error; **(B)** multiplication/FOIL error; **(C)** i-squared sign error.

Powers of i & complex division — Q1
Correct: **(D)**. *Traps:* **(A)** power-cycle error; **(B)** power-cycle error; **(C)** power-cycle error.

Powers of i & complex division — Q2
Correct: **(A)**. *Traps:* **(B)** power-cycle error; **(C)** power-cycle error; **(D)** power-cycle error.

Powers of i & complex division — Q3
Correct: **(D)**. *Traps:* **(A)** conjugate error; **(B)** conjugate error; **(C)** conjugate error.

Powers of i & complex division — Q4
Correct: **(C)**. *Traps:* **(A)** conjugate error; **(B)** conjugate error; **(D)** conjugate error.

Powers of i & complex division — Q5
Correct: **(C)**. *Traps:* **(A)** division-method error; **(B)** division-method error; **(D)** division-method error.

Add & subtract complex numbers — Q1
Correct: **(A)**. *Traps:* **(B)** swapped the parts; **(C)** merged real & imaginary; **(D)** multiplied the i-parts.

Add & subtract complex numbers — Q2
Correct: **(B)**. *Traps:* **(A)** merged real & imaginary; **(C)** multiplied the i-parts; **(D)** swapped the parts.

Add & subtract complex numbers — Q3
Correct: **(A)**. *Traps:* **(B)** merged real & imaginary; **(C)** swapped the parts; **(D)** multiplied the i-parts.

Add & subtract complex numbers — Q4
Correct: **(D)**. *Traps:* **(A)** multiplied the i-parts; **(B)** merged real & imaginary; **(C)** swapped the parts.

Add & subtract complex numbers — Q5
Correct: **(C)**. *Traps:* **(A)** multiplied the i-parts; **(B)** swapped the parts; **(D)** merged real & imaginary.

Multiply complex numbers (FOIL with i) — Q1
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** straight-across product; **(D)** i-squared as +1.

Multiply complex numbers (FOIL with i) — Q2
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** i-squared as +1; **(D)** straight-across product.

Multiply complex numbers (FOIL with i) — Q3
Correct: **(A)**. *Traps:* **(B)** straight-across product; **(C)** one cross term; **(D)** i-squared as +1.

Multiply complex numbers (FOIL with i) — Q4
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** i-squared as +1; **(D)** straight-across product.

Multiply complex numbers (FOIL with i) — Q5
Correct: **(C)**. *Traps:* **(A)** i-squared as +1; **(B)** straight-across product; **(D)** one cross term.

Complex conjugates & division — Q1
Correct: **(D)**. *Traps:* **(A)** dropped the b^2 term; **(B)** i-squared as +1; **(C)** left an imaginary unit.

Complex conjugates & division — Q2
Correct: **(D)**. *Traps:* **(A)** dropped the b^2 term; **(B)** left an imaginary unit; **(C)** i-squared as +1.

Complex conjugates & division — Q3
Correct: **(B)**. *Traps:* **(A)** i-squared as +1; **(C)** left an imaginary unit; **(D)** dropped the b^2 term.

Complex conjugates & division — Q4
Correct: **(A)**. *Traps:* **(B)** i-squared as +1; **(C)** dropped the b^2 term; **(D)** left an imaginary unit.

Complex conjugates & division — Q5
Correct: **(D)**. *Traps:* **(A)** i-squared as +1; **(B)** left an imaginary unit; **(C)** dropped the b^2 term.

Complex solutions of quadratics — Q1
Correct: **(A)**. *Traps:* **(B)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q2
Correct: **(A)**. *Traps:* **(B)** stopped at 'no solution'; **(C)** forgot the square root; **(D)** dropped the imaginary unit.

Complex solutions of quadratics — Q3
Correct: **(B)**. *Traps:* **(A)** forgot the square root; **(C)** dropped the imaginary unit; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q4
Correct: **(B)**. *Traps:* **(A)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q5
Correct: **(B)**. *Traps:* **(A)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

Problem-Solving & Data Analysis

Ratios & equivalent ratios — Q1
Correct: **(A)**. *Traps:* **(B)** order/inversion error; **(C)** additive scaling; **(D)** order/inversion error.

Ratios & equivalent ratios — Q2
Correct: **(C)**. *Traps:* **(A)** order/inversion error; **(B)** part-part vs part-whole confusion; **(D)** part-part vs part-whole confusion.

Ratios & equivalent ratios — Q3
Correct: **(D)**. *Traps:* **(A)** order/inversion error; **(B)** simplification error; **(C)** order/inversion error.

Ratios & equivalent ratios — Q4
Correct: **(D)**. *Traps:* **(A)** additive scaling; **(B)** order/inversion error; **(C)** additive scaling.

Ratios & equivalent ratios — Q5
Correct: **(B)**. *Traps:* **(A)** additive scaling; **(C)** order/inversion error; **(D)** additive scaling.

Unit rates — Q1
Correct: **(D)**. *Traps:* **(A)** failure to reduce to per-one; **(B)** rate inversion; **(C)** units/label error.

Unit rates — Q2
Correct: **(D)**. *Traps:* **(A)** rate inversion; **(B)** failure to reduce to per-one; **(C)** coefficient/division error.

Unit rates — Q3
Correct: **(A)**. *Traps:* **(B)** compare-without-normalizing; **(C)** compare-without-normalizing; **(D)** rate inversion.

Unit rates — Q4
Correct: **(A)**. *Traps:* **(B)** failure to reduce to per-one; **(C)** failure to reduce to per-one; **(D)** rate inversion.

Unit rates — Q5
Correct: **(D)**. *Traps:* **(A)** rate inversion; **(B)** failure to reduce to per-one; **(C)** failure to reduce to per-one.

Proportions & solving — Q1
Correct: **(A)**. *Traps:* **(B)** additive proportion error; **(C)** cross-multiply structure error; **(D)** coefficient/division error.

Proportions & solving — Q2
Correct: **(D)**. *Traps:* **(A)** misaligned setup; **(B)** additive proportion error; **(C)** misaligned setup.

Proportions & solving — Q3
Correct: **(C)**. *Traps:* **(A)** cross-multiply structure error; **(B)** additive proportion error; **(D)** coefficient/division error.

Proportions & solving — Q4
Correct: **(C)**. *Traps:* **(A)** misaligned setup; **(B)** additive proportion error; **(D)** cross-multiply structure error.

Proportions & solving — Q5
Correct: **(D)**. *Traps:* **(A)** additive proportion error; **(B)** cross-multiply structure error; **(C)** misaligned setup.

Proportional relationships (y = kx) — Q1
Correct: **(D)**. *Traps:* **(A)** proportional vs non-proportional confusion; **(B)** constant-of-proportionality error; **(C)** application error.

Proportional relationships (y = kx) — Q2
Correct: **(B)**. *Traps:* **(A)** proportional vs non-proportional confusion; **(C)** proportional vs non-proportional confusion; **(D)** proportional vs non-proportional confusion.

Proportional relationships (y = kx) — Q3
Correct: **(B)**. *Traps:* **(A)** application error; **(C)** constant-of-proportionality error; **(D)** proportional vs non-proportional confusion.

Proportional relationships (y = kx) — Q4
Correct: **(C)**. *Traps:* **(A)** constant-of-proportionality error; **(B)** application error; **(D)** application error.

Proportional relationships (y = kx) — Q5
Correct: **(A)**. *Traps:* **(B)** constant-of-proportionality error; **(C)** application error; **(D)** proportional vs non-proportional confusion.

Percentages — Q1
Correct: **(D)**. *Traps:* **(A)** percent-decimal conversion error; **(B)** base/part inversion; **(C)** percent-decimal conversion error.

Percentages — Q2
Correct: **(A)**. *Traps:* **(B)** reverse-percent operation error; **(C)** percent-decimal conversion error; **(D)** base/part inversion.

Percentages — Q3
Correct: **(B)**. *Traps:* **(A)** percent-decimal conversion error; **(C)** percent-decimal conversion error; **(D)** percent-decimal conversion error.

Percentages — Q4
Correct: **(B)**. *Traps:* **(A)** reverse-percent operation error; **(C)** reverse-percent operation error; **(D)** reverse-percent operation error.

Percentages — Q5
Correct: **(A)**. *Traps:* **(B)** base/part inversion; **(C)** reverse-percent operation error; **(D)** reverse-percent operation error.

Percent change — Q1
Correct: **(C)**. *Traps:* **(A)** wrong base; **(B)** wrong base; **(D)** wrong base.

Percent change — Q2
Correct: **(A)**. *Traps:* **(B)** direction/value error; **(C)** direction/value error; **(D)** direction/value error.

Percent change — Q3
Correct: **(C)**. *Traps:* **(A)** wrong base; **(B)** successive-change additive error; **(D)** successive-change additive error.

Percent change — Q4
Correct: **(B)**. *Traps:* **(A)** direction/value error; **(C)** direction/value error; **(D)** successive-change additive error.

Percent change — Q5
Correct: **(A)**. *Traps:* **(B)** successive-change additive error; **(C)** wrong base; **(D)** wrong base.

Apply ratios, rates & percent in context (multi-step) — Q1
Correct: **(A)**. *Traps:* **(B)** multi-step setup error; **(C)** intermediate-value-as-final; **(D)** intermediate-value-as-final.

Apply ratios, rates & percent in context (multi-step) — Q2
Correct: **(B)**. *Traps:* **(A)** multi-step setup error; **(C)** intermediate-value-as-final; **(D)** answer-interpretation/units.

Apply ratios, rates & percent in context (multi-step) — Q3
Correct: **(B)**. *Traps:* **(A)** intermediate-value-as-final; **(C)** intermediate-value-as-final; **(D)** multi-step setup error.

Apply ratios, rates & percent in context (multi-step) — Q4
Correct: **(B)**. *Traps:* **(A)** intermediate-value-as-final; **(C)** answer-interpretation/units; **(D)** multi-step setup error.

Apply ratios, rates & percent in context (multi-step) — Q5
Correct: **(A)**. *Traps:* **(B)** multi-step setup error; **(C)** answer-interpretation/units; **(D)** intermediate-value-as-final.

Unit conversion & dimensional analysis — Q1
Correct: **(B)**. *Traps:* **(A)** factor-orientation error; **(C)** conversion-factor value error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q2
Correct: **(B)**. *Traps:* **(A)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q3
Correct: **(A)**. *Traps:* **(B)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q4
Correct: **(D)**. *Traps:* **(A)** conversion-factor value error; **(B)** factor-orientation error; **(C)** conversion-factor value error.

Unit conversion & dimensional analysis — Q5
Correct: **(A)**. *Traps:* **(B)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** unit-tracking/label error.

One-variable data: measures of center & spread — Q1
Correct: **(C)**. *Traps:* **(A)** range/spread vs center confusion; **(B)** range/spread vs center confusion; **(D)** mean/median confusion.

One-variable data: measures of center & spread — Q2
Correct: **(B)**. *Traps:* **(A)** median-without-ordering error; **(C)** median-without-ordering error; **(D)** median-without-ordering error.

One-variable data: measures of center & spread — Q3
Correct: **(C)**. *Traps:* **(A)** outlier-resistance error; **(B)** outlier-resistance error; **(D)** outlier-resistance error.

One-variable data: measures of center & spread — Q4
Correct: **(B)**. *Traps:* **(A)** mean/median confusion; **(C)** range/spread vs center confusion; **(D)** range/spread vs center confusion.

One-variable data: measures of center & spread — Q5
Correct: **(B)**. *Traps:* **(A)** range/spread vs center confusion; **(C)** mean/median confusion; **(D)** mean/median confusion.

Distributions & standard deviation (comparing spread) — Q1
Correct: **(A)**. *Traps:* **(B)** SD-direction error; **(C)** SD-direction error; **(D)** SD-direction error.

Distributions & standard deviation (comparing spread) — Q2
Correct: **(B)**. *Traps:* **(A)** range-as-SD error; **(C)** range-as-SD error; **(D)** range-as-SD error.

Distributions & standard deviation (comparing spread) — Q3
Correct: **(C)**. *Traps:* **(A)** skew/mean-median error; **(B)** skew/mean-median error; **(D)** skew/mean-median error.

Distributions & standard deviation (comparing spread) — Q4
Correct: **(A)**. *Traps:* **(B)** spread-invariance error; **(C)** spread-invariance error; **(D)** spread-invariance error.

Distributions & standard deviation (comparing spread) — Q5
Correct: **(B)**. *Traps:* **(A)** SD-direction error; **(C)** spread-invariance error; **(D)** SD-direction error.

Two-variable data: scatterplots & line of best fit — Q1
Correct: **(D)**. *Traps:* **(A)** best-fit slope interpretation error; **(B)** best-fit slope interpretation error; **(C)** best-fit slope interpretation error.

Two-variable data: scatterplots & line of best fit — Q2
Correct: **(B)**. *Traps:* **(A)** intercept/prediction misuse; **(C)** intercept/prediction misuse; **(D)** intercept/prediction misuse.

Two-variable data: scatterplots & line of best fit — Q3
Correct: **(A)**. *Traps:* **(B)** model-type choice error; **(C)** model-type choice error; **(D)** model-type choice error.

Two-variable data: scatterplots & line of best fit — Q4
Correct: **(D)**. *Traps:* **(A)** association/causation misread; **(B)** association/causation misread; **(C)** association/causation misread.

Two-variable data: scatterplots & line of best fit — Q5
Correct: **(C)**. *Traps:* **(A)** intercept/prediction misuse; **(B)** best-fit slope interpretation error; **(D)** intercept/prediction misuse.

Probability & two-way tables — Q1
Correct: **(B)**. *Traps:* **(A)** table-reading error; **(C)** ratio-inversion/complement error; **(D)** ratio-inversion/complement error.

Probability & two-way tables — Q2
Correct: **(B)**. *Traps:* **(A)** table-reading error; **(C)** joint/conditional confusion; **(D)** conditional-denominator error.

Probability & two-way tables — Q3
Correct: **(D)**. *Traps:* **(A)** ratio-inversion/complement error; **(B)** conditional-denominator error; **(C)** joint/conditional confusion.

Probability & two-way tables — Q4
Correct: **(A)**. *Traps:* **(B)** table-reading error; **(C)** table-reading error; **(D)** joint/conditional confusion.

Probability & two-way tables — Q5
Correct: **(D)**. *Traps:* **(A)** ratio-inversion/complement error; **(B)** table-reading error; **(C)** conditional-denominator error.

Inference, margin of error & evaluating statistical claims — Q1
Correct: **(B)**. *Traps:* **(A)** margin-of-error meaning error; **(C)** margin-of-error meaning error; **(D)** margin-of-error meaning error.

Inference, margin of error & evaluating statistical claims — Q2
Correct: **(B)**. *Traps:* **(A)** margin-of-error meaning error; **(C)** margin-of-error meaning error; **(D)** margin-of-error meaning error.

Inference, margin of error & evaluating statistical claims — Q3
Correct: **(B)**. *Traps:* **(A)** generalization-scope error; **(C)** generalization-scope error; **(D)** generalization-scope error.

Inference, margin of error & evaluating statistical claims — Q4
Correct: **(B)**. *Traps:* **(A)** causation-from-observation error; **(C)** causation-from-observation error; **(D)** causation-from-observation error.

Inference, margin of error & evaluating statistical claims — Q5
Correct: **(A)**. *Traps:* **(B)** selection-vs-assignment confusion; **(C)** selection-vs-assignment confusion; **(D)** selection-vs-assignment confusion.

Weighted averages & combined means — Q1
Correct: **(A)**. *Traps:* **(B)** total-vs-mean error; **(C)** average-of-averages error; **(D)** total-vs-mean error.

Weighted averages & combined means — Q2
Correct: **(B)**. *Traps:* **(A)** weight/value swap; **(C)** total-vs-mean error; **(D)** average-of-averages error.

Weighted averages & combined means — Q3
Correct: **(C)**. *Traps:* **(A)** weight/value swap; **(B)** average-of-averages error; **(D)** total-vs-mean error.

Weighted averages & combined means — Q4
Correct: **(C)**. *Traps:* **(A)** reverse weighted-average error; **(B)** reverse weighted-average error; **(D)** reverse weighted-average error.

Weighted averages & combined means — Q5
Correct: **(D)**. *Traps:* **(A)** total-vs-mean error; **(B)** average-of-averages error; **(C)** total-vs-mean error.

Line of best fit: prediction & residuals — Q1
Correct: **(D)**. *Traps:* **(A)** prediction computation error; **(B)** prediction computation error; **(C)** prediction computation error.

Line of best fit: prediction & residuals — Q2
Correct: **(A)**. *Traps:* **(B)** residual sign/direction error; **(C)** residual sign/direction error; **(D)** residual sign/direction error.

Line of best fit: prediction & residuals — Q3
Correct: **(C)**. *Traps:* **(A)** residual sign/direction error; **(B)** residual sign/direction error; **(D)** residual sign/direction error.

Line of best fit: prediction & residuals — Q4
Correct: **(C)**. *Traps:* **(A)** extrapolation-reliability error; **(B)** extrapolation-reliability error; **(D)** extrapolation-reliability error.

Line of best fit: prediction & residuals — Q5
Correct: **(B)**. *Traps:* **(A)** slope-as-rate misread; **(C)** prediction computation error; **(D)** slope-as-rate misread.

Compound & conditional probability — Q1
Correct: **(C)**. *Traps:* **(A)** and/or operation swap; **(B)** and/or operation swap; **(D)** and/or operation swap.

Compound & conditional probability — Q2
Correct: **(D)**. *Traps:* **(A)** independence/overlap error; **(B)** independence/overlap error; **(C)** and/or operation swap.

Compound & conditional probability — Q3
Correct: **(C)**. *Traps:* **(A)** complement error; **(B)** complement error; **(D)** complement error.

Compound & conditional probability — Q4
Correct: **(A)**. *Traps:* **(B)** conditional-denominator error; **(C)** conditional-denominator error; **(D)** conditional-denominator error.

Compound & conditional probability — Q5
Correct: **(D)**. *Traps:* **(A)** independence/overlap error; **(B)** independence/overlap error; **(C)** and/or operation swap.

Geometry & Trigonometry

Angles & lines — Q1
Correct: **(B)**. *Traps:* **(A)** wrong relationship/setup; **(C)** wrong relationship/setup; **(D)** complementary/supplementary swap.

Angles & lines — Q2
Correct: **(A)**. *Traps:* **(B)** wrong relationship/setup; **(C)** wrong relationship/setup; **(D)** complementary/supplementary swap.

Angles & lines — Q3
Correct: **(A)**. *Traps:* **(B)** wrong relationship/setup; **(C)** parallel-line angle-pair error; **(D)** complementary/supplementary swap.

Angles & lines — Q4
Correct: **(B)**. *Traps:* **(A)** wrong relationship/setup; **(C)** parallel-line angle-pair error; **(D)** complementary/supplementary swap.

Angles & lines — Q5
Correct: **(A)**. *Traps:* **(B)** wrong relationship/setup; **(C)** complementary/supplementary swap; **(D)** wrong relationship/setup.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q1
Correct: **(B)**. *Traps:* **(A)** angle-sum error; **(C)** isosceles base-angle error; **(D)** angle-sum error.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q2
Correct: **(A)**. *Traps:* **(B)** isosceles base-angle error; **(C)** angle-sum error; **(D)** isosceles base-angle error.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q3
Correct: **(B)**. *Traps:* **(A)** angle-sum error; **(C)** exterior-angle error; **(D)** exterior-angle error.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q4
Correct: **(D)**. *Traps:* **(A)** triangle-inequality error; **(B)** triangle-inequality error; **(C)** triangle-inequality error.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q5
Correct: **(B)**. *Traps:* **(A)** angle-sum error; **(C)** angle-sum error; **(D)** angle-sum error.

Right triangles & the Pythagorean theorem — Q1
Correct: **(B)**. *Traps:* **(A)** missing square or square-root; **(C)** missing square or square-root; **(D)** missing square or square-root.

Right triangles & the Pythagorean theorem — Q2
Correct: **(D)**. *Traps:* **(A)** missing square or square-root; **(B)** hypotenuse/leg confusion; **(C)** missing square or square-root.

Right triangles & the Pythagorean theorem — Q3
Correct: **(A)**. *Traps:* **(B)** missing square or square-root; **(C)** missing square or square-root; **(D)** hypotenuse/leg confusion.

Right triangles & the Pythagorean theorem — Q4
Correct: **(A)**. *Traps:* **(B)** special-triangle ratio error; **(C)** missing square or square-root; **(D)** special-triangle ratio error.

Right triangles & the Pythagorean theorem — Q5
Correct: **(C)**. *Traps:* **(A)** special-triangle ratio error; **(B)** special-triangle ratio error; **(D)** special-triangle ratio error.

Similarity & congruence — Q1
Correct: **(D)**. *Traps:* **(A)** additive vs multiplicative scaling; **(B)** corresponding-side mismatch; **(C)** corresponding-side mismatch.

Similarity & congruence — Q2
Correct: **(D)**. *Traps:* **(A)** additive vs multiplicative scaling; **(B)** corresponding-side mismatch; **(C)** corresponding-side mismatch.

Similarity & congruence — Q3
Correct: **(D)**. *Traps:* **(A)** corresponding-side mismatch; **(B)** corresponding-side mismatch; **(C)** additive vs multiplicative scaling.

Similarity & congruence — Q4
Correct: **(C)**. *Traps:* **(A)** area scale-factor error; **(B)** area scale-factor error; **(D)** area scale-factor error.

Similarity & congruence — Q5
Correct: **(C)**. *Traps:* **(A)** area scale-factor error; **(B)** corresponding-side mismatch; **(D)** area scale-factor error.

Area & perimeter — Q1
Correct: **(A)**. *Traps:* **(B)** area/perimeter confusion; **(C)** wrong height; **(D)** area/perimeter confusion.

Area & perimeter — Q2
Correct: **(A)**. *Traps:* **(B)** wrong height; **(C)** wrong height; **(D)** triangle 1/2 omission.

Area & perimeter — Q3
Correct: **(B)**. *Traps:* **(A)** area/perimeter confusion; **(C)** wrong height; **(D)** triangle 1/2 omission.

Area & perimeter — Q4
Correct: **(C)**. *Traps:* **(A)** composite-figure error; **(B)** composite-figure error; **(D)** composite-figure error.

Area & perimeter — Q5
Correct: **(B)**. *Traps:* **(A)** composite-figure error; **(C)** area/perimeter confusion; **(D)** triangle 1/2 omission.

Circles — Q1
Correct: **(A)**. *Traps:* **(B)** circumference/area formula swap; **(C)** radius/diameter confusion; **(D)** radius/diameter confusion.

Circles — Q2
Correct: **(A)**. *Traps:* **(B)** circumference/area formula swap; **(C)** radius/diameter confusion; **(D)** circumference/area formula swap.

Circles — Q3
Correct: **(A)**. *Traps:* **(B)** arc/sector fraction error; **(C)** arc/sector fraction error; **(D)** circumference/area formula swap.

Circles — Q4
Correct: **(A)**. *Traps:* **(B)** circle-equation error; **(C)** circle-equation error; **(D)** circle-equation error.

Circles — Q5
Correct: **(A)**. *Traps:* **(B)** radius/diameter confusion; **(C)** circle-equation error; **(D)** circle-equation error.

Volume & surface area — Q1
Correct: **(B)**. *Traps:* **(A)** volume/surface-area confusion; **(C)** volume/surface-area confusion; **(D)** volume/surface-area confusion.

Volume & surface area — Q2
Correct: **(B)**. *Traps:* **(A)** volume/surface-area confusion; **(C)** radius/diameter & power error; **(D)** radius/diameter & power error.

Volume & surface area — Q3
Correct: **(D)**. *Traps:* **(A)** radius/diameter & power error; **(B)** missing fractional coefficient; **(C)** radius/diameter & power error.

Volume & surface area — Q4
Correct: **(D)**. *Traps:* **(A)** radius/diameter & power error; **(B)** missing fractional coefficient; **(C)** missing fractional coefficient.

Volume & surface area — Q5
Correct: **(C)**. *Traps:* **(A)** volume/surface-area confusion; **(B)** volume/surface-area confusion; **(D)** volume/surface-area confusion.

Right-triangle trigonometry — Q1
Correct: **(D)**. *Traps:* **(A)** wrong/inverted ratio; **(B)** wrong/inverted ratio; **(C)** wrong/inverted ratio.

Right-triangle trigonometry — Q2
Correct: **(B)**. *Traps:* **(A)** wrong/inverted ratio; **(C)** opposite/adjacent identification error; **(D)** wrong/inverted ratio.

Right-triangle trigonometry — Q3
Correct: **(D)**. *Traps:* **(A)** wrong/inverted ratio; **(B)** wrong/inverted ratio; **(C)** opposite/adjacent identification error.

Right-triangle trigonometry — Q4
Correct: **(C)**. *Traps:* **(A)** cofunction error; **(B)** wrong/inverted ratio; **(D)** cofunction error.

Right-triangle trigonometry — Q5
Correct: **(B)**. *Traps:* **(A)** wrong/inverted ratio; **(C)** wrong/inverted ratio; **(D)** opposite/adjacent identification error.

Radian measure & trigonometric relationships — Q1
Correct: **(C)**. *Traps:* **(A)** degree-radian conversion error; **(B)** degree-radian conversion error; **(D)** degree-radian conversion error.

Radian measure & trigonometric relationships — Q2
Correct: **(B)**. *Traps:* **(A)** degree-radian conversion error; **(C)** degree-radian conversion error; **(D)** degree-radian conversion error.

Radian measure & trigonometric relationships — Q3
Correct: **(B)**. *Traps:* **(A)** complementary/cofunction error; **(C)** complementary/cofunction error; **(D)** complementary/cofunction error.

Radian measure & trigonometric relationships — Q4
Correct: **(D)**. *Traps:* **(A)** Pythagorean-identity error; **(B)** Pythagorean-identity error; **(C)** Pythagorean-identity error.

Radian measure & trigonometric relationships — Q5
Correct: **(D)**. *Traps:* **(A)** arc-length radian/degree error; **(B)** arc-length radian/degree error; **(C)** arc-length radian/degree error.

Coordinate geometry: distance & midpoint — Q1
Correct: **(D)**. *Traps:* **(A)** distance-formula error; **(B)** distance-formula error; **(C)** distance-formula error.

Coordinate geometry: distance & midpoint — Q2
Correct: **(A)**. *Traps:* **(B)** distance/midpoint operation swap; **(C)** midpoint error; **(D)** midpoint error.

Coordinate geometry: distance & midpoint — Q3
Correct: **(D)**. *Traps:* **(A)** distance/midpoint operation swap; **(B)** distance/midpoint operation swap; **(C)** distance/midpoint operation swap.

Coordinate geometry: distance & midpoint — Q4
Correct: **(D)**. *Traps:* **(A)** sign/pairing error; **(B)** sign/pairing error; **(C)** sign/pairing error.

Coordinate geometry: distance & midpoint — Q5
Correct: **(D)**. *Traps:* **(A)** midpoint error; **(B)** distance-formula error; **(C)** distance-formula error.

Circle angles: central, inscribed & arcs — Q1
Correct: **(C)**. *Traps:* **(A)** arc-measure vs central-angle confusion; **(B)** arc-measure vs central-angle confusion; **(D)** arc-measure vs central-angle confusion.

Circle angles: central, inscribed & arcs — Q2
Correct: **(C)**. *Traps:* **(A)** inscribed/central factor error; **(B)** inscribed/central factor error; **(D)** inscribed/central factor error.

Circle angles: central, inscribed & arcs — Q3
Correct: **(D)**. *Traps:* **(A)** inscribed/central factor error; **(B)** inscribed/central factor error; **(C)** inscribed/central factor error.

Circle angles: central, inscribed & arcs — Q4
Correct: **(A)**. *Traps:* **(B)** semicircle error; **(C)** semicircle error; **(D)** semicircle error.

Circle angles: central, inscribed & arcs — Q5
Correct: **(B)**. *Traps:* **(A)** inscribed/central factor error; **(C)** inscribed/central factor error; **(D)** arc-measure vs central-angle confusion.

Scale factor: length, area & volume — Q1
Correct: **(A)**. *Traps:* **(B)** linear-factor-to-area/volume error; **(C)** area/volume exponent swap; **(D)** linear-factor-to-area/volume error.

Scale factor: length, area & volume — Q2
Correct: **(D)**. *Traps:* **(A)** area/volume exponent swap; **(B)** linear-factor-to-area/volume error; **(C)** area/volume exponent swap.

Scale factor: length, area & volume — Q3
Correct: **(C)**. *Traps:* **(A)** area/volume exponent swap; **(B)** linear-factor-to-area/volume error; **(D)** linear-factor-to-area/volume error.

Scale factor: length, area & volume — Q4
Correct: **(A)**. *Traps:* **(B)** reverse scale error; **(C)** reverse scale error; **(D)** reverse scale error.

Scale factor: length, area & volume — Q5
Correct: **(B)**. *Traps:* **(A)** reverse scale error; **(C)** reverse scale error; **(D)** area/volume exponent swap.

Circle equations: center & radius by completing the square — Q1
Correct: **(C)**. *Traps:* **(A)** half-then-square error; **(B)** half-then-square error; **(D)** half-then-square error.

Circle equations: center & radius by completing the square — Q2
Correct: **(C)**. *Traps:* **(A)** center-sign/radius error; **(B)** center-sign/radius error; **(D)** center-sign/radius error.

Circle equations: center & radius by completing the square — Q3
Correct: **(D)**. *Traps:* **(A)** center-sign/radius error; **(B)** center-sign/radius error; **(C)** center-sign/radius error.

Circle equations: center & radius by completing the square — Q4
Correct: **(A)**. *Traps:* **(B)** balance error; **(C)** balance error; **(D)** balance error.

Circle equations: center & radius by completing the square — Q5
Correct: **(A)**. *Traps:* **(B)** half-then-square error; **(C)** center-sign/radius error; **(D)** center-sign/radius error.

Student-Produced Response

SPR answer entry: format & symbols — Q1
Correct: **(D)**. *Traps:* **(A)** mixed-number entry error; **(B)** mixed-number entry error; **(C)** mixed-number entry error.

SPR answer entry: format & symbols — Q2
Correct: **(A)**. *Traps:* **(B)** symbol inclusion error; **(C)** symbol inclusion error; **(D)** symbol inclusion error.

SPR answer entry: format & symbols — Q3
Correct: **(C)**. *Traps:* **(A)** decimal precision error; **(B)** decimal precision error; **(D)** decimal precision error.

SPR answer entry: format & symbols — Q4
Correct: **(C)**. *Traps:* **(A)** percent-not-converted error; **(B)** percent-not-converted error; **(D)** percent-not-converted error.

SPR answer entry: format & symbols — Q5
Correct: **(B)**. *Traps:* **(A)** mixed-number entry error; **(C)** decimal precision error; **(D)** mixed-number entry error.

Entering negatives & decimals — Q1
Correct: **(C)**. *Traps:* **(A)** space not decimal; **(B)** dropped the decimal part; **(D)** dropped the sign.

Entering negatives & decimals — Q2
Correct: **(B)**. *Traps:* **(A)** space not decimal; **(C)** dropped the decimal part; **(D)** dropped the sign.

Entering negatives & decimals — Q3
Correct: **(C)**. *Traps:* **(A)** dropped the sign; **(B)** space not decimal; **(D)** dropped the decimal part.

Entering negatives & decimals — Q4
Correct: **(A)**. *Traps:* **(B)** dropped the decimal part; **(C)** dropped the sign; **(D)** space not decimal.

Entering negatives & decimals — Q5
Correct: **(C)**. *Traps:* **(A)** space not decimal; **(B)** dropped the sign; **(D)** dropped the decimal part.

Fractions vs decimals: rounding & truncation — Q1
Correct: **(D)**. *Traps:* **(A)** ratio notation; **(B)** inverted; **(C)** truncated too short.

Fractions vs decimals: rounding & truncation — Q2
Correct: **(A)**. *Traps:* **(B)** inverted; **(C)** truncated too short; **(D)** ratio notation.

Fractions vs decimals: rounding & truncation — Q3
Correct: **(B)**. *Traps:* **(A)** ratio notation; **(C)** inverted; **(D)** truncated too short.

Fractions vs decimals: rounding & truncation — Q4
Correct: **(B)**. *Traps:* **(A)** ratio notation; **(C)** inverted; **(D)** truncated too short.

Fractions vs decimals: rounding & truncation — Q5
Correct: **(D)**. *Traps:* **(A)** inverted; **(B)** ratio notation; **(C)** truncated too short.

Converting percents & mixed numbers — Q1
Correct: **(A)**. *Traps:* **(B)** no conversion; **(C)** kept the % sign; **(D)** divided by 10.

Converting percents & mixed numbers — Q2
Correct: **(C)**. *Traps:* **(A)** divided by 10; **(B)** kept the % sign; **(D)** no conversion.

Converting percents & mixed numbers — Q3
Correct: **(B)**. *Traps:* **(A)** no conversion; **(C)** kept the % sign; **(D)** divided by 10.

Converting percents & mixed numbers — Q4
Correct: **(B)**. *Traps:* **(A)** kept the % sign; **(C)** no conversion; **(D)** divided by 10.

Converting percents & mixed numbers — Q5
Correct: **(C)**. *Traps:* **(A)** kept the % sign; **(B)** divided by 10; **(D)** no conversion.

Information and Ideas

Central ideas and main purpose — Q1
Correct: **(B)**. *Traps:* **(A)** Half-Right; **(C)** Too Narrow; **(D)** Too Broad.

Central ideas and main purpose — Q2
Correct: **(B)**. *Traps:* **(A)** Wrong Action; **(C)** Too Broad; **(D)** Too Narrow.

Central ideas and main purpose — Q3
Correct: **(A)**. *Traps:* **(B)** Too Narrow; **(C)** Half-Right; **(D)** Too Broad.

Central ideas and main purpose — Q4
Correct: **(B)**. *Traps:* **(A)** Too Broad; **(C)** Wrong Action; **(D)** Too Narrow.

Central ideas and main purpose — Q5
Correct: **(D)**. *Traps:* **(A)** Too Narrow; **(B)** Too Broad; **(C)** Half-Right.

Supporting details (locating a specific detail) — Q1
Correct: **(C)**. *Traps:* **(A)** Extreme Wording; **(B)** Outside Knowledge; **(D)** Detail Swap.

Supporting details (locating a specific detail) — Q2
Correct: **(D)**. *Traps:* **(A)** Detail Swap; **(B)** Detail Swap; **(C)** Extreme Wording.

Supporting details (locating a specific detail) — Q3
Correct: **(D)**. *Traps:* **(A)** Extreme Wording; **(B)** Detail Swap; **(C)** Close but Off.

Supporting details (locating a specific detail) — Q4
Correct: **(A)**. *Traps:* **(B)** Detail Swap; **(C)** Outside Knowledge; **(D)** Close but Off.

Supporting details (locating a specific detail) — Q5
Correct: **(D)**. *Traps:* **(A)** Detail Swap; **(B)** Close but Off; **(C)** Outside Knowledge.

Command of textual evidence — Q1
Correct: **(D)**. *Traps:* **(A)** Off-Claim; **(B)** Neutral Fact; **(C)** Backwards.

Command of textual evidence — Q2
Correct: **(D)**. *Traps:* **(A)** Neutral Fact; **(B)** Neutral Fact; **(C)** Off-Claim.

Command of textual evidence — Q3
Correct: **(C)**. *Traps:* **(A)** Off-Claim; **(B)** Neutral Fact; **(D)** Off-Claim.

Command of textual evidence — Q4
Correct: **(C)**. *Traps:* **(A)** Off-Claim; **(B)** Overreach; **(D)** Neutral Fact.

Command of textual evidence — Q5
Correct: **(B)**. *Traps:* **(A)** Off-Claim; **(C)** Neutral Fact; **(D)** Neutral Fact.

Command of quantitative evidence — Q1
Correct: **(B)**. *Traps:* **(A)** Number Not There; **(C)** Flipped Comparison; **(D)** Number Not There.

Command of quantitative evidence — Q2
Correct: **(C)**. *Traps:* **(A)** Number Not There; **(B)** Number Not There; **(D)** Flipped Comparison.

Command of quantitative evidence — Q3
Correct: **(D)**. *Traps:* **(A)** Flipped Comparison; **(B)** Number Not There; **(C)** Unsupported Leap.

Command of quantitative evidence — Q4
Correct: **(C)**. *Traps:* **(A)** Number Not There; **(B)** Right Data, Wrong Point; **(D)** Flipped Comparison.

Command of quantitative evidence — Q5
Correct: **(B)**. *Traps:* **(A)** Number Not There; **(C)** Flipped Comparison; **(D)** Number Not There.

Inferences — Q1
Correct: **(B)**. *Traps:* **(A)** Contradiction; **(C)** Overstated; **(D)** Reversed Logic.

Inferences — Q2
Correct: **(A)**. *Traps:* **(B)** Overstated; **(C)** Contradiction; **(D)** Needs More.

Inferences — Q3
Correct: **(D)**. *Traps:* **(A)** Overstated; **(B)** Needs More; **(C)** Contradiction.

Inferences — Q4
Correct: **(C)**. *Traps:* **(A)** Reversed Logic; **(B)** Needs More; **(D)** Contradiction.

Inferences — Q5
Correct: **(D)**. *Traps:* **(A)** Contradiction; **(B)** Contradiction; **(C)** Needs More.

Craft and Structure

Words in context — Q1
Correct: **(A)**. *Traps:* **(B)** Near-opposite; **(C)** Clue-blind; **(D)** Right neighborhood, wrong nuance.

Words in context — Q2
Correct: **(C)**. *Traps:* **(A)** Right neighborhood, wrong nuance; **(B)** Clue-blind; **(D)** Near-opposite.

Words in context — Q3
Correct: **(A)**. *Traps:* **(B)** Near-opposite; **(C)** Right neighborhood, wrong nuance; **(D)** Clue-blind.

Words in context — Q4
Correct: **(C)**. *Traps:* **(A)** Right neighborhood, wrong nuance; **(B)** Wrong sense; **(D)** Near-opposite.

Words in context — Q5
Correct: **(D)**. *Traps:* **(A)** Near-opposite; **(B)** Right neighborhood, wrong nuance; **(C)** Wrong sense.

Words in context: precise connotation — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong charge; **(B)** Wrong charge; **(C)** Too strong.

Words in context: precise connotation — Q2
Correct: **(C)**. *Traps:* **(A)** Wrong charge; **(B)** Denotation-only; **(D)** Too weak.

Words in context: precise connotation — Q3
Correct: **(C)**. *Traps:* **(A)** Wrong charge; **(B)** Wrong charge; **(D)** Too strong.

Words in context: precise connotation — Q4
Correct: **(B)**. *Traps:* **(A)** Wrong charge; **(C)** Denotation-only; **(D)** Wrong charge.

Words in context: precise connotation — Q5
Correct: **(C)**. *Traps:* **(A)** Too weak; **(B)** Wrong charge; **(D)** Wrong charge.

Text structure and purpose — Q1
Correct: **(B)**. *Traps:* **(A)** Half-right; **(C)** Reversed move; **(D)** Overreach.

Text structure and purpose — Q2
Correct: **(B)**. *Traps:* **(A)** Half-right; **(C)** Reversed move; **(D)** Wrong location.

Text structure and purpose — Q3
Correct: **(B)**. *Traps:* **(A)** Half-right; **(C)** Content, not function; **(D)** Reversed move.

Text structure and purpose — Q4
Correct: **(C)**. *Traps:* **(A)** Wrong location; **(B)** Wrong location; **(D)** Wrong location.

Text structure and purpose — Q5
Correct: **(A)**. *Traps:* **(B)** Half-right; **(C)** Reversed move; **(D)** Content, not function.

Purpose of a detail — Q1
Correct: **(C)**. *Traps:* **(A)** Off-tone; **(B)** Literal echo; **(D)** Too broad.

Purpose of a detail — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong claim; **(C)** Off-tone; **(D)** True but idle.

Purpose of a detail — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong claim; **(B)** Wrong claim; **(C)** Off-tone.

Purpose of a detail — Q4
Correct: **(C)**. *Traps:* **(A)** True but idle; **(B)** Off-tone; **(D)** Wrong claim.

Purpose of a detail — Q5
Correct: **(A)**. *Traps:* **(B)** Wrong claim; **(C)** Wrong claim; **(D)** Literal echo.

Cross-text connections — Q1
Correct: **(B)**. *Traps:* **(A)** Outside view; **(C)** Flipped speaker; **(D)** Overstated clash.

Cross-text connections — Q2
Correct: **(C)**. *Traps:* **(A)** Topic-only match; **(B)** Outside view; **(D)** Outside view.

Cross-text connections — Q3
Correct: **(C)**. *Traps:* **(A)** Outside view; **(B)** Topic-only match; **(D)** Flipped speaker.

Cross-text connections — Q4
Correct: **(A)**. *Traps:* **(B)** Overstated clash; **(C)** Outside view; **(D)** One-passage answer.

Cross-text connections — Q5
Correct: **(B)**. *Traps:* **(A)** Outside view; **(C)** Flipped speaker; **(D)** Overstated clash.

Expression of Ideas

Transitions: choosing the logical connector — Q1
Correct: **(A)**. *Traps:* **(B)** Wrong family; **(C)** False contrast; **(D)** False contrast.

Transitions: choosing the logical connector — Q2
Correct: **(C)**. *Traps:* **(A)** False contrast; **(B)** Fake causation; **(D)** Wrong family.

Transitions: choosing the logical connector — Q3
Correct: **(C)**. *Traps:* **(A)** Fake causation; **(B)** Wrong family; **(D)** Wrong family.

Transitions: choosing the logical connector — Q4
Correct: **(C)**. *Traps:* **(A)** Fake causation; **(B)** Redundant addition; **(D)** False contrast.

Transitions: choosing the logical connector — Q5
Correct: **(C)**. *Traps:* **(A)** Wrong family; **(B)** False contrast; **(D)** False contrast.

Transitions: diagnosing the relationship — Q1
Correct: **(C)**. *Traps:* **(A)** Overreading contrast; **(B)** Surface-word matching; **(D)** Overreading contrast.

Transitions: diagnosing the relationship — Q2
Correct: **(D)**. *Traps:* **(A)** Overreading contrast; **(B)** Overreading contrast; **(C)** Surface-word matching.

Transitions: diagnosing the relationship — Q3
Correct: **(D)**. *Traps:* **(A)** Consequence mistaken for addition; **(B)** Surface-word matching; **(C)** Restatement mistaken for example.

Transitions: diagnosing the relationship — Q4
Correct: **(A)**. *Traps:* **(B)** Restatement mistaken for example; **(C)** Overreading contrast; **(D)** Consequence mistaken for addition.

Transitions: diagnosing the relationship — Q5
Correct: **(B)**. *Traps:* **(A)** Overreading contrast; **(C)** Overreading contrast; **(D)** Time-word overreach.

Rhetorical synthesis: state a main point from notes — Q1
Correct: **(D)**. *Traps:* **(A)** One-note answer; **(B)** Too narrow; **(C)** Unsupported addition.

Rhetorical synthesis: state a main point from notes — Q2
Correct: **(B)**. *Traps:* **(A)** Too narrow; **(C)** One-note answer; **(D)** Vague filler.

Rhetorical synthesis: state a main point from notes — Q3
Correct: **(A)**. *Traps:* **(B)** One-note answer; **(C)** Too narrow; **(D)** Vague filler.

Rhetorical synthesis: state a main point from notes — Q4
Correct: **(B)**. *Traps:* **(A)** One-note answer; **(C)** One-note answer; **(D)** Too narrow.

Rhetorical synthesis: emphasize a similarity or difference — Q1
Correct: **(B)**. *Traps:* **(A)** Off-axis comparison; **(C)** Opposite relationship; **(D)** Single-item answer.

Rhetorical synthesis: emphasize a similarity or difference — Q2
Correct: **(C)**. *Traps:* **(A)** Single-item answer; **(B)** Opposite relationship; **(D)** Opposite relationship.

Rhetorical synthesis: emphasize a similarity or difference — Q3
Correct: **(B)**. *Traps:* **(A)** Opposite relationship; **(C)** Opposite relationship; **(D)** Single-item answer.

Rhetorical synthesis: emphasize a similarity or difference — Q4
Correct: **(C)**. *Traps:* **(A)** Single-item answer; **(B)** Opposite relationship; **(D)** Opposite relationship.

Rhetorical synthesis: introduce a topic to a specific audience — Q1
Correct: **(C)**. *Traps:* **(A)** Wrong pitch; **(B)** Off-topic opener; **(D)** Detail without frame.

Rhetorical synthesis: introduce a topic to a specific audience — Q2
Correct: **(D)**. *Traps:* **(A)** Detail without frame; **(B)** Wrong job; **(C)** Off-topic opener.

Rhetorical synthesis: introduce a topic to a specific audience — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong pitch; **(B)** Wrong job; **(C)** Off-topic opener.

Rhetorical synthesis: introduce a topic to a specific audience — Q4
Correct: **(B)**. *Traps:* **(A)** Off-topic opener; **(C)** Detail without frame; **(D)** Wrong pitch.

Standard English Conventions

Sentence Boundaries — Q1
Correct: **(D)**. *Traps:* **(A)** Comma splice; **(B)** Fused sentence (run-on); **(C)** Missing conjunction.

Sentence Boundaries — Q2
Correct: **(B)**. *Traps:* **(A)** Fused sentence (run-on); **(C)** Comma splice; **(D)** Illegal full stop.

Sentence Boundaries — Q3
Correct: **(C)**. *Traps:* **(A)** Fused sentence (run-on); **(B)** Missing conjunction; **(D)** Comma splice.

Sentence Boundaries — Q4
Correct: **(C)**. *Traps:* **(A)** Illegal full stop; **(B)** Missing conjunction; **(D)** Illegal full stop.

Sentence Boundaries — Q5
Correct: **(B)**. *Traps:* **(A)** Fused sentence (run-on); **(C)** Illegal full stop; **(D)** Missing conjunction.

Punctuation Within a Sentence — Q1
Correct: **(C)**. *Traps:* **(A)** Unmatched single comma; **(B)** Colon after a fragment (semicolon misused as an introducing mark); **(D)** Colon after a fragment.

Punctuation Within a Sentence — Q2
Correct: **(A)**. *Traps:* **(B)** Mismatched pair; **(C)** Mismatched pair; **(D)** Mismatched pair.

Punctuation Within a Sentence — Q3
Correct: **(C)**. *Traps:* **(A)** Unmatched single comma; **(B)** Colon after a fragment; **(D)** Restrictive over-punctuation (nonessential left unmarked).

Punctuation Within a Sentence — Q4
Correct: **(A)**. *Traps:* **(B)** Colon after a fragment ("such as" as an improper introducer); **(C)** Colon after a fragment (semicolon before a non-clause); **(D)** Unmatched single comma.

Punctuation Within a Sentence — Q5
Correct: **(D)**. *Traps:* **(A)** Restrictive over-punctuation; **(B)** Mismatched pair (lone dash); **(C)** Colon after a fragment.

Subject–Verb Agreement — Q1
Correct: **(A)**. *Traps:* **(B)** Intervening-phrase attraction; **(C)** Intervening-phrase attraction; **(D)** Intervening-phrase attraction.

Subject–Verb Agreement — Q2
Correct: **(D)**. *Traps:* **(A)** Inverted-order slip; **(B)** Inverted-order slip; **(C)** Inverted-order slip.

Subject–Verb Agreement — Q3
Correct: **(D)**. *Traps:* **(A)** Collective miscount; **(B)** Collective miscount; **(C)** Collective miscount.

Subject–Verb Agreement — Q4
Correct: **(A)**. *Traps:* **(B)** Or/nor mismatch; **(C)** Or/nor mismatch; **(D)** Or/nor mismatch.

Subject–Verb Agreement — Q5
Correct: **(B)**. *Traps:* **(A)** Intervening-phrase attraction; **(C)** Intervening-phrase attraction; **(D)** Intervening-phrase attraction.

Pronoun Agreement and Clarity — Q1
Correct: **(D)**. *Traps:* **(A)** Number disagreement; **(B)** Number disagreement; **(C)** Number disagreement.

Pronoun Agreement and Clarity — Q2
Correct: **(A)**. *Traps:* **(B)** Ambiguous reference; **(C)** Number disagreement; **(D)** Missing antecedent.

Pronoun Agreement and Clarity — Q3
Correct: **(B)**. *Traps:* **(A)** Collective disagreement; **(C)** Collective disagreement; **(D)** Number disagreement (its/it's form error).

Pronoun Agreement and Clarity — Q4
Correct: **(C)**. *Traps:* **(A)** Number disagreement; **(B)** Number disagreement; **(D)** Number disagreement.

Pronoun Agreement and Clarity — Q5
Correct: **(D)**. *Traps:* **(A)** Ambiguous reference; **(B)** Missing antecedent; **(C)** Number disagreement.

Verb Forms and Tense Consistency — Q1
Correct: **(B)**. *Traps:* **(A)** Sequence mismatch; **(C)** Wrong perfect; **(D)** Unmotivated tense shift.

Verb Forms and Tense Consistency — Q2
Correct: **(B)**. *Traps:* **(A)** Unmotivated tense shift; **(C)** Unmotivated tense shift; **(D)** Unmotivated tense shift.

Verb Forms and Tense Consistency — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong perfect; **(B)** Sequence mismatch; **(C)** Wrong perfect.

Verb Forms and Tense Consistency — Q4
Correct: **(B)**. *Traps:* **(A)** Unmotivated tense shift; **(C)** Wrong perfect; **(D)** Sequence mismatch.

Verb Forms and Tense Consistency — Q5
Correct: **(C)**. *Traps:* **(A)** Sequence mismatch; **(B)** Faulty participle; **(D)** Unmotivated tense shift.

Modifiers and Parallel Structure — Q1
Correct: **(C)**. *Traps:* **(A)** Dangling modifier; **(B)** Dangling modifier; **(D)** Dangling modifier.

Modifiers and Parallel Structure — Q2
Correct: **(C)**. *Traps:* **(A)** Faulty parallelism; **(B)** Faulty parallelism; **(D)** Faulty parallelism.

Modifiers and Parallel Structure — Q3
Correct: **(A)**. *Traps:* **(B)** Dangling modifier; **(C)** Dangling modifier; **(D)** Dangling modifier.

Modifiers and Parallel Structure — Q4
Correct: **(C)**. *Traps:* **(A)** Correlative mismatch; **(B)** Correlative mismatch; **(D)** Correlative mismatch.

Modifiers and Parallel Structure — Q5
Correct: **(C)**. *Traps:* **(A)** Faulty parallelism; **(B)** Faulty parallelism; **(D)** Faulty parallelism.

Part IV

Full-Length Practice Test

PSAT/NMSQT Practice Test

This full-length practice test mirrors the digital PSAT/NMSQT: a Reading and Writing section (two modules of 27 questions, 32 minutes each) and a Math section (two modules of 22 questions, 35 minutes each). Time yourself, then use the answer keys that follow each section—and for every miss, name the trap.

Module 1 — Reading and Writing

1. *When the botanist Elena Vargas described the desert bloom, she was careful not to overstate its scale. The event, she noted, was modest in extent yet remarkable in timing—a brief answer to an unusually wet winter.*

As used in the text, "modest" most nearly means

(A) shy
(B) limited
(C) affordable
(D) humble

2. *The engineer's proposal was not merely ambitious; it was audacious. Where colleagues had suggested reinforcing the old bridge, she argued for demolishing it entirely and beginning again.*

As used in the text, "audacious" most nearly means

(A) dishonest
(B) careless
(C) boldly daring
(D) well funded

3. *Early critics dismissed the composer's late symphonies as chaotic. Later scholars, however, found in the same works a deliberate architecture, arguing that what sounded like disorder was in fact a carefully governed freedom.*

Which choice best describes the overall structure of the text?

(A) It traces a single listener's changing taste over a lifetime.

(B) It presents an initial judgment and then a revised interpretation.

(C) It lists the technical features of a musical form in order.

(D) It compares two composers working in different centuries.

4. *Text 1: Marine biologist Dr. Okafor contends that coral reefs can recover from bleaching if given enough undisturbed time, pointing to reefs that regained color within a decade.*

Text 2: Dr. Lindqvist counters that "recovery" in color is not the same as recovery in function; a reef may look healthy while its web of species remains badly thinned.

Based on the texts, how would Dr. Lindqvist (Text 2) most likely respond to the claim in Text 1?

(A) By agreeing that a decade is sufficient for full recovery.

(B) By noting that visible color can mask an incomplete recovery.

(C) By denying that bleaching harms reefs at all.

(D) By arguing that reefs never regain their original color.

5. *The novelist prized economy above all. A single vivid detail, she believed, could do the work of a paragraph, and she pruned her drafts until only the load-bearing words remained.*

As used in the text, "economy" most nearly means

(A) national wealth

(B) thrift with words

(C) quiet mood

(D) personal profit

6. *The historian opens her chapter with a puzzle: why did a prosperous port city abandon its harbor within a single generation? Only after laying out the mystery does she introduce the silt-choked river that supplies the answer.*

Which choice best describes the function of the underlined portion in the text as a whole? (The chapter's opening question.)

(A) It sets up a problem that the rest of the chapter works to explain.

(B) It summarizes a conclusion the historian has already proven.

(C) It dismisses an idea that other historians have proposed.

(D) It provides statistical support for the city's wealth.

7. *The metallurgist warned that the alloy's strength was provisional. Under ordinary loads it held admirably, but repeated stress, she cautioned, could reveal weaknesses that a single test would never expose.*

As used in the text, "provisional" most nearly means

(A) temporary or conditional

(B) officially approved

(C) widely admired

(D) exceptionally great

8. *Text 1: Urban planner Reyes argues that widening highways eases congestion by adding capacity for a growing population.*

Text 2: Transit researcher Osei points to "induced demand": new lanes, she finds, quickly fill with drivers who previously avoided the road, leaving traffic much as it was.

Based on the texts, Osei (Text 2) would most likely characterize the relief Reyes describes as

(A) permanent and substantial

(B) short-lived, because the new capacity attracts more drivers

(C) impossible to measure by any method

(D) limited only to freight traffic

9. *Fireflies do not glow at random. Each species flashes in a distinct rhythm, and females respond only to the pattern of their own kind. In this way a meadow crowded with several species remains, in effect, several separate conversations.*

Which choice best states the main idea of the text?

(A) Fireflies are found only in crowded meadows.

(B) Species-specific flash patterns keep firefly signals distinct.

(C) Female fireflies cannot produce light of their own.

(D) All firefly species share a single flashing rhythm.

10. *Archaeologists excavating the settlement found grinding stones in nearly every dwelling but ovens in only a few. The pattern suggests that grain was processed at home while baking was a shared, communal task.*

Which finding, if true, would most directly support the researchers' interpretation?

(A) The grinding stones were made of imported rock.

(B) The few ovens were unusually large and centrally located.

(C) The dwellings were built at different times.

(D) The settlement was abandoned during a drought.

11. *A student studying migratory songbirds recorded that individuals raised in captivity, having never seen an experienced adult, still oriented in the correct seasonal direction on their first flight. The student concluded that the birds' basic sense of direction is _____.*

Which choice most logically completes the text?

(A) learned entirely by imitating older birds

(B) inborn rather than taught

(C) unrelated to the changing seasons

(D) different for every individual

12. *In a survey of 400 commuters, those who biked to work reported higher satisfaction than those who drove, even though bikers' trips took, on average, eight minutes longer. Researchers noted that trip length alone did not predict how commuters felt about the journey.*

Which choice best states the main idea of the text?

(A) Biking is always faster than driving to work.

(B) Longer commutes are universally disliked.

(C) Commuter satisfaction depended on more than trip duration.

(D) Most commuters in the survey chose to bike.

13. *The geologist noted a curious detail in the canyon wall: a thin band of white ash lodged between darker layers of sandstone. Because such ash can be dated precisely, the band offered a reliable marker for aligning rock layers across the region.*

According to the text, why is the ash band useful to geologists?

(A) It is the thickest layer in the canyon wall.

(B) It can be dated precisely, allowing layers to be aligned.

(C) It is composed of the same material as sandstone.

(D) It appears only at the very top of the formation.

14. *A team testing a new water filter reported that it removed 99 percent of a common bacterium in laboratory trials. The lead researcher cautioned, however, that field conditions—muddy water, inconsistent handling—might reduce that figure, and she called for trials outside the lab before any broad claims.*

Which choice best describes the researcher's attitude toward the laboratory result?

(A) She regards it as final proof of the filter's success.

(B) She considers it promising but not yet conclusive.

(C) She believes it overstates the filter's dangers.

(D) She dismisses it as irrelevant to real use.

15. *The poet Dawit Bekele rarely published. Over forty years he released just three slim volumes, revising individual poems for years before he judged them finished. Admirers came to see this slowness not as a flaw but as the very source of his precision.*

Which choice best states the main idea of the text?

(A) Bekele's careful, unhurried process was central to the quality of his work.

(B) Bekele published more than most poets of his era.

(C) Bekele regretted revising his poems so often.

(D) Bekele's admirers wished he had written faster.

16. *The observatory's newest telescope, funded by a coalition of universities, _____ images sharp enough to distinguish individual stars in distant galaxies.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) produce

(B) producing

(C) produces

(D) to produce

17. *Because the trail climbed steeply for the first mile_____ the hikers rested at the ridge before continuing.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) mile,

(B) mile

(C) mile;

(D) mile:

18. *The museum acquired three paintings last spring: a landscape by an unknown artist, a portrait once owned by a duchess_____ and a still life recovered from a private collection.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) duchess;

(B) duchess

(C) duchess,

(D) duchess:

19. *Neither the coach nor the players _____ the referee's final call, though several of them muttered as they left the field.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) disputes

(B) disputing

(C) having disputed

(D) disputed

20. *Having studied the currents for weeks_____ the sailors plotted a route that used the tide rather than fighting it.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) , the sailors plotted

(B) the sailors plotted

(C) and the sailors plotted

(D) ; the sailors plotted

21. *The recipe calls for ingredients that many cooks already _____ on hand: flour, salt, and a single egg.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) keeps

(B) keep

(C) keeping

(D) to keep

22. *The city council approved the new park_____ residents had petitioned for it for nearly a decade.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) park,

(B) park; residents

(C) park residents

(D) park: residents,

23. *Solar panels convert sunlight directly into electricity. ____ they generate no exhaust and require no fuel once installed, making them attractive in regions that prize clean air.*

Which choice completes the text with the most logical transition?

(A) Nevertheless,

(B) As a result,

(C) By contrast,

(D) For example,

24. *Many gardeners assume that richer soil always yields larger vegetables. ____ some crops, such as carrots, grow best in lean, loose ground that lets their roots stretch downward.*

Which choice completes the text with the most logical transition?

(A) In fact,

(B) Therefore,

(C) Similarly,

(D) In addition,

25. *The lecture ran nearly two hours over its scheduled time. ____ several attendees stayed afterward to ask the speaker still more questions.*

Which choice completes the text with the most logical transition?

(A) Consequently,

(B) Even so,

(C) For instance,

(D) Meanwhile,

26. *While researching a local river, a student took the following notes:*

- The Maren River flows for 120 kilometers through farmland.
- In the 1970s, runoff from farms lowered the river's water quality.
- Farmers began planting buffer strips of grass along the banks in 1995.
- By 2015, water-quality tests showed marked improvement.

The student wants to emphasize the effect of the buffer strips on the river. Which choice most effectively uses relevant information from the notes to accomplish this goal?

(A) The Maren River flows for 120 kilometers through farmland.

(B) After farmers planted buffer strips of grass beginning in 1995, water-quality tests by 2015 showed marked improvement.

(C) Runoff from farms lowered the river's water quality in the 1970s.

(D) The Maren River passes through farmland on its way downstream.

27. *While preparing a report on a painter, a student took the following notes:*

- Amara Sow was a self-taught painter born in 1948.
- She worked mainly with materials she found or salvaged.
- Critics praised her bold use of reclaimed metal and cloth.
- Her largest work, "Tidewall," was assembled entirely from discarded objects.

The student wants to introduce Amara Sow's approach to materials to an audience unfamiliar with her. Which choice most effectively uses relevant information from the notes to accomplish this goal?

(A) Amara Sow, born in 1948, created her largest work in her studio.

(B) Critics have written a great deal about painters of the twentieth century.

(C) Amara Sow, a self-taught painter, built her acclaimed works—including "Tidewall"—from found and salvaged materials.

(D) Amara Sow was born in 1948 and became a painter.

Module 2 — Reading and Writing

1. *The diplomat was known for her tact. Even when negotiations collapsed, she chose her words with such care that the other side left the table still willing to return to it.*

As used in the text, "tact" most nearly means

(A) physical touch

(B) skill in handling delicate situations

(C) strict honesty

(D) quick temper

2. *The reformer's speeches were famously plain. He shunned ornament, believing that a clear argument delivered in ordinary language would persuade more listeners than any flourish.*

As used in the text, "plain" most nearly means

(A) flat in landscape

(B) unattractive

(C) simple and unadorned

(D) openly visible

3. *The essay begins by picturing a familiar scene—a child stacking blocks—before turning that image into a claim about how all learning proceeds: by building, toppling, and building again.*

Which choice best describes the overall structure of the text?

(A) It moves from a concrete image to a broader general claim.

(B) It lists the stages of childhood development in order.

(C) It contrasts two competing theories of education.

(D) It recounts a single event from the author's own childhood.

4. *Text 1: Economist Haddad argues that remote work benefits small towns, since well-paid employees can now live far from expensive cities and spend locally.*

Text 2: Sociologist Bianchi cautions that these newcomers often keep their old-city habits and salaries, driving up local rents faster than wages for longtime residents rise.

Based on the texts, Bianchi (Text 2) would most likely add which qualification to Haddad's argument?

(A) Remote work harms small towns in every respect.

(B) The local spending Haddad praises may come with rising costs for existing residents.

(C) Remote workers rarely move to small towns at all.

(D) Small towns were already more expensive than cities.

5. *The chemist described the catalyst as indispensable. Without it the reaction crept along for days; with even a trace of it, the same change completed in minutes.*

As used in the text, "indispensable" most nearly means

(A) easily replaced

(B) absolutely necessary

(C) mildly helpful

(D) difficult to obtain

6. *After three paragraphs describing the frog's shrinking habitat, the author abruptly shifts: "But the frogs, it turns out, are not the story's victims—they are its warning." The remainder of the essay treats the species as an early signal of wider trouble.*

Which choice best describes the function of the quoted sentence in the text?

(A) It marks a turn from describing a problem to reframing its significance.

(B) It offers statistical evidence for the frog's decline.

(C) It restates the essay's opening in different words.

(D) It concedes that the author's earlier claims were mistaken.

7. *The critic called the film's pacing languid. Scenes lingered; conversations unfolded in real time; and viewers seeking action grew restless long before the plot began to move.*

As used in the text, "languid" most nearly means

(A) unpleasantly loud

(B) slow and unhurried

(C) brightly colored

(D) tightly plotted

8. *Text 1: Nutritionist Park recommends eating a variety of whole grains, noting that each supplies a different mix of nutrients.*

Text 2: Dietitian Vasquez agrees that variety matters but adds that, for many people, the practical barrier is cost and availability rather than knowledge of which grains to eat.

Based on the texts, how do the two writers' positions relate?

(A) Vasquez rejects Park's recommendation entirely.

(B) Vasquez accepts Park's point but stresses obstacles to acting on it.

(C) Park and Vasquez disagree about whether grains are nutritious.

(D) Park argues that variety is unnecessary.

9. *Sea otters eat enormous quantities of sea urchins. Where otters thrive, urchin numbers stay low, and the kelp forests the urchins would otherwise devour grow thick, sheltering countless other species.*

Which choice best states the main idea of the text?

(A) Sea urchins are the primary predators of sea otters.

(B) By controlling urchins, sea otters help kelp forests support other life.

(C) Kelp forests grow best where otters are absent.

(D) Sea otters prefer to eat kelp rather than urchins.

10. *A linguist studying a small island community observed that its speakers switched languages depending on setting—one tongue at home, another at the market, a third in ceremonies. She argued that this pattern was not confusion but a finely tuned social skill.*

Which detail from the text best supports the linguist's argument?

(A) The community lived on a small island.

(B) Speakers matched each language to a specific setting.

(C) The linguist studied the community for a long time.

(D) Three languages were spoken in total.

11. *Researchers measured tree growth on two slopes of the same mountain. The wetter, north-facing slope produced taller trees, but the drier, south-facing trees had denser wood. The team concluded that the two environments favored _____.*

Which choice most logically completes the text?

(A) identical growth in both height and density

(B) different strengths—height in one, wood density in the other

(C) neither height nor density in either location

(D) taller trees only on the drier slope

12. *In a study of 300 households, families who ate dinner together at least four nights a week reported that their children had stronger vocabularies, even after accounting for the parents' education. The researchers stressed that shared meals, not family background alone, seemed to matter.*

Which choice best states the main idea of the text?

(A) Parents' education was the only factor affecting vocabulary.

(B) Frequent shared meals were linked to stronger child vocabularies.

(C) Children who ate alone had larger vocabularies.

(D) Most families in the study never ate together.

13. *The report noted a striking figure: although the island covers less than one percent of the nation's land, it is home to nearly a quarter of the country's bird species. Conservationists cited this concentration as a reason to protect the island's remaining forest.*

According to the text, why do conservationists want to protect the island's forest?

(A) The island makes up most of the nation's land area.

(B) A small area holds a large share of the country's bird species.

(C) The island has no remaining forest to protect.

(D) Birds on the island are found nowhere else on Earth.

14. *An engineer testing a bridge design reported that a scale model withstood loads far beyond expected traffic. She added a caution, however: scale models do not always behave like full-sized structures, and she urged testing at larger scales before construction.*

Which choice best describes the engineer's view of the model test?

(A) She treats it as sufficient grounds to begin building.

(B) She regards it as encouraging but in need of further testing.

(C) She concludes the design has already failed.

(D) She dismisses scale testing as pointless.

15. *The clockmaker Idris Farah left no written records, yet his surviving clocks reveal his method. Each one uses the same distinctive escapement, refined slightly from the last, so that the clocks together form a kind of silent notebook of his experiments.*

Which choice best states the main idea of the text?

(A) Farah's clocks, through their shared refined design, document his experimental process.

(B) Farah kept detailed written records of his work.

(C) Farah made only one clock in his lifetime.

(D) Farah's escapement changed completely with each clock.

16. *The archive's oldest documents, stored for decades in a cool basement vault_____ remain remarkably legible today.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) vault:

(B) vault,

(C) vault;

(D) vault

17. *The chef prepared a dish that surprised everyone_____ a dessert made entirely from vegetables.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) everyone,

(B) everyone:

(C) everyone

(D) everyone; and

18. *Each of the volunteers who spent the summer restoring the wetland _____ a certificate at the closing ceremony.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) receive
(B) receiving
(C) received
(D) to receive

19. *The report identified three causes of the delay: a shortage of parts, an unusually harsh winter_____ and a change in the project's leadership.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) winter,
(B) winter;
(C) winter
(D) winter:

20. *The orchestra, along with its two guest soloists, _____ scheduled to perform on Friday evening.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) are
(B) were
(C) being
(D) is

21. *Painted more than four centuries ago_____ the fresco still shows traces of its original blue pigment.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) , the fresco
(B) the fresco
(C) and the fresco
(D) ; the fresco

22. *The librarian reshelved the returned books quickly_____ the reading room was scheduled to close in ten minutes.*

Which choice completes the text so that it conforms to the conventions of Standard English?

(A) quickly,
(B) quickly; the

(C) quickly the

(D) quickly: the reading, room

23. *The new bus line runs every ten minutes during rush hour. ____ it stops within a short walk of nearly every neighborhood on the east side, making car-free commuting practical for many residents.*

Which choice completes the text with the most logical transition?

(A) However,

(B) Moreover,

(C) In contrast,

(D) Instead,

24. *Aluminum is one of the most abundant metals in the earth's crust. ____ it was once so difficult to extract that, in the nineteenth century, it was valued more highly than gold.*

Which choice completes the text with the most logical transition?

(A) Likewise,

(B) Yet

(C) Therefore,

(D) For example,

25. *The volunteers cleared the fallen branches from the path by noon. ____ they repaired the wooden railing that the storm had loosened.*

Which choice completes the text with the most logical transition?

(A) Nevertheless,

(B) Then

(C) By contrast,

(D) In conclusion,

26. *While researching an invention, a student took the following notes:*

- The safety elevator was demonstrated by Elisha Otis in 1854.
- Earlier elevators were considered too dangerous for passengers.
- Otis's design used a spring that gripped the rails if the cable broke.
- After the safety brake, tall passenger buildings became practical.

The student wants to explain why Otis's design was significant. Which choice most effectively uses relevant information from the notes to accomplish this goal?

(A) Elisha Otis demonstrated his elevator in the year 1854.

(B) Otis's safety brake, which gripped the rails if the cable broke, made tall passenger buildings practical.

(C) Earlier elevators were considered too dangerous for passengers to use.

(D) Elisha Otis was an inventor who worked in the nineteenth century.

27. *While preparing a presentation about a scientist, a student took the following notes:*

- Chien-Shiung Wu was an experimental physicist.
- In 1956, two theorists proposed that a certain symmetry might not hold.
- Wu designed a careful experiment to test the proposal.
- Her results confirmed the theorists' idea and reshaped particle physics.

The student wants to emphasize Wu's role in testing the theorists' proposal. Which choice most effectively uses relevant information from the notes to accomplish this goal?

(A) Chien-Shiung Wu was an experimental physicist by profession.

(B) In 1956, two theorists proposed that a certain symmetry might not hold.

(C) Wu designed the careful experiment that confirmed the theorists' 1956 proposal and reshaped particle physics.

(D) Particle physics changed a great deal during the twentieth century.

Answer Key — Reading and Writing Practice Test

Module 1, Q1
(B). "Modest in extent" pairs with "limited"; the bloom was small in scale.

Module 1, Q2
(C). "Audacious" extends "ambitious" to mean boldly daring.

Module 1, Q3
(B). The text gives an early judgment ("chaotic") then a revised reading by later scholars.

Module 1, Q4
(B). Text 2 distinguishes color from function—visible color can mask incomplete recovery.

Module 1, Q5
(B). Pruning drafts to essential words defines "economy" as thrift with words.

Module 1, Q6
(A). The opening question frames a problem the chapter then explains via the silted river.

Module 1, Q7
(A). "Provisional" strength holds under ordinary loads but is conditional—temporary or conditional.

Module 1, Q8
(B). "Induced demand" means new lanes fill with added drivers, so relief is short-lived.

Module 1, Q9
(B). The passage's point is that distinct flash patterns keep species' signals separate.

Module 1, Q10
(B). Central ovens support the claim of shared, communal baking.

Module 1, Q11
(B). Birds with no adult model still orient correctly, so direction is inborn, not taught.

Module 1, Q12
(C). Bikers were more satisfied despite longer trips, so duration alone didn't predict feeling.

Module 1, Q13
(B). The text states ash can be dated precisely, making it a marker to align layers.

Module 1, Q14
(B). She calls the lab result promising but insists on field trials—not yet conclusive.

Module 1, Q15
(A). The main idea is that Bekele's slow, careful process produced his precision.

Module 1, Q16
(C). Singular subject "telescope" takes "produces."

Module 1, Q17
(A). A comma correctly separates the introductory dependent clause from the main clause.

Module 1, Q18
(C). A comma continues the list of items in the series before "and."

Module 1, Q19
(D). Past-tense "disputed" is a finite verb; "neither/nor" with plural "players" fits a past-tense verb.

Module 1, Q20
(A). A comma follows the introductory participial phrase before the main clause.

Module 1, Q21
(B). Plural subject "cooks" takes "keep."

Module 1, Q22
(B). A semicolon correctly joins two independent clauses.

Module 1, Q23
(B). Clean air follows from having no exhaust—"As a result" signals cause and effect.

Module 1, Q24
(A). The second sentence corrects the assumption; "In fact" marks the contrast.

Module 1, Q25
(B). Staying to ask more despite the overrun is a concession—"Even so."

Module 1, Q26
(B). Only this choice links the buffer strips to the later water-quality improvement.

Module 1, Q27
(C). This choice introduces Sow and highlights her use of found and salvaged materials.

Module 2, Q1
(B). "Tact" is skill in handling delicate situations, shown by careful word choice.

Module 2, Q2
(C). "Plain" speech that "shuns ornament" means simple and unadorned.

Module 2, Q3
(A). The essay moves from a concrete image (a child's blocks) to a general claim about learning.

Module 2, Q4
(B). Bianchi qualifies Haddad by warning of rising costs for longtime residents.

Module 2, Q5
(B). Without the catalyst the reaction barely proceeds, so it is absolutely necessary.

Module 2, Q6
(A). The quoted line turns from describing decline to reframing the frog as a warning.

Module 2, Q7
(B). Lingering scenes and restless viewers indicate slow, unhurried pacing.

Module 2, Q8
(B). Vasquez accepts Park's point but stresses cost and availability as obstacles.

Module 2, Q9
(B). Otters control urchins, letting kelp forests thrive and shelter other species.

Module 2, Q10
(B). Matching each language to a setting supports the "finely tuned social skill" claim.

Module 2, Q11
(B). Each slope favored a different strength: height on one, density on the other.

Module 2, Q12
(B). Shared meals, independent of parents' education, were tied to stronger vocabularies.

Module 2, Q13
(B). A small area holding many bird species is the stated reason to protect the forest.

Module 2, Q14
(B). She finds the model encouraging but calls for larger-scale testing first.

Module 2, Q15
(A). The clocks' shared, gradually refined design records Farah's experiments.

Module 2, Q16
(B). A comma closes the nonessential phrase before the verb "remain."

Module 2, Q17
(B). A colon introduces the explanation of the surprising dish.

Module 2, Q18
(C). Singular subject "Each" takes singular past "received."

Module 2, Q19
(A). A comma continues the list before "and" in the series.

Module 2, Q20
(D). Singular subject "orchestra" ("along with" is parenthetical) takes "is."

Module 2, Q21
(A). A comma follows the introductory participial phrase before the main clause.

Module 2, Q22
(B). A semicolon joins two independent clauses.

Module 2, Q23
(B). Frequent service and wide coverage both add reasons—"Moreover."

Module 2, Q24
(B). Abundant yet once hard to extract is a contrast—"Yet."

Module 2, Q25
(B). Repairing the railing after clearing branches is sequential—"Then."

Module 2, Q26
(B). This choice ties the safety brake to making tall passenger buildings practical.

Module 2, Q27
(C). This choice foregrounds Wu's experiment as the test that confirmed the proposal.

Module 1 — Math

1. *A technician charges a flat fee plus an hourly rate. For a repair, the equation* $3x + 7 = 22$ *models the number of hours* x *billed on one job.*

   What is the value of x?

   (A) 3
   (B) 7
   (C) 15
   (D) 5

2. *A machine's output is modeled by the function* $f(x) = 4x - 9$*, where* x *is the number of input units.*

What is the value of $f(6)$?

(A) 15
(B) 24
(C) -9
(D) 33

3. *A jacket that originally costs \$80 is placed on sale at* 25% *off.*

What is the sale price of the jacket?

(A) \$20
(B) \$60
(C) \$75
(D) \$55

4. *A line in the* xy*-plane passes through the points* $(2, 3)$ *and* $(6, 11)$*.*

What is the slope of the line?

(A) $\frac{1}{2}$
(B) 4
(C) 2
(D) 8

5. *A recipe requires* 3 *cups of flour for every* 2 *cups of sugar. A baker uses* 12 *cups of flour and keeps the same ratio.*

How many cups of sugar are needed?

(A) 6
(B) 9
(C) 18
(D) 8

6. *Consider the expression* $x^2 - 2x + 5$ *evaluated at* $x = 3$*.*

What is the value of the expression?

(A) 8
(B) 14
(C) 20
(D) 2

7. *A triangular garden plot has a base of* 10 *meters and a height of* 6 *meters.*

What is the area of the plot, in square meters?

(A) 16
(B) 60
(C) 30
(D) 32

8. *Two numbers satisfy the system* $x + y = 10$ *and* $x - y = 4$.

What is the value of x?

(A) 3
(B) 5
(C) 6
(D) 7

9. *A car travels* 150 *miles in* 3 *hours at a constant speed.*

At this rate, how far will the car travel in 5 hours, in miles?

(A) 200
(B) 250
(C) 225
(D) 300

10. *Consider the expression* $\frac{x^7}{x^3}$, *where* $x \neq 0$.

Which of the following is an equivalent expression?

(A) x^4
(B) x^{10}
(C) x^{21}
(D) x^2

11. *A circular fountain has a radius of* 5 *feet.*

What is the circumference of the fountain, in feet?

(A) 5π
(B) 25π
(C) 100π
(D) 10π

12. *A student solves the inequality* $2x - 3 < 7$.

Which of the following describes all solutions?

(A) $x < 2$

(B) $x > 5$

(C) $x < 5$

(D) $x > 2$

13. *A data set consists of the four values* 4, 8, 10, *and* 14.

What is the mean of the data set?

(A) 9

(B) 8

(C) 10

(D) 36

14. *A phone plan charges \$30 per month plus \$0.10 for each text message sent.*

Which expression gives the total monthly cost, in dollars, for t text messages?

(A) $30t + 0.10$

(B) $30 + 10t$

(C) $3t$

(D) $0.10t + 30$

15. *A right triangle has legs of length* 6 *and* 8.

What is the length of the hypotenuse?

(A) 14

(B) 10

(C) 48

(D) $\sqrt{28}$

16. *Consider the quadratic equation* $x^2 - 5x + 6 = 0$.

What are its solutions?

(A) $x = 1,\ 6$

(B) $x = -2,\ -3$

(C) $x = 2,\ 3$

(D) $x = -1,\ -6$

17. *A bag contains* 3 *red marbles,* 5 *blue marbles, and* 2 *green marbles. One marble is drawn at random.*

What is the probability that the marble is blue?

(A) $\frac{1}{5}$

(B) $\frac{3}{10}$

(C) $\frac{5}{2}$

(D) $\frac{1}{2}$

18. *A gym charges a \$50 sign-up fee plus \$20 per month of membership.*

After how many months will the total cost equal \$170?

(A) 6

(B) 4

(C) 5

(D) 8

19. *A town's population of* 200 *residents increases by* 10% *over one year.*

What is the new population?

(A) 210

(B) 220

(C) 230

(D) 240

20. *A number x satisfies the equation* $5x - 4 = 26$.

What is the value of x? Grid in your answer.

21. *Two angles are complementary, and one of them measures* $35°$.

What is the measure, in degrees, of the other angle? Grid in your answer.

22. *A proportion is given by* $\frac{x}{8} = \frac{3}{4}$.

What is the value of x? Grid in your answer.

Module 2 — Math

1. *A ball is launched upward, and its height in feet after t seconds is* $h(t) = -16t^2 + 32t + 5$.

At what time t, in seconds, does the ball reach its maximum height?

(A) 0.5

(B) 2

(C) 1

(D) 5

2. *The area of a trapezoid is given by* $A = \frac{1}{2}(b_1 + b_2)h$.

Which of the following correctly solves this formula for h?

(A) $\frac{A}{2(b_1+b_2)}$

(B) $\frac{b_1+b_2}{2A}$

(C) $2A(b_1 + b_2)$

(D) $\frac{2A}{b_1+b_2}$

3. *A bacteria culture doubles in size every hour and starts with* 50 *cells.*

How many cells are present after 3 hours?

(A) 150

(B) 300

(C) 400

(D) 800

4. *In a survey of* 400 *people,* 60% *said they preferred tea over coffee.*

How many of the surveyed people preferred tea?

(A) 160

(B) 200

(C) 340

(D) 240

5. *Two similar triangles have a scale factor of* $3 : 5$. *The area of the smaller triangle is* 18 *square units.*

What is the area, in square units, of the larger triangle?

(A) 50

(B) 30

(C) 45

(D) 90

6. *A parabola and a line intersect where* $y = x^2$ *and* $y = 4x - 3$.

What are the x-coordinates of the points of intersection?

(A) $0,\ 4$

(B) $1,\ 3$

(C) $-1,\ -3$

(D) $3,\ -1$

7. *An equation is given by* $|2x - 1| = 7$.

What are all values of x that satisfy the equation?

(A) $4,\ -3$

(B) $3,\ -4$

(C) $4,\ 3$

(D) $-4,\ 3$

8. *A printer produces* 24 *pages per minute at a constant rate.*

How many minutes does it take to print 300 pages?

(A) 12.5

(B) 10

(C) 15

(D) 12

9. *The product* $(x + 3)(x - 4)$ *can be written in the form* $x^2 + bx - 12$.

What is the value of b?

(A) 1

(B) -7

(C) 7

(D) -1

10. *A cylindrical tank has a radius of* 3 *feet and a height of* 10 *feet.*

What is the volume of the tank, in cubic feet?

(A) 90π

(B) 30π

(C) 900π

(D) 60π

11. *In a right triangle, an acute angle* θ *has an opposite side of length* 3 *and a hypotenuse of length* 5.

What is the value of $\sin\theta$?

(A) $\frac{4}{5}$

(B) $\frac{3}{4}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

12. *Consider the quadratic equation* $x^2 + 4x + 5 = 0$.

How many real solutions does this equation have?

(A) 1

(B) 2

(C) infinitely many

(D) 0

13. *A line of best fit for a scatterplot is* $y = 2.5x + 12$*, where* x *is the number of hours studied and* y *is the predicted exam score.*

What does the number 2.5 represent in this model?

(A) the predicted score for a student who studies 0 hours

(B) the predicted increase in score for each additional hour of study

(C) the maximum possible score

(D) the total number of hours studied

14. *At a fair, adult tickets cost \$12 and child tickets cost \$8. A total of* 20 *tickets were sold for \$204.*

How many adult tickets were sold?

(A) 11

(B) 8

(C) 9

(D) 12

15. *In a circle of radius* 4*, a central angle of* $90°$ *intercepts an arc.*

What is the length of the intercepted arc?

(A) π

(B) 4π

(C) 8π

(D) 2π

16. *An equation is given by* $\frac{x}{x-2} = 3$.

What is the value of x?

(A) 2

(B) 3

(C) 6

(D) -3

17. *A stock priced at \$100 rises by* 20% *one day and then falls by* 20% *the next day.*

What is the final price of the stock?

(A) \$104
(B) \$100
(C) \$96
(D) \$120

18. *The graph of* $y = x^2$ *is shifted* 3 *units to the right and* 2 *units up.*

Which equation represents the new graph?

(A) $(x+3)^2 + 2$
(B) $(x-3)^2 + 2$
(C) $(x-3)^2 - 2$
(D) $(x-2)^2 + 3$

19. *In a* 30°–60°–90° *right triangle, the side opposite the* 30° *angle has length* 5.

What is the length of the hypotenuse?

(A) 10
(B) $5\sqrt{3}$
(C) $5\sqrt{2}$
(D) 15

20. *A number* x *satisfies the equation* $2^x = 32$.

What is the value of x? Grid in your answer.

21. *A rectangle has a length of* 12 *and a width of* 5.

What is the length of the rectangle's diagonal? Grid in your answer.

22. *The quadratic equation* $x^2 - 10x + 21 = 0$ *has two real solutions.*

What is the sum of the two solutions? Grid in your answer.

Answer Key — Math Practice Test

Module 1, Q1
(D). $3x + 7 = 22 \Rightarrow 3x = 15 \Rightarrow x = 5$.

Module 1, Q2
(A). $f(6) = 4(6) - 9 = 24 - 9 = 15$.

Module 1, Q3
(B). $80 \times (1 - 0.25) = 80 \times 0.75 = \$60$.

Module 1, Q4
(C). Slope $= \frac{11-3}{6-2} = \frac{8}{4} = 2$.

Module 1, Q5
(D). $3 : 2 = 12 : x \Rightarrow x = 8$ cups of sugar.

Module 1, Q6
(A). $3^2 - 2(3) + 5 = 9 - 6 + 5 = 8$.

Module 1, Q7
(C). Area $= \frac{1}{2}(10)(6) = 30$ square meters.

Module 1, Q8
(D). Adding the equations: $2x = 14 \Rightarrow x = 7$.

Module 1, Q9
(B). Rate $= 50$ mph; $50 \times 5 = 250$ miles.

Module 1, Q10
(A). $\frac{x^7}{x^3} = x^{7-3} = x^4$.

Module 1, Q11
(D). $C = 2\pi r = 2\pi(5) = 10\pi$ feet.

Module 1, Q12
(C). $2x - 3 < 7 \Rightarrow 2x < 10 \Rightarrow x < 5$.

Module 1, Q13
(A). Mean $= \frac{4+8+10+14}{4} = \frac{36}{4} = 9$.

Module 1, Q14
(D). Cost $= 0.10t + 30$ (per-text charge plus flat fee).

Module 1, Q15
(B). $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

Module 1, Q16
(C). $x^2 - 5x + 6 = (x - 2)(x - 3) = 0 \Rightarrow x = 2, 3$.

Module 1, Q17
(D). $P(\text{blue}) = \frac{5}{3+5+2} = \frac{5}{10} = \frac{1}{2}$.

Module 1, Q18
(A). $50 + 20m = 170 \Rightarrow 20m = 120 \Rightarrow m = 6$.

Module 1, Q19
(B). $200 \times 1.10 = 220$.

Module 1, Q20
6. $5x - 4 = 26 \Rightarrow 5x = 30 \Rightarrow x = 6$. Grid: `6`.

Module 1, Q21
55. Complementary angles sum to $90°$; $90 - 35 = 55$. Grid: `55`.

Module 1, Q22
6. $\frac{x}{8} = \frac{3}{4} \Rightarrow x = \frac{3}{4} \times 8 = 6$. Grid: `6`.

Module 2, Q1
(C). Vertex at $t = -\frac{b}{2a} = -\frac{32}{2(-16)} = 1$ second.

Module 2, Q2
(D). $A = \frac{1}{2}(b_1 + b_2)h \Rightarrow h = \frac{2A}{b_1 + b_2}$.

Module 2, Q3
(C). $50 \times 2^3 = 50 \times 8 = 400$ cells.

Module 2, Q4
(D). $0.60 \times 400 = 240$ people.

Module 2, Q5
(A). Area ratio $= (3:5)^2 = 9:25$; $18 \times \frac{25}{9} = 50$.

Module 2, Q6
(B). $x^2 = 4x - 3 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3) = 0 \Rightarrow x = 1, 3$.

Module 2, Q7
(A). $2x - 1 = 7 \Rightarrow x = 4$; $2x - 1 = -7 \Rightarrow x = -3$.

Module 2, Q8
(A). $\frac{300}{24} = 12.5$ minutes.

Module 2, Q9
(D). $(x+3)(x-4) = x^2 - x - 12$, so $b = -1$.

Module 2, Q10
(A). $V = \pi r^2 h = \pi(3)^2(10) = 90\pi$ cubic feet.

Module 2, Q11
(C). $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$.

Module 2, Q12
(D). Discriminant $= 4^2 - 4(1)(5) = 16 - 20 = -4 < 0$, so 0 real solutions.

Module 2, Q13
(B). The slope 2.5 is the predicted score increase per additional study hour.

Module 2, Q14
(A). $a + c = 20$, $12a + 8c = 204 \Rightarrow 4a = 44 \Rightarrow a = 11$ adult tickets.

Module 2, Q15
(D). Arc $= \frac{90}{360} \cdot 2\pi(4) = \frac{1}{4}(8\pi) = 2\pi$.

Module 2, Q16
(B). $x = 3(x-2) = 3x - 6 \Rightarrow -2x = -6 \Rightarrow x = 3$.

Module 2, Q17
(C). $100 \times 1.2 = 120$; $120 \times 0.8 = \$96$.

Module 2, Q18
(B). Right 3, up 2: $y = (x-3)^2 + 2$.

Module 2, Q19
(A). Hypotenuse $= 2 \times$ (side opposite $30°$) $= 2 \times 5 = 10$.

Module 2, Q20
5. $2^x = 32 = 2^5 \Rightarrow x = 5$. Grid: `5`.

Module 2, Q21
13. Diagonal $= \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$. Grid: `13`.

Module 2, Q22
10. Roots are 3 and 7; sum $= 10$ (also $-\frac{b}{a} = 10$). Grid: `10`.