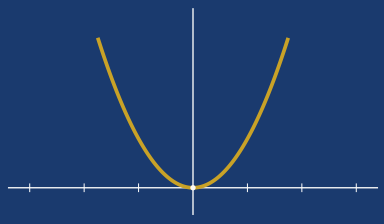
PHYSICS FIRST SERIES

SAT
MATHEMATICS

A Diagnostic Manual

Every wrong answer has a name.



Algebra · Advanced Math

Problem-Solving & Data Analysis · Geometry & Trigonometry

"Pure mathematics is, in its way, the poetry of logical ideas."

— Albert Einstein

Physics First Series · quantumtutoring.ai

Generated from the SAT-math misconception-graph (`math/sat/nodes`).

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How to use this manual

The Digital SAT Math section is **44 questions** in two adaptive modules of 22, 35 minutes each. About **three questions in four are multiple choice** and **one in four is student-produced response** (a grid-in with no choices). A Desmos graphing calculator and a formula reference sheet are available on *every* question, and within each module the questions run easy to hard. Roughly a third are set in a real-world context; the rest are context-free.

This manual is built on one idea: **on this test, a wrong answer is not random — it is a specific, nameable mistake.** For every skill you will find:

- **The skill** — the rule or method, stated plainly.
- **The Approach** — *Read, Plan, Solve, Check* (RPSC), the same four-step habit used across the Physics First Series. Most points are lost not to missing knowledge but to careless slips — a sign error, a misread of what is asked, the wrong grid-in form; the *Check* step is where you catch them.
- **Common traps** — the handful of named wrong-answer patterns for that skill. Learn to recognize the trap and you eliminate the distractor.
- **Practice** — items graded easy to hard. Answers, with the trap each wrong choice represents, are in the **Answer Key** at the end of each chapter.

Do not just note whether you were right. For each miss, name the trap you fell for. That is the whole method.

How to Approach a Problem

You have about a minute and a half per question. You do not need cleverness; you need a **habit** that keeps you from beating yourself. Every skill in this manual runs on the same four steps — *Read, Plan, Solve, Check* — and most lost points are caught at the last one.

Read — what is given, and what is asked

Name the quantities and, above all, **what the question actually wants**. The most common way to miss an easy question is to solve for x when it asked for $2x$, or to find a price before tax when it asked for the total. Underline the asked-for quantity before you compute anything.

Plan — choose the method or formula

Decide the route before you calculate: substitute, factor, use the formula sheet, set up a proportion, or let Desmos graph it. A short plan beats a long scramble. The reference sheet is provided — your job is to *select* the right formula, not to recall it (except surface area and trigonometry, which are *not* on the sheet; memorize those).

Solve — execute carefully

Do the algebra one step at a time, watching signs, units, and grouping. Carry units through so the answer's units make sense. On a grid-in there are no choices to catch a slip, so slow down by a beat.

Check — the step that saves the points

Ask three things: Does the answer **answer the question** that was asked? Is it **reasonable** in size and sign? Is it in the **right form** — and for a grid-in, does it fit the entry rules (a fraction or decimal, no mixed numbers, enough decimal places)?

Worked example: Read · Plan · Solve · Check

A jacket is discounted 20% to a sale price of $48. What was the original price?

Read. Asked: the *original* price, not the discount. **Plan.** Sale price is 80% of the original, so original $= 48 \div 0.80$.

Solve. $48 \div 0.80 = 60$. **Check.** A $60 jacket, 20% off, drops $12 to $48 — consistent, and $60 is larger than the sale price, as it must be. **Answer: $60.**

The trap here is answering **$57.60** — taking 20% *off the $48* instead of recognizing $48 as the already-discounted price. Naming that trap is what the *Check* step is for.

Coming next: the adaptive tutor

This manual shows you the procedure. An interactive tutor (in development) will serve these skills adaptively, name the exact trap behind each wrong answer, and follow your progress skill by skill — turning *show* into *do*.

DOMAIN 1

Algebra

The grammar of the whole section

> *"As long as algebra is taught in school, there will be prayer in school."*
> — Cokie Roberts

About 35% of the section — tied for the largest domain. Linear equations, inequalities, systems, and graphs; equivalent expressions; and absolute value.

Solve linear equations in one variable

node: sat.linear.L1

The skill. Solving a linear equation means finding the one value of the variable that makes both sides equal. You do it by undoing whatever has been done to the variable — if a number is added, subtract it; if the variable is multiplied by a number, divide by that number — always doing the same thing to both sides so the equation stays balanced. Clear any parentheses and combine like terms first. Most linear equations have exactly one answer, but a few are special: if the variable disappears and you are left with a false statement like $3 = 5$, there is no solution; if you are left with a true statement like $0 = 0$, every number works.

Worked example

Solve $4x - 3 = 2x + 7$.

Read. We have $4x - 3 = 2x + 7$. The variable x appears on both sides, so first we gather the x-terms on one side and the plain numbers on the other.

Plan.

1. Subtract $2x$ from both sides to collect the x-terms on the left.
2. Add 3 to both sides to move the number to the right.
3. Divide both sides by 2 to get x alone.

Solve.

$4x - 3 = 2x + 7$	the original equation
$2x - 3 = 7$	subtract 2x from both sides
$2x = 10$	add 3 to both sides
$x = 5$	divide both sides by 2

Check. Put x = 5 back in: the left side is 4(5) - 3 = 17 and the right side is 2(5) + 7 = 17. Both sides match, so x = 5 is correct.

Answer: x = 5.

Approach Read · Plan · Solve · Check

Read. Clear any parentheses and fractions first, then gather the variable terms on one side and the plain numbers on the other.

Plan.

1. Undo addition or subtraction by doing the opposite to both sides.
2. Undo multiplication or division by doing the opposite to both sides.
3. Keep going until the variable stands alone.

Solve.

1. Do each operation to both sides.
2. Watch the sign of every term you move.
3. Simplify until the variable is by itself.

Check. Put your answer back into the original equation to check it works. If the variable cancels out, a false statement like 3 = 5 means no solution and a true statement like 0 = 0 means every number works.

Common traps — every wrong answer is one of these

Moving a term without flipping its sign. 1. Starts with x + 5 = 12.

2. Moves the 5 across the equals sign but keeps it positive: x = 12 + 5.
3. Gets x = 17 instead of the correct x = 7.
4. Fix: to move a term, do the opposite operation — subtract 5 from both sides.

Forgetting to distribute to every term.
Multiplies the number in front of parentheses by only the first term inside, turning 3(x + 2) into 3x + 2 instead of 3x + 6. Multiply the outside number by every term inside, and watch the signs when that number is negative.

Doing something to only one side.
Adds, subtracts, or divides on just one side of the equation, or adds variable terms that should be subtracted when collecting them. Whatever you do to one side you must do to the other, or

the two sides stop being equal.

Dividing the wrong way.
From 3x = 15, writes 15 - 3 or 3/15 instead of 15/3, or forgets to divide by the number in front of x at all. To undo multiplication by 3, divide both sides by 3.

Missing the special cases.
Treats an impossible equation like 2x + 1 = 2x + 3 as if it has an answer, or treats an always-true equation like 2x + 2 = 2(x + 1) as having a single answer instead of infinitely many. When the variable cancels, check whether what remains is false (no solution) or true (all numbers).

Practice

1. *Solve 3x + 5 = 20.*

What is x?

(A) 45
(B) 25/3
(C) 5
(D) 15

2. *Solve 5x - 3 = 2x + 9.*

What is x?

(A) 12
(B) 4
(C) 12/7
(D) 2

3. *Solve 2(x + 3) = 16.*

What is x?

(A) 10
(B) 11
(C) 5
(D) 6.5

4. *Solve 10 - 2(x - 1) = 4.*

What is x?

(A) 8

(B) -8

(C) 2

(D) 4

5. *The equation 3x + 6 = 3(x + 2).*

How many solutions does it have?

(A) No solution

(B) One solution, x = 2

(C) One solution, x = 0

(D) Infinitely many (all real numbers)

Slope as ratio (rise/run, geometric)

node: sat.linear.L2

The skill. The slope of a line measures how steep it is and which way it tilts. It is the change in y divided by the change in x between any two points on the line — often said as "rise over run." A positive slope rises as you move to the right, a negative slope falls, a flat (horizontal) line has slope 0, and a straight up-and-down (vertical) line has an undefined slope. Because a line is perfectly straight, every pair of points on it gives the same slope.

Worked example

Find the slope of the line through (1, 3) and (4, 9).

Read. We are given two points, (1, 3) and (4, 9). Call (1, 3) point 1 and (4, 9) point 2. We want the slope — how much y changes for each step to the right in x.

Plan. Slope is the change in y over the change in x. Subtract the y-values for the top and the x-values for the bottom, keeping the points in the same order, then simplify the fraction.

Solve.

$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} && \text{write the definition} \\ &= \frac{9 - 3}{4 - 1} && \text{substitute the two points} \\ &= \frac{6}{3} && \text{subtract on top and bottom} \\ &= 2 && \text{reduce the fraction} \end{aligned}$$

Check. The line goes up as it moves to the right, so a positive slope is expected. A slope of 2 means y rises by 2 each time x increases by 1.

Answer: the slope is 2.

Approach Read · Plan · Solve · Check

Read. Find two points on the line and write each as an ordered pair, (x1, y1) and (x2, y2). Decide which one is point 1 and keep that choice for the whole calculation.

Plan.

1. Subtract the two y-values to get the change in y (this is the top of the fraction).
2. Subtract the two x-values in the SAME order to get the change in x (the bottom).
3. Divide the change in y by the change in x.

Solve.

1. Do the subtraction on top.
2. Do the subtraction on the bottom.
3. Reduce the fraction to lowest terms.

Check. Make sure the sign matches the picture: up to the right is positive, down to the right is negative. A horizontal line gives 0; a vertical line is undefined. Any two points on the line should give the same slope.

Common traps — every wrong answer is one of these

Flipping the fraction (run over rise).
Divides the change in x by the change in y instead of the other way around. This is the single most common slope error — it turns a slope of 2 into 1/2. Always put the change in y on top.

Wrong sign.
1. Subtracts the y-values top-to-bottom: y1 - y2.
2. But subtracts the x-values the other way: x2 - x1.
3. The mismatched order flips the sign, so a positive slope comes out negative.
4. Fix: subtract both top and bottom in the same order, then the sign is automatic.

Horizontal and vertical swapped.
Says a flat line has an undefined slope and an up-and-down line has slope 0. It is the reverse: a horizontal line has slope 0, and a vertical line has an undefined slope.

Using one point instead of two.
Takes y/x from a single point, or reads one coordinate as the slope. Slope always compares two points — it is a difference divided by a difference, never a single coordinate.

Thinking the slope depends on the points.
Believes the slope changes depending on which two points you choose. On a straight line every pair of points gives exactly the same slope — that is what makes it straight.

Practice

1. *A line passes through (1, 3) and (3, 7).*

What is the slope of the line?

(A) -2

(B) 3

(C) 1/2

(D) 2

2. *A line passes through (-2, 5) and (3, -5).*

What is the slope of the line?

(A) 2

(B) -1/2

(C) -2

(D) -2.5

3. *A line passes through (-3, 4) and (5, 4).*

What is the slope of the line?

(A) 8

(B) undefined

(C) 4

(D) 0

4. *A line passes through (2, -1) and (2, 6).*

What is the slope of the line?

(A) 7

(B) 0

(C) undefined

(D) 2

5. *Points (0, 1), (2, 4), and (4, 7) all lie on the same line.*

What is the slope of the line?

(A) 7/4

(B) 3/2, and any pair of the three points gives the same value

(C) It depends on which two points you choose

(D) 2/3

Slope as rate of change (contextual)

node: sat.linear.L3

The skill. A rate of change tells you how much one quantity goes up or down each time the other goes up by one — like dollars per month or centimeters per minute. You find it the same way as slope: pick two points, divide the change in the top quantity by the change in the bottom quantity, and reduce it to a per-one-unit amount. Always state it with its units, in the order y-units per x-unit, and give it a negative sign if the quantity is going down.

Worked example

A gym charges a sign-up fee plus a fixed monthly rate. After 2 months a member has paid \$190, and after 5 months has paid \$340. What is the monthly rate?

Read. We have two points: (2 months, \$190) and (5 months, \$340). The monthly rate is how many dollars the total goes up for each extra month.

Plan.

1. Subtract the two amounts paid to get the change in dollars.
2. Subtract the two month-counts to get the change in months.
3. Divide dollars by months to get the rate per month.

Solve.

$$\begin{aligned} \text{rate} &= \frac{340 - 190}{5 - 2} && \text{change in dollars over change in months} \\ &= \frac{150}{3} && \text{subtract on top and bottom} \\ &= 50 && \text{divide} \end{aligned}$$

Check. The total climbs by \$150 over 3 months, which is \$50 for each month. The amount is going up, so a positive rate makes sense.

Answer: \$50 per month.

Approach Read · Plan · Solve · Check

Read. Figure out what the two quantities are and their units. Words like per, each, or every are signals that a rate is wanted.

Plan.

1. Pick two clear points from the table, graph, or wording.
2. Subtract the two y-values to get the change in y.
3. Subtract the two x-values in the same order to get the change in x.
4. Divide the change in y by the change in x, keeping the units.

Solve.

1. Do the two subtractions.

2. Divide to get the amount per one unit.
3. Attach the units as y-units per x-unit.

Check. Check the sign against the story — going down is negative, going up is positive. Make sure the units read y per x, not x per y, and that the size is reasonable.

Common traps — every wrong answer is one of these

Dividing the rate upside down.
Divides the change in x by the change in y, giving minutes per dollar when the question asks for dollars per minute. Put the quantity the question asks about on top.

Thinking the rate is not constant.
Believes the rate only holds between neighboring rows or that it changes along the table — or the opposite mistake, using any two points on data that is not actually linear. On a straight-line relationship every pair of points gives the same rate; on non-linear data no single rate exists.

Using one point as the rate.
Takes a single point and divides its y by its x instead of using the change between two points. That only works when the line passes through the origin; with a starting fee or offset it gives the wrong rate.

Wrong sign for the direction.
Reports a falling quantity with a positive rate, or a rising one with a negative rate. Match the sign to whether the quantity grows or shrinks.

Units flipped or missing.
Gives a bare number with no units, or the right number with the per backwards. State the rate as y-units per x-unit.

Ignoring the real step size.
On a table where x jumps by 10 each row, treats the change in x as 1 instead of 10. Use the actual gap between the x-values.

Reading a data point as the starting value.
Takes the y-value of some point in the middle of the data as the fixed or initial amount. The starting value is y when x = 0, not just any point's height.

Practice

1. *A candle burns at a constant rate. t (min): 0, 10, 20, 30 -> H (cm): 20, 17, 14, 11.*

What is the rate of change of height with respect to time?

(A) -3 cm per minute

(B) -0.3 minutes per cm

(C) +0.3 cm per minute

(D) -0.3 cm per minute

2. *Same candle table.*

Which two times must you use to find the rate?

(A) Any two rows give the same rate

(B) Only the first and last rows

(C) The starting value 20 is the rate

(D) Only consecutive rows (0 and 10)

3. *A gym charges a sign-up fee plus a monthly rate. After 2 months a member has paid $190; after 5 months, $340.*

What is the monthly rate?

(A) $95 per month

(B) $68 per month

(C) $50 per month

(D) $50 (no time unit)

4. *Same gym scenario.*

What is the sign-up fee?

(A) $0

(B) $90

(C) $50

(D) $190

5. *A table that looks linear but whose successive differences are not constant.*

What is the rate of change of this relationship?

(A) A single rate computed from the first two points

(B) A single rate computed from the last two points

(C) The relationship is not linear, so it has no single rate of change

(D) A single rate computed from the first and last points

Forms of a line (slope-intercept, point-slope, standard)

node: sat.linear.L4

The skill. A straight line can be written three common ways, and each shows different information at a glance. Slope-intercept form, $y = mx + b$, shows the slope m and the y-intercept b directly.

Point-slope form, y - y1 = m(x - x1), is built around the slope m and one point (x1, y1) the line passes through. Standard form, Ax + By = C, hides the slope — to get it, solve for y, which gives a slope of -A/B. To find where a line crosses an axis, set the other variable to zero: the x-intercept has y = 0, the y-intercept has x = 0.

Worked example

Find the slope of the line 2x + 3y = 12.

Read. The line is in standard form, Ax + By = C, which does not show the slope directly. We rewrite it as y = mx + b so the slope shows.

Plan.

1. Subtract 2x from both sides to start isolating y.
2. Divide every term by 3 to get y by itself.
3. Read the slope as the number in front of x.

Solve.

$$\begin{aligned} 2x + 3y &= 12 && \text{the original equation} \\ 3y &= -2x + 12 && \text{subtract } 2x \text{ from both sides} \\ y &= -\frac{2}{3}x + 4 && \text{divide every term by } 3 \end{aligned}$$

Check. Now the equation is in y = mx + b form, so the slope is the coefficient of x, which is -2/3. The negative sign fits a line that falls as it moves to the right.

Answer: the slope is -2/3.

Approach Read · Plan · Solve · Check

Read. First notice which of the three forms the equation is written in, because that decides how you read it.

Plan.

1. In slope-intercept form y = mx + b, read the slope as the number in front of x and the intercept as the constant.
2. In standard form Ax + By = C, solve for y first, then read the slope.
3. In point-slope form, read the point (x1, y1) using the opposite signs of what is written.

Solve.

1. Rearrange into the form you need.
2. Track the sign of every term as you move it.
3. Read off the slope or intercept.

Check. Does the slope's sign match which way the line tilts? For an intercept, make sure you set the correct variable to zero — y = 0 for the x-intercept, x = 0 for the y-intercept.

Common traps — every wrong answer is one of these

Mixing up slope and intercept.
In y = mx + b, reads the constant b as the slope, or the coefficient m as the intercept. The slope is always the number multiplying x; the y-intercept is the number by itself.

Reading slope straight off standard form.
From Ax + By = C, grabs the x-coefficient A as the slope without first solving for y. In standard form the slope is -A/B, not A.

Sign or flip error solving for y.
Solves standard form for y but reports the slope as A/B, B/A, or +A/B instead of -A/B. Moving the Ax term to the other side makes it negative, and then you divide by B.

Sign slip in point-slope form.
In y - y1 = m(x - x1), keeps the wrong signs for the point or swaps its x and y. The form subtracts each coordinate, so a point (-3, 5) shows up as (x + 3) and (y - 5).

Mixing up the two intercepts.
Sets the wrong variable to zero, or reads the constant C straight off as an intercept. The x-intercept comes from y = 0 and the y-intercept from x = 0.

Practice

1. *The line $y = 3x - 5$.*

What is the y-intercept?

(A) -5
(B) 5/3
(C) 3
(D) (-5, 0)

2. *The line $2x + 3y = 12$.*

What is the slope?

(A) -3/2
(B) -2/3
(C) 2
(D) 2/3

3. *The line $2x + 3y = 12$.*

What is the x-intercept?

(A) 3

(B) 6

(C) 12

(D) 4

4. *A line has slope 2 and passes through (-3, 5).*

Which is its point-slope equation?

(A) y - 3 = 2(x + 5)

(B) y - 5 = 2(x - 3)

(C) y + 5 = 2(x - 3)

(D) y - 5 = 2(x + 3)

5. *The line 4x - 2y = 6.*

Which equation describes the same line?

(A) y = 2x - 3

(B) y = -2x - 3

(C) y = 2x + 6

(D) y = -2x + 3

Write a line from conditions (two points / point+slope / table / parallel-perpendicular)

node: sat.linear.L5

The skill. To write the equation of a line you need two things: its slope and its y-intercept. The slope might be given, or you find it from two points (change in y over change in x), or from a parallel line (same slope) or a perpendicular line (flip the slope and change its sign). Once you have the slope, plug one known point into y = mx + b and solve for b, the y-intercept. Then write the finished equation.

Worked example

Write the equation of the line with slope 3 that passes through the point (2, 7).

Read. We know the slope is m = 3 and the line goes through (2, 7). We need the y-intercept b, then we can write y = 3x + b.

Plan.

1. Substitute the slope 3 and the point (2, 7) into y = mx + b.
2. Solve the resulting equation for b.
3. Write the final equation with the slope and b.

Solve.

$y = mx + b$	slope-intercept form
$7 = 3(2) + b$	substitute m = 3 and the point (2, 7)
$7 = 6 + b$	multiply
$b = 1$	subtract 6 from both sides

Check. The equation is y = 3x + 1. Testing the point: 3(2) + 1 = 7, which matches the given y-value, so the line passes through (2, 7).

Answer: y = 3x + 1.

Approach Read · Plan · Solve · Check

Read. Look at what you are given: a slope, two points, a table, or a line to be parallel or perpendicular to.

Plan.

1. Find the slope — use it directly, compute it from two points, or flip-and-negate it for a perpendicular line.
2. Substitute one known point and the slope into y = mx + b.
3. Solve for b, the y-intercept.
4. Write the equation as y = mx + b.

Solve.

1. Carry the signs through the substitution.
2. Solve for b.
3. If you started from point-slope form, distribute fully to finish.

Check. Check that the line really passes through the given point by plugging that point back in and seeing that both sides match.

Common traps — every wrong answer is one of these

Using the point's y-value as the intercept. 1. Has slope 3 and the point (2, 7).
2. Writes y = 3x + 7, using the 7 from the point as the intercept.
3. But the intercept b is not the point's y-value — it must be solved for.
4. Fix: substitute (2, 7): 7 = 3(2) + b gives b = 1, so y = 3x + 1.

Sign or distribution slip.
Distributes or handles a sign wrong when expanding point-slope form into y = mx + b, or when solving for b. Multiply the slope through every term inside the parentheses and keep track of negatives.

Wrong slope for parallel or perpendicular.
Uses the same slope for a perpendicular line, or flips the slope without changing its sign, or changes the sign without flipping. Parallel lines share the slope; perpendicular slopes are negative reciprocals — flip and change the sign.

Practice

1. *A line has slope 3 and passes through (2, 7).*

Which is its equation?

(A) y = 3x + 13
(B) y = 3x + 7
(C) y = 3x + 1
(D) y = 3x + 2

2. *A line passes through (1, 4) and (3, 10).*

Which is its equation?

(A) y = 3x + 4
(B) y = (1/3)x + 4
(C) y = 3x - 1
(D) y = 3x + 1

3. *The point-slope equation y - 5 = 3(x - 2).*

Write it in slope-intercept form.

(A) y = 3x - 1
(B) y = 3x - 11
(C) y = 3x + 5
(D) y = 3x + 3

4. *A line passes through (0, 2) and is perpendicular to y = 2x + 1.*

Which is its equation?

(A) y = -1/2 x + 2
(B) y = -2x + 2
(C) y = 2x + 2
(D) y = 1/2 x + 2

5. *A line passes through (-1, 3) and (2, -3).*

Which is its equation?

(A) y = -2x - 3

(B) y = -2x + 3

(C) y = -2x + 1

(D) y = 2x + 1

Interpret slope & intercept of a linear model in context

node: sat.linear.L6

The skill. When a straight-line equation y = mx + b models a real situation, each number has a real-world meaning. The slope m is a rate — how much y changes for each one-unit increase in x — and it carries units like dollars per hour. The intercept b is the value of y when x = 0, the fixed or starting amount before any change happens. Reading a model well means matching each number to what it stands for and stating its units.

Worked example

A repair shop's total charge is C = 45h + 75 dollars, where h is the number of hours worked. What does each number mean?

Read. The equation is in y = mx + b form, with C as the total charge and h as the hours. The 45 multiplies h and the 75 stands alone.

Plan.

1. Identify the slope as the number multiplying h.
2. Identify the intercept as the constant term.
3. State what each one means, with units.

Solve.

$C = 45h + 75$	the model, in y = mx + b form
$m = 45$	the slope is the coefficient of h
$b = 75$	the intercept is the constant term

Check. At h = 0 the charge is C = 75, the fixed fee before any labor. Each extra hour adds 45, so 45 is the hourly rate in dollars per hour.

Answer: 45 is the hourly labor rate (dollars per hour) and 75 is the fixed fee (dollars).

Approach Read · Plan · Solve · Check

Read. Work out what x and y stand for in the situation and what units each one has.

Plan.

1. The slope m is the per-unit rate, in units of y per unit of x.
2. The intercept b is the value of y when x = 0, the fixed or starting amount.
3. Match each number in the equation to its real-world meaning.

Solve. Read the slope off as the coefficient of x and the intercept off as the constant term.

Check. Check the sign — a negative slope means the quantity is dropping. Remember b is a one-time or starting amount while m is a per-each amount.

Common traps — every wrong answer is one of these

Swapping the roles of slope and intercept.
Reads the constant b as the rate and the slope m as the fixed amount. The number multiplying x is the rate; the number by itself is the starting value.

Treating the rate as a total.
Uses the slope m as the value after one unit and forgets to add the starting amount b. The total after one unit is m + b, not just m.

Confusing a number with the variable.
Says a coefficient is the variable — for example calling the 45 in 45h the hours rather than the rate per hour. The number is the rate; h is the count of hours.

Wrong sign or direction.
Reads a negative slope as an increase, or says a quantity is some amount when it actually decreases by that amount each step. A negative slope means the quantity falls.

Dropping the units.
Gives the right number with the wrong dimension, like dollars instead of dollars per hour. A rate needs the full per-unit label.

Practice

1. *A technician's total charge C (dollars) is C = 45h + 75, where h = hours.*

What does 45 represent?

(A) The hourly labor rate
(B) The charge for a one-hour repair
(C) The number of hours
(D) The fixed service fee

2. *Same technician scenario.*

What does 75 represent?

(A) The charge when no labor time is needed

(B) The total for the first hour

(C) The hourly rate

(D) The total charge for the longest possible job

3. *A pool drains so its depth D (inches) after t minutes is D = 60 - 1.5t.*

What does -1.5 represent?

(A) The depth rises by 1.5 inches per minute

(B) The depth drops by 1.5 inches per minute

(C) The pool starts at 1.5 inches

(D) The depth is 1.5 inches

4. *Same pool scenario.*

Which gives the correct interpretation of the slope, with units?

(A) 1.5 minutes per inch

(B) 1.5 inches

(C) The depth is -1.5 inches at the start

(D) The depth decreases by 1.5 inches per minute

5. *Two phone plans: P = 30 + 12n and Q = 18n, where n = gigabytes.*

Which interpretation correctly compares the plans' fixed costs and per-GB rates?

(A) P has a $30 fixed cost and $12/GB; Q has no fixed cost and $18/GB; they cross at n = 5

(B) The 30 and 18 are both per-GB rates

(C) P costs $12 per GB regardless of n, so P is the flat plan

(D) Q is always cheaper because 18 > 12

Solve linear systems (substitution / elimination)

node: sat.linear.L7

The skill. A system of two linear equations asks for the one pair of values (x, y) that makes both equations true at once. Two methods work. In substitution, you solve one equation for a variable and put that expression into the OTHER equation, so only one variable is left. In elimination, you scale the equations so one variable's coefficients are opposites, then add them to cancel that variable. Either way you end with a single-variable equation; solve it, then put the answer back to find the other variable. There is usually one solution, but parallel lines give none and identical lines give infinitely many.

Worked example

Solve the system $y = 2x + 1$ and $3x + y = 11$.

Read. The first equation already gives y by itself, so substitution is the quick route. We put $2x + 1$ in for y in the second equation.

Plan.

1. Substitute $2x + 1$ for y in the second equation.
2. Solve the resulting equation for x.
3. Put x back into $y = 2x + 1$ to find y.

Solve.

$3x + (2x + 1) = 11$	substitute $y = 2x + 1$ into $3x + y = 11$
$5x + 1 = 11$	combine like terms
$5x = 10$	subtract 1 from both sides
$x = 2$	divide both sides by 5
$y = 2(2) + 1 = 5$	back-substitute into $y = 2x + 1$

Check. Test (2, 5) in both equations: $5 = 2(2) + 1$ is true, and $3(2) + 5 = 11$ is true. Both hold, so the solution is (2, 5).

Answer: (x, y) = (2, 5).

Approach Read · Plan · Solve · Check

Read. Choose a method: substitution works well when a variable is already by itself, elimination when the coefficients line up to cancel easily.

Plan.

1. Substitution: solve one equation for a variable, then put that into the other equation.
2. Elimination: scale the equations so one variable's coefficients are opposites, then add.
3. Solve the single-variable equation you are left with.
4. Put that value back into an original equation to get the other variable.

Solve.

1. Reduce the system to one variable and solve it.
2. Back-substitute to find the second variable.
3. Write the answer as an ordered pair (x, y).

Check. Check your (x, y) in both original equations. If every variable cancels, a false statement like $0 = 5$ means no solution and a true statement like $0 = 0$ means infinitely many.

Common traps — every wrong answer is one of these

Substituting back into the same equation. 1. Solves y = 2x + 1 for y.
2. Puts that y back into y = 2x + 1, the equation it came from.
3. Gets 0 = 0 and wrongly concludes infinitely many solutions.
4. Fix: substitute the expression into the OTHER equation, not the one it came from.

Sign or scaling slip in elimination.
Adds the equations when they should be subtracted, or scales only one term or one side instead of the whole equation. To cancel a variable, multiply every term of an equation by the same number.

Stopping early or mixing up x and y.
Solves for one variable and forgets the other, mis-signs the back-substitution, or reports the pair with x and y swapped. Find both values and keep them in (x, y) order.

Misreading the number of solutions.
Reaches parallel or identical lines but still reports a single point, or mixes up no-solution with infinitely-many. Parallel lines (0 = nonzero) give no solution; identical lines (0 = 0) give infinitely many.

Practice

1. *Solve the system: $y = 2x + 1$ and $3x + y = 11$.*

What is the solution (x, y)?

(A) x = 2 (no y found)
(B) (2, 5)
(C) (10, 21)
(D) Infinitely many solutions

2. *Solve the system: $2x + 3y = 12$ and $2x - y = 4$.*

What is the solution (x, y)?

(A) (1, 2)
(B) (2, 3)
(C) (4, 4)
(D) (3, 2)

3. *Solve the system: $3x + 2y = 16$ and $x + y = 6$.*

What is the solution (x, y)?

(A) (4, 2)

(B) (2, 4)

(C) (10, -4)

(D) (4, 6)

4. *Solve the system: $y = 3x - 2$ and $y = x + 4$.*

What is the solution (x, y)?

(A) (3, 7)

(B) (1, 5)

(C) (6, 10)

(D) (3, 11)

5. *The system: $y = 2x + 3$ and $2y = 4x + 6$.*

How many solutions does it have?

(A) Exactly one solution

(B) Infinitely many (the two equations describe the same line)

(C) (0, 3)

(D) No solution

Systems in context & number of solutions

node: sat.linear.L8

The skill. Many word problems turn into a system: each condition in the story becomes one equation. Once you have the system, solve it and then read the answer back in the words of the problem, with units — making sure you report the quantity that was actually asked for. Some questions instead ask how many solutions a system has, and you can answer that from its structure: different slopes cross once (one solution), the same slope with different intercepts never meet (no solution, parallel), and the same slope and intercept are the same line (infinitely many).

Worked example

A shop sells small drinks for $3 and large drinks for $5. One day it sold 80 drinks and took in $310. How many large drinks were sold?

Read. Let s be the number of small drinks and l the number of large. There were 80 drinks in all, and the money adds to $310. We want l, the number of large drinks.

Plan.

1. Write $s + l = 80$ for the count and $3s + 5l = 310$ for the money.
2. Solve the first equation for s and substitute into the second.
3. Solve for l.

Solve.

$s = 80 - l$	solve the count equation s + l = 80 for s
$3(80 - l) + 5l = 310$	substitute into the money equation
$240 + 2l = 310$	distribute and combine like terms
$2l = 70$	subtract 240 from both sides
$l = 35$	divide both sides by 2

Check. With l = 35, then s = 80 - 35 = 45. Money check: 3(45) + 5(35) = 135 + 175 = 310, which matches. So 35 large drinks were sold.

Answer: 35 large drinks.

Approach Read · Plan · Solve · Check

Read. Name each unknown with its units, then write one equation for each condition the problem states.

Plan.

1. Solve the system by substitution or elimination.
2. For a how-many-solutions question instead, compare the slopes and intercepts without solving.
3. Map the numeric answer back to the exact quantity the question asks for.

Solve.

1. Carry the solve through to a number.
2. Attach the correct units.
3. State the quantity the question asked about.

Check. Does your answer name the right quantity with the right units? Does a no-solution or infinitely-many result actually make sense in the story?

Common traps — every wrong answer is one of these

Building the wrong equations.
Sets up the system wrong: swaps the count equation with the total equation, attaches a price to the wrong item, or leaves out a condition. Each sentence of the problem should map to one correct equation.

Judging solution count by appearance.
Decides the number of solutions by how different the equations look instead of comparing slopes and intercepts. Two equations that look different can still be the same line or parallel lines.

Reporting the wrong quantity.
Solves correctly but gives the other variable, the total, or the crossing point instead of what was asked, or leaves off the units. Re-read the question and report exactly the quantity it wants.

Practice

1. *A shop sells small drinks for \$3 and large for \$5. One day it sold 80 drinks for \$310 total. Let s = small drinks, l = large drinks.*

Which system represents this situation?

(A) 3s + 5l = 80; s + l = 310

(B) s + l = 80; 3s + 5l = 310

(C) s + l = 310; 3s + 5l = 80

(D) 5s + 3l = 310; s + l = 80

2. *From the same shop: $s + l = 80$ and $3s + 5l = 310$ (so $s = 45$, $l = 35$).*

How many large drinks were sold?

(A) 45

(B) 35

(C) 80

(D) 70

3. *The system $2x + 4y = 8$ and $x + 2y = 5$.*

How many solutions does it have?

(A) One solution at (1, 2)

(B) Infinitely many solutions

(C) No solution (the lines are parallel)

(D) Exactly one solution

4. *Plan A charges \$40 plus \$10 per month; Plan B charges \$25 plus \$15 per month.*

After how many months do the two plans cost the same?

(A) 13 months

(B) 3 months

(C) -3 months

(D) \$70

5. *Plan X costs $30 + 20m$ dollars; Plan Y costs $20m + 30$ dollars, where m is months.*

For how many values of m do the plans cost the same?

(A) All values of m – the plans always cost the same

(B) Only m = 0

(C) No value of m

(D) Exactly one value of m

Linear inequalities & systems of inequalities

node: sat.linear.L9

The skill. A linear inequality is solved almost exactly like an equation — undo operations to isolate the variable — with one special rule: whenever you multiply or divide both sides by a negative number, flip the inequality sign. To picture the answer on a graph, draw the boundary line (solid if the inequality includes or-equal-to, dashed if it is strict) and shade the side that works, which you can confirm by testing a point. When several inequalities must all hold, the solution is the region where all their shaded parts overlap.

Worked example

Solve the inequality -3x + 2 <= 14.

Read. We want all values of x that make this true. We isolate x, and because the coefficient of x is negative we will need the sign-flip rule.

Plan.

1. Subtract 2 from both sides.
2. Divide both sides by -3.
3. Flip the inequality sign because we divided by a negative.

Solve.

$-3x + 2 \le 14$	the original inequality
$-3x \le 12$	subtract 2 from both sides
$x \ge -4$	divide by -3 and flip the sign

Check. Test x = 0, which satisfies x >= -4: the original gives -3(0) + 2 = 2, and 2 <= 14 is true. Test x = -10, which should fail: -3(-10) + 2 = 32, and 32 <= 14 is false. So x >= -4 behaves correctly.

Answer: x >= -4.

Approach Read · Plan · Solve · Check

Read. Decide what you are after: a solution set on a number line, a single shaded region, or the overlap of several regions.

Plan.

1. Isolate the variable just as in an equation.
2. Flip the inequality sign if you multiply or divide by a negative.
3. For a graph, draw the boundary — solid if or-equal-to, dashed if strict.
4. Test a point to decide which side to shade.

Solve.

1. Carry the solve through, applying the flip rule when needed.

2. For a system, shade each inequality's region.
3. Keep only the region common to all of them.

Check. Pick a point in your shaded region and check it really satisfies every inequality. Make sure included endpoints are solid and excluded ones are open.

Common traps — every wrong answer is one of these

Forgetting to flip the sign. 1. Solves -2x > 6 by dividing both sides by -2.
2. Keeps the sign the same and writes x > -3.
3. But dividing by a negative reverses the inequality.
4. Fix: flip the sign when dividing by a negative, giving x < -3.

Wrong boundary type.
Mixes up strict and inclusive: an open dot where a closed one belongs, a dashed line where a solid one belongs, or including an endpoint that should be excluded. Use solid or closed for or-equal-to and dashed or open for a strict inequality.

Shading the wrong side.
Shades the wrong half-plane, often by assuming greater-than always means above the line without testing a point, or by testing a point incorrectly. Pick a test point, check it in the inequality, and shade the side that works.

Wrong region for a system.
Takes the union of the regions, or the area outside them, instead of the overlap, or keeps only one of the inequalities. The solution to a system is where all the shaded regions overlap.

Practice

1. *Solve -2x + 3 >= 11.*

What is the solution set?

(A) x >= -4
(B) x <= -4
(C) x <= -7
(D) x <= 4

2. *Solve and graph x - 2 <= 3 on a number line.*

Which graph is correct?

(A) Closed dot at 5, shaded to the left
(B) Open dot at 5, shaded to the left
(C) Closed dot at 5, shaded to the right

(D) Open dot at 5, shaded to the right

3. *The inequality $y > 2x - 1$ in the xy-plane.*

Which region is its graph?

(A) Above the line, boundary included (solid)

(B) Below the line, boundary excluded (dashed)

(C) Above the line, boundary excluded (dashed)

(D) Below the line, boundary included (solid)

4. *The system $y <= x + 2$ and $y >= -x$.*

Which region is the solution?

(A) The overlap (intersection) of the two shaded regions

(B) The region outside both

(C) Only where y <= x + 2

(D) The union of the two shaded regions

5. *The compound inequality $-2 < x + 1 <= 3$.*

For how many integer values of x does it hold?

(A) 5

(B) 4

(C) 6

(D) Infinitely many

Literal equations & rearranging formulas

node: sat.linear.L10

The skill. A literal equation is a formula with several letters, like P = 2L + 2W, and solving for one of them means rearranging so that letter stands alone on one side. You do it exactly as with numbers: treat all the other letters as constants and undo the operations in reverse order. Two cautions matter: when you divide by something, divide every term on that side, not just one; and if the letter you want appears in two places, gather those terms and factor it out so it appears only once.

Worked example

The formula F = (9/5)C + 32 converts Celsius to Fahrenheit. Solve it for C.

Read. We want C by itself. We treat F as a constant and undo what is being done to C: it is multiplied by 9/5, then 32 is added.

Plan.

1. Subtract 32 from both sides.
2. Multiply both sides by 5/9 to undo the 9/5.
3. Read off C.

Solve.

$$F = \frac{9}{5}C + 32 \qquad \text{the original formula}$$
$$F - 32 = \frac{9}{5}C \qquad \text{subtract 32 from both sides}$$
$$\frac{5}{9}(F - 32) = C \qquad \text{multiply both sides by 5/9}$$

Check. Try F = 212: C = (5/9)(212 - 32) = (5/9)(180) = 100, the correct boiling point in Celsius. The rearrangement works.

Answer: C = (5/9)(F - 32).

Approach Read · Plan · Solve · Check

Read. Decide which letter to isolate, and treat all the other letters as ordinary constants.

Plan.

1. Undo addition and subtraction first, then multiplication and division.
2. Divide every term on the side by the coefficient, not just one term.
3. If the target letter appears twice, gather its terms and factor it out.

Solve.

1. Apply each inverse operation to the whole side.
2. Simplify until the target letter stands alone.

Check. Check that every term got divided and that the target letter is by itself, appearing nowhere on the other side.

Common traps — every wrong answer is one of these

Wrong operation or order.
Undoes the operations in the wrong order or uses the wrong inverse — multiplying when it should divide, or moving a term with the wrong sign. Undo addition and subtraction before multiplication and division, and use the opposite operation each time.

Dividing only one term. 1. Solving P = 2L + 2W for L, divides by 2.
2. Divides only the 2L term and writes L = P/2 - 2W.
3. But the whole side had to be divided, including the P.
4. Fix: divide every term by 2, giving L = (P - 2W)/2.

Not factoring a repeated variable. 1. Solving ax = bx + c for x, tries to divide before

collecting.
2. Leaves x on both sides, as in x = (c + bx)/a.
3. The variable is not isolated because it still appears on the right.
4. Fix: move bx over, factor x from ax - bx, then divide: x = c/(a - b).

Practice

1. *The perimeter of a rectangle is P = 2L + 2W.*

Solve for L.

(A) L = P/2 - 2W
(B) L = 2(P - 2W)
(C) L = P - 2W - 2
(D) L = (P - 2W)/2

2. *y = x/3 + 4.*

Solve for x.

(A) x = (y - 4)/3
(B) x = y/3 - 4
(C) x = 3(y - 4)
(D) x = 3y - 4

3. *ax = bx + c.*

Solve for x.

(A) x = c/(ab)
(B) x = c/(a - b)
(C) x = (c + bx)/a
(D) x = c/(a + b)

4. *The temperature conversion formula is F = (9/5)C + 32.*

Solve for C.

(A) C = (5/9)F - 32
(B) C = (5/9)F + 32
(C) C = (9/5)(F - 32)
(D) C = (5/9)(F - 32)

5. *The area of a trapezoid is A = (1/2)(b1 + b2)h.*

Solve for h.

(A) h = A/(2(b1 + b2))

(B) h = 2A - (b1 + b2)

(C) h = 2A/b1 + b2

(D) h = 2A/(b1 + b2)

Build & interpret linear models in context

node: sat.linear.L11

The skill. Building a linear model means turning a situation into an equation of the form y = (rate)x + (starting amount). The quantity that repeats for each unit — dollars per month, liters per minute — is the slope, and the one-time or starting value is the y-intercept. If the quantity goes down, the rate is negative. When the problem sets a limit rather than an exact value, use an inequality: at most means <=, at least means >=.

Worked example

A tank holds 200 liters of water and drains at 8 liters per minute. Write an equation for the volume y after x minutes.

Read. The starting amount is 200 liters, at x = 0. The water leaves at 8 liters each minute, so the volume goes down — the rate is negative.

Plan.

1. Use 200 as the starting amount (the constant).
2. Use 8 as the per-minute rate, made negative because the tank is draining.
3. Assemble y = starting amount plus rate times x.

Solve.

$y = 200 + (\text{rate})x$	start with the fixed amount plus a rate term
$y = 200 - 8x$	the rate is -8 liters per minute (draining)

Check. At x = 0 the tank holds 200 liters, correct. After 10 minutes y = 200 - 8(10) = 120 liters, which is 80 liters gone in 10 minutes at 8 per minute. It fits.

Answer: y = 200 - 8x.

Approach Read · Plan · Solve · Check

Read. Pick out the variable, the per-unit rate, the fixed or starting amount, and whether the problem states an exact value or a limit.

Plan.

1. Put the per-unit rate as the coefficient of the variable.
2. Put the fixed or starting amount as the constant.
3. Make the rate negative if the quantity decreases.
4. Choose =, <=, or >= to match the wording.

Solve. Assemble the equation y = rate times x plus the starting amount, or the matching inequality.

Check. Is the per-unit amount the slope and the one-time amount the constant, not swapped? Is the sign right, and is the inequality pointing the correct way?

Common traps — every wrong answer is one of these

Swapping the rate and the starting amount.

1. A gym charges $30 to sign up plus $20 per month.
2. Writes y = 30x + 20, using the one-time fee as the rate.
3. But the per-month $20 is the rate and the $30 is the starting amount.
4. Fix: put the rate on x and the fixed fee as the constant: y = 20x + 30.

Wrong sign or combining the pieces.
Models a decrease as an increase (wrong sign on the rate), or adds the rate and the fixed amount into a single number. Keep the rate and the fixed amount as separate terms, and make the rate negative when the quantity falls.

Wrong inequality or using an equation.
Points the inequality the wrong way (<= versus >=), or writes an equals sign where the situation sets a limit. At most is <=, at least is >=, and a cap on a total calls for an inequality, not an equation.

Practice

1. *A gym charges a $30 one-time sign-up fee plus $20 per month.*

Which equation gives the total cost y for x months?

(A) y = 20x - 30
(B) y = 30x + 20
(C) y = 50x
(D) y = 20x + 30

2. *A tank holds 200 liters and drains at 8 liters per minute.*

Which equation gives the volume y after x minutes?

(A) y = 200 - 8x

(B) y = 200 + 8x

(C) y = 8x - 200

(D) y = 8x + 200

3. *Maria has $50 and each ticket costs $4.*

Which inequality shows she can buy x tickets without exceeding her money?

(A) 4x <= 50

(B) 4x = 50

(C) 50x <= 4

(D) 4x >= 50

4. *A plumber charges a $75 visit fee plus $60 per hour, and a job costs $315.*

Which equation finds the number of hours x?

(A) 135x = 315

(B) 75x + 60 = 315

(C) 60x - 75 = 315

(D) 60x + 75 = 315

5. *A printing service charges $0.10 per page plus a $5 setup fee, and a school's budget is at most $50.*

Which inequality gives the number of pages x they can print?

(A) 5x + 0.10 <= 50

(B) 0.10x + 5 >= 50

(C) 0.10x + 5 = 50

(D) 0.10x + 5 <= 50

Parametric linear equations & systems (solve for the constant controlling solution count)

node: sat.linear.L12

The skill. Some problems give you an equation or system with an unknown constant and ask which value of that constant produces a certain number of solutions. For a one-variable equation ax = c, there is one solution when a is not zero, no solution when a is zero but c is not (since 0 cannot equal a nonzero number), and infinitely many when both a and c are zero. For a two-variable system, the lines cross once when their slopes differ; they are parallel with no solution when the coefficients are in proportion but the constants are not; and they are the same line with infinitely many solutions

when everything is in proportion, constants included. You set up the condition that matches the target count and solve for the unknown.

Worked example

The system kx - 3y = 4 and 4x - 5y = 7 has no solution. What is the value of k?

Read. No solution means the two lines are parallel: their x- and y-coefficients are in the same proportion, but the constants are not. We match the coefficients to find k.

Plan.

1. Set the ratio of x-coefficients equal to the ratio of y-coefficients: k/4 = (-3)/(-5).
2. Simplify the right side.
3. Solve for k.

Solve.

$$\frac{k}{4} = \frac{-3}{-5} \qquad \text{coefficients in proportion for parallel lines}$$

$$\frac{k}{4} = \frac{3}{5} \qquad \text{simplify the right side}$$

$$k = \frac{12}{5} \qquad \text{multiply both sides by 4}$$

Check. With k = 12/5 the first equation's coefficients are in the ratio 3/5 with the second's, but the constants give 4/7, which is different — so the lines are parallel with no solution, as required.

Answer: k = 12/5.

Approach Read · Plan · Solve · Check

Read. Identify the target count — one, none, or infinitely many — and which coefficient or constant is unknown.

Plan.

1. For ax = c, focus on when the coefficient a becomes zero.
2. For a system with no solution, set the slopes equal (coefficients in proportion) but the constants not matching.
3. For a system with infinitely many solutions, set every part in proportion, constants included.
4. Solve the resulting condition for the unknown constant.

Solve.

1. Write the proportion and solve it for the constant.
2. Check whether the constants also match, which tells no-solution apart from infinitely-many.

Check. Did you use the right condition for the target count? No solution means equal slopes but NOT identical lines; infinitely many means fully identical; ax = c fails only when a = 0.

Common traps — every wrong answer is one of these

Wrong condition for the target count.
Uses the wrong structure for the count wanted — requiring identical equations for a no-solution answer, or only equal slopes for an infinitely-many answer. Parallel lines (equal slopes, different constants) give no solution; identical lines (everything proportional) give infinitely many.

Missing when ax = c has no solution.
For ax = c, forgets that no solution happens only when the coefficient a = 0, and instead picks a nonzero value or thinks a nonzero coefficient can give no solution. Any nonzero a gives the single solution x = c/a.

Setting up the proportion wrong.
Matches the wrong terms, cross-multiplies incorrectly, makes a sign error, or forgets to scale the constant when scaling the equation. Line up x-coefficient with x-coefficient, y with y, and constant with constant.

Practice

1. *In the equation ax = 5, a is a constant.*

For what value of a does the equation have no solution?

(A) 0
(B) 10
(C) 5
(D) 1

2. *The system kx - 3y = 4 and 4x - 5y = 7 has no solution, where k is a constant.*

What is the value of k?

(A) 16/7
(B) -16/7
(C) -12/5
(D) 12/5

3. *The system 3s - 2t = a and -15s + bt = -7 has infinitely many solutions, where a and b are constants.*

What is the value of a?

(A) -7/5
(B) -35
(C) 7

(D) 7/5

4. *The system $2x + 3y = 6$ and $4x + 6y = k$ has a constant k.*

For what value of k does the system have infinitely many solutions?

(A) 3

(B) 12

(C) Any value except 12

(D) 6

5. *In the equation $4x + c = 4x + 7$, c is a constant.*

For what value of c does the equation have infinitely many solutions?

(A) 0

(B) 7

(C) Any value of c

(D) No value of c

Solve a linear system graphically (intersection point)

node: sat.systems.S1

The skill. The solution of a two-line system is the single point where the two lines cross, because that point is the one (x, y) that lies on both lines at once. You can find it by graphing both lines and reading off where they meet, or match a system to its picture by comparing slopes and intercepts. If the lines are parallel — same slope, different intercepts — they never cross, so there is no solution. If the two equations describe the very same line, they overlap everywhere and there are infinitely many solutions.

Worked example

Two lines are graphed: y = x + 1 and y = -x + 5. At what point do they intersect?

Read. The intersection is the point that lies on both lines, so its x and y satisfy both equations. Setting the two expressions for y equal will find the x-coordinate.

Plan.

1. Set the two right-hand sides equal: x + 1 = -x + 5.
2. Solve for x.
3. Put x into either equation to get y.

Solve.

$x + 1 = -x + 5$	both equations equal y, so set them equal
$2x + 1 = 5$	add x to both sides
$2x = 4$	subtract 1 from both sides
$x = 2$	divide both sides by 2
$y = 2 + 1 = 3$	substitute x = 2 into y = x + 1

Check. The point (2, 3) should be on both lines: y = x + 1 gives 2 + 1 = 3, and y = -x + 5 gives -2 + 5 = 3. Both give y = 3, so the lines cross at (2, 3).

Answer: (2, 3).

Approach Read · Plan · Solve · Check

Read. The solution is the point where the two lines meet — the one point that lies on both lines.

Plan.

1. Put each equation in slope-intercept form.
2. Graph both lines, or compare their slopes and intercepts.
3. Read off the point where they cross.

Solve.

1. Locate the intersection point (x, y).
2. Check it by substituting into both equations.

Check. Same slope with different intercepts means no solution; identical lines mean infinitely many. Make sure you read the x-coordinate first, then y.

Common traps — every wrong answer is one of these

Swapping the coordinates.
Reads the crossing point as (y, x) instead of (x, y), swapping the two numbers. Always give the horizontal coordinate first, then the vertical one.

Naming a point for parallel lines.
Sees two parallel lines but still picks a point on one of them as the solution. Parallel lines never meet, so the system has no solution at all.

Using just one line.
Reports a point on only one of the lines — often its intercept — as the solution, ignoring the second equation. The solution must lie on both lines at once.

Mismatching slope or intercept.
Matches the system to the wrong graph by mixing up which line has which slope or intercept.

Check that each equation's slope and intercept match the line you assign it.

Practice

1. *Two lines cross at the point 3 right and 2 up from the origin.*

What is the solution (x, y)?

(A) (3, 2)
(B) (2, 3)
(C) (0, 2)
(D) (3, 0)

2. *A system graphs as two distinct parallel lines.*

How many solutions does the system have?

(A) Infinitely many
(B) Exactly two
(C) No solution
(D) One solution at their y-intercepts

3. *Lines $y = x + 1$ and $y = -x + 5$ are graphed.*

At what point do they intersect?

(A) (3, 2)
(B) (0, 1)
(C) (0, 5)
(D) (2, 3)

4. *The system $y = 2x - 3$ and $y = 2x + 4$ is graphed.*

What is the solution?

(A) (0, -3)
(B) No solution (parallel lines)
(C) Infinitely many
(D) (0, 4)

5. *A system's two equations reduce to the SAME line.*

How many solutions?

(A) No solution

(B) Two

(C) Exactly one

(D) Infinitely many solutions

Equivalent linear expressions (distribute and combine like terms)

node: sat.expressions.X1

The skill. Simplifying a linear expression means writing it in its shortest equal form. First distribute: multiply each number in front of parentheses by every term inside, not just the first. Then combine like terms — add the coefficients of the matching variable, and add the plain numbers separately, but never mix an x-term with a constant. A minus sign in front of parentheses acts like multiplying every term inside by -1. Two expressions are equivalent when they simplify to exactly the same thing.

Worked example

Simplify the expression 4x + 7 - 2(x + 3).

Read. There is a set of parentheses with a -2 in front of it. We distribute the -2 across both terms inside, then combine the like terms.

Plan.

1. Distribute -2 to both x and 3 inside the parentheses.
2. Group the x-terms and the constants.
3. Combine each group.

Solve.

$$\begin{aligned} 4x + 7 - 2(x + 3) &= 4x + 7 - 2x - 6 && \text{distribute -2 to both terms} \\ &= (4x - 2x) + (7 - 6) && \text{group like terms} \\ &= 2x + 1 && \text{combine each group} \end{aligned}$$

Check. Test x = 5: the original is 4(5) + 7 - 2(5 + 3) = 20 + 7 - 16 = 11, and the simplified form is 2(5) + 1 = 11. They match, so 2x + 1 is equivalent.

Answer: 2x + 1.

Approach Read · Plan · Solve · Check

Read. Rewrite the expression in its simplest form; two expressions are equivalent only if they simplify to the same thing.

Plan.

1. Distribute each factor across every term inside its parentheses.

2. Group the variable terms together and the constants together.
3. Add within each group.

Solve.

1. Multiply each factor through its parentheses.
2. Add the coefficients of the variable terms.
3. Add the constants separately.

Check. Check that you distributed to every term, kept the sign on a negative factor, and never added an x-term to a plain number.

Common traps — every wrong answer is one of these

Distributing to only the first term.
Multiplies the outside factor by just the first term inside, turning 3(2x - 4) into 6x - 4 instead of 6x - 12. The factor must multiply every term inside the parentheses.

Dropping a negative sign.
When a minus sign is in front of parentheses, flips the sign of only the first term, writing -(2x - 5) as -2x - 5 instead of -2x + 5. A leading minus changes the sign of every term inside.

Combining unlike terms.
Adds a variable term to a constant as if they were alike, turning 6x + 9 into 15x. Only x-terms combine with x-terms, and numbers with numbers.

Adding coefficients wrong.
Combines the right terms but does the arithmetic wrong, writing 5x - 2x as 7x instead of 3x. Add or subtract the coefficients carefully, watching the signs.

Practice

1. Which is equivalent to 3(2x - 4) + 5?

(A) 6x - 12
(B) 6x + 1
(C) 6x - 7
(D) 6x - 4 + 5

2. Which is equivalent to -(2x - 5) + 3x?

(A) 5x + 5
(B) x + 5
(C) x - 5
(D) -5x + 5

3. Simplify: 4x + 7 - 2(x + 3).

(A) 2x + 1

(B) 2x + 13

(C) 6x + 1

(D) 2x + 4

4. Which is equivalent to 5(x + 2) - 3x?

(A) 12x

(B) 2x + 2

(C) 2x + 10

(D) 8x + 10

5. Simplify: 2(3x - 1) - (x - 4).

(A) 5x - 6

(B) 5x - 2

(C) 7x + 2

(D) 5x + 2

Equivalent linear expressions by factoring a common factor

node: sat.expressions.X2

The skill. Factoring a linear expression is the reverse of distributing: you pull out a number that divides every term and write it outside a set of parentheses. For ax + b you find the greatest common factor k — the biggest number that divides both a and b — and write k(px + q), where p and q are what is left after dividing each term by k. You can always check by distributing back: multiplying out should return the original expression. If you factor out a negative, the sign of every remaining term flips.

Worked example

Factor the expression 12x + 8 completely.

Read. We look for the greatest number that divides both 12 and 8. That number comes out front, and what is left goes inside the parentheses.

Plan.

1. Find the greatest common factor of 12 and 8.
2. Divide each term by that factor.
3. Write the factor outside and the quotients inside.

Solve.

$$\begin{aligned} 12x + 8 &= 4 \cdot 3x + 4 \cdot 2 && \text{4 is the greatest common factor of 12 and 8} \\ &= 4(3x + 2) && \text{pull the 4 outside the parentheses} \end{aligned}$$

Check. Distribute back: 4(3x + 2) = 12x + 8, the original expression. So the factored form is correct.

Answer: 4(3x + 2).

Approach Read · Plan · Solve · Check

Read. Rewrite the expression as (common factor)(what is left) without changing its value.

Plan.

1. Find the greatest number that divides every term.
2. Write that factor outside the parentheses.
3. Divide each term by the factor and put the results inside.

Solve.

1. Divide each term by the common factor.
2. Place the quotients inside the parentheses.

Check. Distribute back to make sure you recover the original. Check that the factor divided every term, including the signs.

Common traps — every wrong answer is one of these

Factoring only one term.
Pulls the factor from just one term and leaves the other alone, writing 6x + 9 as 3(2x) + 9 instead of 3(2x + 3). Every term inside the parentheses must be the result of dividing by the factor.

Factor does not divide every term.
Pulls out a number that does not divide every term evenly, forcing a fraction, like 6x + 8 = 3(2x + 8/3). The common factor must divide all terms with no remainder — here that is 2, not 3.

Sign error factoring a negative. 1. Factors -6x + 9 by pulling out -3.
2. Writes -3(2x + 3), keeping the +3 inside.
3. But -3 times +3 gives -9, not the original +9.
4. Fix: flip the sign of every remaining term: -3(2x - 3).

Dividing a term wrong.
Divides one term by the common factor incorrectly, writing 12x + 8 as 4(3x + 4) when 8 divided by 4 is 2, not 4. Divide each term carefully and check by multiplying back.

Practice

1. Which is equivalent to 6x + 9?

   (A) 3(2x + 3)
   (B) 3(2x) + 9
   (C) 2(3x + 4.5)
   (D) 3(2x + 6)

2. Factor completely: 12x + 8.

   (A) 4(3x + 4)
   (B) 2(6x + 8)
   (C) 4(3x + 8)
   (D) 4(3x + 2)

3. Which is equivalent to -6x + 9?

   (A) -3(2x - 3)
   (B) -3(2x + 9)
   (C) -3(2x + 3)
   (D) 3(2x - 3)

4. Which is equivalent to 10x - 15?

   (A) 5(2x - 3)
   (B) 5(2x - 15)
   (C) 5(2x - 5)
   (D) 5(2x + 3)

5. Factor: 9x + 6.

   (A) 3(3x + 6)
   (B) 2(4.5x + 3)
   (C) 3(3x + 2)
   (D) 3(3x + 3)

Absolute-value equations & functions

node: sat.absval.A1

The skill. The absolute value of a number is its distance from zero, so it is never negative. To solve |expression| = k when k is positive, remember that two different insides sit k units from zero, so you split into two cases: expression = k and expression = -k, and solve each as an ordinary linear

equation. If k is negative there is no solution, because a distance can't be negative; if k is zero there is exactly one solution. The graph of y = |x - h| + k is a V that opens upward with its corner, the vertex, at (h, k). For inequalities, |x| < k means x is within k of zero, so -k < x < k (between), while |x| > k means x is more than k from zero, so x < -k or x > k (outside).

Worked example

Solve |x - 3| = 7.

Read. The absolute value of x - 3 equals 7. Since 7 is positive, the inside can be 7 or -7, so there are two cases.

Plan.

1. Set the inside equal to 7 and solve.
2. Set the inside equal to -7 and solve.
3. Report both answers.

Solve.

$$x - 3 = 7 \quad \text{or} \quad x - 3 = -7 \qquad \text{split into two cases}$$

$$x = 10 \quad \text{or} \quad x = -4 \qquad \text{add 3 to both sides}$$

Check. Check: |10 - 3| = |7| = 7 and |-4 - 3| = |-7| = 7. Both answers work.
Answer: x = 10 or x = -4.

Approach Read · Plan · Solve · Check

Read. Decide whether the problem is an equation, an inequality, or a graph, and check the sign of the number on the right before solving.

Plan.

1. For an equation with a positive right side, write two cases: inside = k and inside = -k.
2. If the right side is negative, there is no solution.
3. For a graph, read the vertex (h, k) from y = |x - h| + k.
4. For an inequality, 'less than' means between and 'greater than' means outside.

Solve. Solve each case as a linear equation, or apply the vertex or between/outside rule.

Check. Did you keep both cases, check that the right side is not negative, read the vertex sign correctly, and match between/outside to less-than/greater-than?

Common traps — every wrong answer is one of these

Dropped the negative case.
Solves |expression| = k using only expression = k and reports a single answer instead of two. Fix: also solve expression = -k, since the inside can be positive or negative.

Missed the impossible equation.
Treats |expression| = (a negative number) as solvable and sets up cases anyway. Fix: an absolute value can never be negative, so there is no solution.

Vertex sign misread.
Misreads the vertex of y = |x - h| + k — for example taking the x-coordinate as -h instead of h, getting the sign of k wrong, or thinking the V opens downward. Fix: the vertex is (h, k) and the V opens upward.

Between and outside swapped.
Uses the outside (or) form for |x| < k, or the between (and) form for |x| > k. Fix: less-than is between, -k < x < k; greater-than is outside, x < -k or x > k.

Practice

1. *Solve |x| = 5.*

What are all solutions?

(A) x = 0
(B) x = 5
(C) x = -5
(D) x = 5 or x = -5

2. *Solve |x - 3| = 7.*

What are all solutions?

(A) x = 10 or x = 4
(B) x = 7 or x = -7
(C) x = 10
(D) x = 10 or x = -4

3. *Solve |2x + 1| = -3.*

What are all solutions?

(A) No solution
(B) x = -3
(C) x = -2 or x = 1
(D) x = -2

4. *The graph of y = |x - 4| + 2.*

Where is its vertex?

(A) (4, 2)

(B) (-4, 2)

(C) (-4, -2)

(D) (4, -2)

5. *Solve the inequality $|x| < 4$.*

Which describes all solutions?

(A) $x > 4$

(B) $x < -4$ or $x > 4$

(C) $x < 4$

(D) $-4 < x < 4$

Absolute-value equations with a coefficient

node: sat.absval.A2

The skill. When an absolute-value equation has a coefficient, like $|ax + b| = k$ with k positive, the same two-case idea applies: the inside can equal k or -k. Write $ax + b = k$ and $ax + b = -k$, then solve each linear equation for x — which includes dividing by the coefficient a at the end. Both cases must be kept, and the coefficient must be divided out to finish.

Worked example

Solve $|2x + 2| = 6$.

Read. The absolute value of $2x + 2$ equals 6. Since 6 is positive, the inside is 6 or -6, giving two cases.

Plan.

1. Set $2x + 2 = 6$ and solve for x.
2. Set $2x + 2 = -6$ and solve for x.
3. Remember to divide by 2 in each case.

Solve.

$$\begin{aligned} 2x + 2 = 6 \quad &\text{or} \quad 2x + 2 = -6 && \text{split into two cases} \\ 2x = 4 \quad &\text{or} \quad 2x = -8 && \text{subtract 2 from both sides} \\ x = 2 \quad &\text{or} \quad x = -4 && \text{divide both sides by 2} \end{aligned}$$

Check. Check: $|2(2) + 2| = |6| = 6$ and $|2(-4) + 2| = |-6| = 6$. Both answers work.
Answer: $x = 2$ or $x = -4$.

Approach Read · Plan · Solve · Check

Read. Confirm the right side is positive, so the equation splits into two cases.

Plan.

1. Write ax + b = k and solve for x.
2. Write ax + b = -k and solve for x.
3. Divide by the coefficient a in each case.

Solve. Solve each linear equation, remembering to divide by a.

Check. Did you keep both cases and divide by the coefficient a?

Common traps — every wrong answer is one of these

Dropped the negative case.
Solves only ax + b = k and misses the other answer. Fix: also solve ax + b = -k.

Both signs flipped.
Reports the negatives of the two correct answers. Fix: solve each case carefully; the two solutions are not simply the negatives of each other.

Coefficient ignored.
Solves as if a were 1 and never divides by the coefficient. Fix: after isolating ax, divide both sides by a.

Practice

1. *Solve $|2x + 2| = 6$.*

Which is correct?

(A) x = 2 or x = -4
(B) x = 2
(C) x = -2 or x = 4
(D) x = 6

2. *Solve $|3x + 6| = 6$.*

Which is correct?

(A) x = 0
(B) x = 0 or x = 4
(C) x = 6
(D) x = 0 or x = -4

3. *Solve |3x + 3| = 6.*

Which is correct?

(A) x = 1 or x = -3
(B) x = 6
(C) x = -1 or x = 3
(D) x = 1

4. *Solve |3x + 6| = 9.*

Which is correct?

(A) x = 1 or x = -5
(B) x = -1 or x = 5
(C) x = 1
(D) x = 9

5. *Solve |3x - 6| = 9.*

Which is correct?

(A) x = 5
(B) x = 5 or x = -1
(C) x = 15 or x = -3
(D) x = -5 or x = 1

Absolute-value inequalities: less-than (between)

node: sat.absval.A3

The skill. The inequality |x - h| < k says the distance between x and h is less than k, so x lands between two values: h - k < x < h + k. A 'less than' absolute-value inequality always gives a single range written as one compound (and) inequality, not two separate pieces joined by 'or'. To find the range, subtract k from h and add k to h.

Worked example

Solve |x - 3| < 4.

Read. The distance from x to 3 is less than 4, so x is between two values. This is a 'less than' inequality, which gives a single range.

Plan.

1. Write -4 < x - 3 < 4.
2. Add 3 to all three parts.
3. Read off the range.

Solve.

$$\begin{aligned} -4 < x - 3 < 4 \quad & \text{less than means between} \\ -4 + 3 < x < 4 + 3 \quad & \text{add 3 to all three parts} \\ -1 < x < 7 \quad & \text{simplify} \end{aligned}$$

Check. Check the endpoints: x = -1 gives |-1 - 3| = 4, and x = 7 gives |7 - 3| = 4, both equal to 4 and correctly just outside the range. A midpoint like x = 3 gives 0 < 4.

Answer: -1 < x < 7.

Approach Read · Plan · Solve · Check

Read. A 'less than' inequality means the values are between two bounds.

Plan.

1. Write the compound inequality -k < x - h < k.
2. Add h to all three parts.
3. This gives h - k < x < h + k.

Solve. Add h to each part to get the two bounds.

Check. Is the answer a single 'between' range, and did you add h to both bounds?

Common traps — every wrong answer is one of these

Used the outside form.
Writes x < h - k or x > h + k, the 'greater than' shape. Fix: 'less than' gives a single between-range, h - k < x < h + k.

Kept only one bound.
Reports just one side, like x < h + k, dropping the lower bound. Fix: a between-range needs both bounds.

Forgot the shift.
Leaves the answer as -k < x < k and never adds h. Fix: add h to both bounds to account for the shift.

Practice

1. *Solve |x - 3| < 4.*

Which is correct?

(A) x < 7

(B) x < -1 or x > 7

(C) -1 < x < 7

(D) -4 < x < 4

2. *Solve |x - 5| < 4.*

Which is correct?

(A) x < 1 or x > 9
(B) -4 < x < 4
(C) x < 9
(D) 1 < x < 9

3. *Solve |x - 2| < 2.*

Which is correct?

(A) x < 4
(B) -2 < x < 2
(C) x < 0 or x > 4
(D) 0 < x < 4

4. *Solve |x - 2| < 4.*

Which is correct?

(A) -4 < x < 4
(B) -2 < x < 6
(C) x < 6
(D) x < -2 or x > 6

5. *Solve |x - 2| < 5.*

Which is correct?

(A) -3 < x < 7
(B) x < 7
(C) -5 < x < 5
(D) x < -3 or x > 7

Absolute-value inequalities: greater-than (outside)

node: sat.absval.A4

The skill. The inequality |x - h| > k says the distance between x and h is more than k, so x falls outside the middle region: x < h - k or x > h + k. A 'greater than' absolute-value inequality always splits into two separate pieces joined by 'or', one going low and one going high. To find the two bounds, subtract k from h and add k to h.

Worked example

Solve |x - 3| > 2.

Read. The distance from x to 3 is more than 2, so x is outside the middle. This is a 'greater than' inequality, which splits into two pieces.

Plan.

1. Write x - 3 < -2 or x - 3 > 2.
2. Add 3 to each piece.
3. Read off the two pieces.

Solve.

$$x - 3 < -2 \quad \text{or} \quad x - 3 > 2 \qquad \text{greater than means outside}$$
$$x < 1 \quad \text{or} \quad x > 5 \qquad \text{add 3 to each piece}$$

Check. Check: x = 0 gives |0 - 3| = 3 > 2, and x = 6 gives |6 - 3| = 3 > 2, so both outside pieces work; x = 3 gives 0, correctly excluded.

Answer: x < 1 or x > 5.

Approach Read · Plan · Solve · Check

Read. A 'greater than' inequality means the values are outside two bounds.

Plan.

1. Write the two pieces x - h < -k or x - h > k.
2. Add h to each piece.
3. This gives x < h - k or x > h + k.

Solve. Add h to each piece to get the two bounds.

Check. Are there two pieces joined by 'or', and did you add h to each?

Common traps — every wrong answer is one of these

Used the between form.
Writes h - k < x < h + k, the 'less than' shape. Fix: 'greater than' splits into two outside pieces joined by 'or'.

Kept only one piece.
Reports just one inequality, like x > h + k, dropping the other piece. Fix: 'greater than' needs both the low and the high piece.

Forgot the shift.
Leaves the answer as x < -k or x > k and never adds h. Fix: add h to each bound to account for the shift.

Practice

1. *Solve |x - 3| > 2.*

Which is correct?

(A) x < 1 or x > 5
(B) x < -2 or x > 2
(C) 1 < x < 5
(D) x > 5

2. *Solve |x - 3| > 4.*

Which is correct?

(A) x > 7
(B) -1 < x < 7
(C) x < -4 or x > 4
(D) x < -1 or x > 7

3. *Solve |x - 2| > 2.*

Which is correct?

(A) x > 4
(B) 0 < x < 4
(C) x < 0 or x > 4
(D) x < -2 or x > 2

4. *Solve |x - 5| > 4.*

Which is correct?

(A) x > 9
(B) x < 1 or x > 9
(C) 1 < x < 9
(D) x < -4 or x > 4

5. *Solve |x - 1| > 3.*

Which is correct?

(A) -2 < x < 4
(B) x > 4
(C) x < -2 or x > 4
(D) x < -3 or x > 3

Graphs of absolute-value functions (vertex form)

node: sat.absval.A5

The skill. The graph of y = |x + h| - k is a V-shape, and its corner, the vertex, sits where the inside of the absolute value is zero. Setting x + h = 0 gives x = -h, so the x-coordinate of the vertex is the opposite of the inside sign. The number outside, -k, is the y-coordinate directly, so the vertex is (-h, -k). In short: the horizontal position flips the inside sign, while the outside constant gives the height as written.

Worked example

Find the vertex of y = |x + 4| - 2.

Read. This is a V-shaped graph. The vertex is where the inside, x + 4, equals zero, and the outside constant gives the height.

Plan.

1. Set x + 4 = 0 and solve for x.
2. Read the outside constant -2 as the y-coordinate.
3. Write the vertex.

Solve.

$x + 4 = 0$	the inside equals zero at the vertex
$x = -4$	subtract 4 from both sides
$y = -2$	the outside constant is the y-coordinate

Check. At x = -4 the inside is 0, so y = |0| - 2 = -2, the lowest point of the V. The vertex (-4, -2) checks out.

Answer: the vertex is (-4, -2).

Approach Read · Plan · Solve · Check

Read. The vertex is the point where the inside of the absolute value equals zero.

Plan.

1. Set the inside equal to zero and solve for x.
2. Read the outside constant as the y-coordinate.
3. Write the vertex as (x, y).

Solve. Solve x + h = 0 for the x-coordinate and take the outside constant for the y-coordinate.

Check. Did the x-coordinate flip the inside sign, and did the y-coordinate come straight from the outside constant?

Common traps — every wrong answer is one of these

Wrong sign on x.
Uses +h instead of -h for the vertex x-coordinate. Fix: solve inside = 0, so x + h = 0 gives x = -h.

Wrong sign on y.
Uses +k instead of -k for the vertex y-coordinate. Fix: the outside constant is the y-value as written, so -k gives y = -k.

Both signs wrong.
Reports (h, k), flipping both coordinates. Fix: the vertex is (-h, -k) for y = |x + h| - k.

Practice

1. *What is the vertex of $y = |x + 4| - 2$?*

Which is correct?

(A) (4, -2)
(B) (4, 2)
(C) (-4, -2)
(D) (-4, 2)

2. *What is the vertex of $y = |x + 2| - 5$?*

Which is correct?

(A) (-2, -5)
(B) (2, 5)
(C) (2, -5)
(D) (-2, 5)

3. *What is the vertex of $y = |x + 3| - 2$?*

Which is correct?

(A) (3, 2)
(B) (-3, -2)
(C) (3, -2)
(D) (-3, 2)

4. *What is the vertex of $y = |x + 3| - 5$?*

Which is correct?

(A) (3, -5)

(B) (-3, 5)

(C) (-3, -5)

(D) (3, 5)

5. *Find the vertex of $y = |x - 5| + 3$.*

Which is correct?

(A) (-5, -3)

(B) (5, 3)

(C) (5, -3)

(D) (-5, 3)

Answer Key — Algebra

Solve linear equations in one variable — Q1
Correct: **(C)**. *Traps:* **(A)** Dividing the wrong way; **(B)** Moving a term without flipping its sign; **(D)** Dividing the wrong way.

Solve linear equations in one variable — Q2
Correct: **(B)**. *Traps:* **(A)** Dividing the wrong way; **(C)** Doing something to only one side; **(D)** Moving a term without flipping its sign.

Solve linear equations in one variable — Q3
Correct: **(C)**. *Traps:* **(A)** Dividing the wrong way; **(B)** Moving a term without flipping its sign; **(D)** Forgetting to distribute to every term.

Solve linear equations in one variable — Q4
Correct: **(D)**. *Traps:* **(A)** Dividing the wrong way; **(B)** Moving a term without flipping its sign; **(C)** Forgetting to distribute to every term.

Solve linear equations in one variable — Q5
Correct: **(D)**. *Traps:* **(A)** Missing the special cases; **(B)** Missing the special cases; **(C)** Missing the special cases.

Slope as ratio (rise/run, geometric) — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong sign; **(B)** Using one point instead of two; **(C)** Flipping the fraction (run over rise).

Slope as ratio (rise/run, geometric) — Q2
Correct: **(C)**. *Traps:* **(A)** Wrong sign; **(B)** Flipping the fraction (run over rise); **(D)** Using one point instead of two.

Slope as ratio (rise/run, geometric) — Q3
Correct: **(D)**. *Traps:* **(A)** Flipping the fraction (run over rise); **(B)** Horizontal and vertical swapped; **(C)** Using one point instead of two.

Slope as ratio (rise/run, geometric) — Q4
Correct: **(C)**. *Traps:* **(A)** Flipping the fraction (run over rise); **(B)** Horizontal and vertical swapped; **(D)** Using one point instead of two.

Slope as ratio (rise/run, geometric) — Q5
Correct: **(B)**. *Traps:* **(A)** Using one point instead of two; **(C)** Thinking the slope depends on the points; **(D)** Flipping the fraction (run over rise).

Slope as rate of change (contextual) — Q1
Correct: **(D)**. *Traps:* **(A)** Ignoring the real step size; **(B)** Units flipped or missing; **(C)** Wrong sign for the direction.

Slope as rate of change (contextual) — Q2
Correct: **(A)**. *Traps:* **(B)** Thinking the rate is not constant; **(C)** Swapping the roles of slope and intercept; **(D)** Thinking the rate is not constant.

Slope as rate of change (contextual) — Q3
Correct: **(C)**. *Traps:* **(A)** Using one point as the rate; **(B)** Using one point as the rate; **(D)** Units flipped or missing.

Slope as rate of change (contextual) — Q4
Correct: **(B)**. *Traps:* **(A)** Using one point as the rate; **(C)** Swapping the roles of slope and intercept; **(D)** Reading a data point as the starting value.

Slope as rate of change (contextual) — Q5
Correct: **(C)**. *Traps:* **(A)** Thinking the rate is not constant; **(B)** Thinking the rate is not constant; **(D)** Thinking the rate is not constant.

Forms of a line (slope-intercept, point-slope, standard) — Q1
Correct: **(A)**. *Traps:* **(B)** Mixing up the two intercepts; **(C)** Mixing up slope and intercept; **(D)** Mixing up the two intercepts.

Forms of a line (slope-intercept, point-slope, standard) — Q2
Correct: **(B)**. *Traps:* **(A)** Sign or flip error solving for y; **(C)** Reading slope straight off standard form; **(D)** Sign or flip error solving for y.

Forms of a line (slope-intercept, point-slope, standard) — Q3
Correct: **(B)**. *Traps:* **(A)** Reading slope straight off standard form; **(C)** Mixing up the two intercepts; **(D)** Mixing up the two intercepts.

Forms of a line (slope-intercept, point-slope, standard) — Q4
Correct: **(D)**. *Traps:* **(A)** Sign slip in point-slope form; **(B)** Sign slip in point-slope form; **(C)** Sign slip in point-slope form.

Forms of a line (slope-intercept, point-slope, standard) — Q5
Correct: **(A)**. *Traps:* **(B)** Sign or flip error solving for y; **(C)** Mixing up the two intercepts; **(D)** Sign or flip error solving for y.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q1
Correct: **(C)**. *Traps:* **(A)** Sign or distribution slip; **(B)** Using the point's y-value as the intercept; **(D)** Using the point's y-value as the intercept.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q2
Correct: **(D)**. *Traps:* **(A)** Using the point's y-value as the intercept; **(B)** Flipping the fraction (run over rise); **(C)** Sign or distribution slip.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q3
Correct: **(A)**. *Traps:* **(B)** Sign or distribution slip; **(C)** Using the point's y-value as the intercept; **(D)** Sign or distribution slip.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q4
Correct: **(A)**. *Traps:* **(B)** Wrong slope for parallel or perpendicular; **(C)** Wrong slope for parallel or perpendicular; **(D)** Wrong slope for parallel or perpendicular.

Write a line from conditions (two points / point+slope / table / parallel-perpendicular) — Q5
Correct: **(C)**. *Traps:* **(A)** Using the point's y-value as the intercept; **(B)** Using the point's y-value as the intercept; **(D)** Wrong sign.

Interpret slope & intercept of a linear model in context — Q1
Correct: **(A)**. *Traps:* **(B)** Treating the rate as a total; **(C)** Confusing a number with the variable; **(D)** Swapping the roles of slope and intercept.

Interpret slope & intercept of a linear model in context — Q2
Correct: **(A)**. *Traps:* **(B)** Treating the rate as a total; **(C)** Swapping the roles of slope and intercept; **(D)** Treating the rate as a total.

Interpret slope & intercept of a linear model in context — Q3
Correct: **(B)**. *Traps:* **(A)** Wrong sign or direction; **(C)** Swapping the roles of slope and intercept; **(D)** Dropping the units.

Interpret slope & intercept of a linear model in context — Q4
Correct: **(D)**. *Traps:* **(A)** Dropping the units; **(B)** Dropping the units; **(C)** Swapping the roles of slope and intercept.

Interpret slope & intercept of a linear model in context — Q5
Correct: **(A)**. *Traps:* **(B)** Confusing a number with the variable; **(C)** Treating the rate as a total; **(D)** Swapping the roles of slope and intercept.

Solve linear systems (substitution / elimination) — Q1
Correct: **(B)**. *Traps:* **(A)** Stopping early or mixing up x and y; **(C)** Dividing the wrong way; **(D)** Substituting back into the same equation.

Solve linear systems (substitution / elimination) — Q2
Correct: **(D)**. *Traps:* **(A)** Stopping early or mixing up x and y; **(B)** Stopping early or mixing up x and y; **(C)** Sign or scaling slip in elimination.

Solve linear systems (substitution / elimination) — Q3
Correct: **(A)**. *Traps:* **(B)** Stopping early or mixing up x and y; **(C)** Sign or scaling slip in elimination; **(D)** Stopping early or mixing up x and y.

Solve linear systems (substitution / elimination) — Q4
Correct: **(A)**. *Traps:* **(B)** Moving a term without flipping its sign; **(C)** Dividing the wrong way; **(D)** Stopping early or mixing up x and y.

Solve linear systems (substitution / elimination) — Q5
Correct: **(B)**. *Traps:* **(A)** Misreading the number of solutions; **(C)** Misreading the number of solutions; **(D)** Misreading the number of solutions.

Systems in context & number of solutions — Q1
Correct: **(B)**. *Traps:* **(A)** Building the wrong equations; **(C)** Building the wrong equations; **(D)** Building the wrong equations.

Systems in context & number of solutions — Q2
Correct: **(B)**. *Traps:* **(A)** Reporting the wrong quantity; **(C)** Reporting the wrong quantity; **(D)** Dividing the wrong way.

Systems in context & number of solutions — Q3
Correct: **(C)**. *Traps:* **(A)** Judging solution count by appearance; **(B)** Judging solution count by appearance; **(D)** Judging solution count by appearance.

Systems in context & number of solutions — Q4
Correct: **(B)**. *Traps:* **(A)** Building the wrong equations; **(C)** Moving a term without flipping its sign; **(D)** Reporting the wrong quantity.

Systems in context & number of solutions — Q5
Correct: **(A)**. *Traps:* **(B)** Judging solution count by appearance; **(C)** Judging solution count by appearance; **(D)** Judging solution count by appearance.

Linear inequalities & systems of inequalities — Q1
Correct: **(B)**. *Traps:* **(A)** Forgetting to flip the sign; **(C)** Moving a term without flipping its sign; **(D)** Dividing the wrong way.

Linear inequalities & systems of inequalities — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong boundary type; **(C)** Shading the wrong side; **(D)** Wrong boundary type.

Linear inequalities & systems of inequalities — Q3
Correct: **(C)**. *Traps:* **(A)** Wrong boundary type; **(B)** Shading the wrong side; **(D)** Shading the wrong side.

Linear inequalities & systems of inequalities — Q4
Correct: **(A)**. *Traps:* **(B)** Wrong region for a system; **(C)** Wrong region for a system; **(D)** Wrong region for a system.

Linear inequalities & systems of inequalities — Q5
Correct: **(A)**. *Traps:* **(B)** Wrong boundary type; **(C)** Wrong boundary type; **(D)** Wrong boundary type.

Literal equations & rearranging formulas — Q1
Correct: **(D)**. *Traps:* **(A)** Dividing only one term; **(B)** Wrong operation or order; **(C)** Wrong operation or order.

Literal equations & rearranging formulas — Q2
Correct: **(C)**. *Traps:* **(A)** Wrong operation or order; **(B)** Wrong operation or order; **(D)** Dividing only one term.

Literal equations & rearranging formulas — Q3
Correct: **(B)**. *Traps:* **(A)** Not factoring a repeated variable; **(C)** Not factoring a repeated variable; **(D)** Wrong operation or order.

Literal equations & rearranging formulas — Q4
Correct: **(D)**. *Traps:* **(A)** Dividing only one term; **(B)** Wrong operation or order; **(C)** Wrong operation or order.

Literal equations & rearranging formulas — Q5
Correct: **(D)**. *Traps:* **(A)** Wrong operation or order; **(B)** Wrong operation or order; **(C)** Dividing only one term.

Build & interpret linear models in context — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong sign or combining the pieces; **(B)** Swapping the rate and the starting amount; **(C)** Wrong sign or combining the pieces.

Build & interpret linear models in context — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong sign or combining the pieces; **(C)** Swapping the rate and the starting amount; **(D)** Swapping the rate and the starting amount.

Build & interpret linear models in context — Q3
Correct: **(A)**. *Traps:* **(B)** Wrong inequality or using an equation; **(C)** Swapping the rate and the starting amount; **(D)** Wrong inequality or using an equation.

Build & interpret linear models in context — Q4
Correct: **(D)**. *Traps:* **(A)** Wrong sign or combining the pieces; **(B)** Swapping the rate and the starting amount; **(C)** Wrong sign or combining the pieces.

Build & interpret linear models in context — Q5
Correct: **(D)**. *Traps:* **(A)** Swapping the rate and the starting amount; **(B)** Wrong inequality or using an equation; **(C)** Wrong inequality or using an equation.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q1
Correct: **(A)**. *Traps:* **(B)** Missing when $ax = c$ has no solution; **(C)** Missing when $ax = c$ has no solution; **(D)** Missing when $ax = c$ has no solution.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q2

Correct: **(D)**. *Traps:* **(A)** Setting up the proportion wrong; **(B)** Setting up the proportion wrong; **(C)** Setting up the proportion wrong.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q3
Correct: **(D)**. *Traps:* **(A)** Setting up the proportion wrong; **(B)** Setting up the proportion wrong; **(C)** Wrong condition for the target count.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q4
Correct: **(B)**. *Traps:* **(A)** Wrong condition for the target count; **(C)** Wrong condition for the target count; **(D)** Wrong condition for the target count.

Parametric linear equations & systems (solve for the constant controlling solution count) — Q5
Correct: **(B)**. *Traps:* **(A)** Missing when ax = c has no solution; **(C)** Wrong condition for the target count; **(D)** Wrong condition for the target count.

Solve a linear system graphically (intersection point) — Q1
Correct: **(A)**. *Traps:* **(B)** Swapping the coordinates; **(C)** Using just one line; **(D)** Using just one line.

Solve a linear system graphically (intersection point) — Q2
Correct: **(C)**. *Traps:* **(A)** Mismatching slope or intercept; **(B)** Naming a point for parallel lines; **(D)** Naming a point for parallel lines.

Solve a linear system graphically (intersection point) — Q3
Correct: **(D)**. *Traps:* **(A)** Swapping the coordinates; **(B)** Using just one line; **(C)** Using just one line.

Solve a linear system graphically (intersection point) — Q4
Correct: **(B)**. *Traps:* **(A)** Naming a point for parallel lines; **(C)** Mismatching slope or intercept; **(D)** Naming a point for parallel lines.

Solve a linear system graphically (intersection point) — Q5
Correct: **(D)**. *Traps:* **(A)** Mismatching slope or intercept; **(B)** Naming a point for parallel lines; **(C)** Mismatching slope or intercept.

Equivalent linear expressions (distribute and combine like terms) — Q1
Correct: **(C)**. *Traps:* **(A)** Combining unlike terms; **(B)** Adding coefficients wrong; **(D)** Distributing to only the first term.

Equivalent linear expressions (distribute and combine like terms) — Q2
Correct: **(B)**. *Traps:* **(A)** Adding coefficients wrong; **(C)** Dropping a negative sign; **(D)** Combining unlike terms.

Equivalent linear expressions (distribute and combine like terms) — Q3
Correct: **(A)**. *Traps:* **(B)** Dropping a negative sign; **(C)** Adding coefficients wrong; **(D)** Distributing to only the first term.

Equivalent linear expressions (distribute and combine like terms) — Q4
Correct: **(C)**. *Traps:* **(A)** Combining unlike terms; **(B)** Distributing to only the first term; **(D)** Adding coefficients wrong.

Equivalent linear expressions (distribute and combine like terms) — Q5
Correct: **(D)**. *Traps:* **(A)** Dropping a negative sign; **(B)** Adding coefficients wrong; **(C)** Adding coefficients wrong.

Equivalent linear expressions by factoring a common factor — Q1
Correct: **(A)**. *Traps:* **(B)** Factoring only one term; **(C)** Factor does not divide every term; **(D)** Dividing a term wrong.

Equivalent linear expressions by factoring a common factor — Q2
Correct: **(D)**. *Traps:* **(A)** Dividing a term wrong; **(B)** Factoring only one term; **(C)** Factoring only one term.

Equivalent linear expressions by factoring a common factor — Q3
Correct: **(A)**. *Traps:* **(B)** Dividing a term wrong; **(C)** Sign error factoring a negative; **(D)** Sign error factoring a negative.

Equivalent linear expressions by factoring a common factor — Q4
Correct: **(A)**. *Traps:* **(B)** Factoring only one term; **(C)** Dividing a term wrong; **(D)** Sign error factoring a negative.

Equivalent linear expressions by factoring a common factor — Q5
Correct: **(C)**. *Traps:* **(A)** Factoring only one term; **(B)** Factor does not divide every term; **(D)** Dividing a term wrong.

Absolute-value equations & functions — Q1
Correct: **(D)**. *Traps:* **(A)** Dropped the negative case; **(B)** Dropped the negative case; **(C)** Dropped the negative case.

Absolute-value equations & functions — Q2
Correct: **(D)**. *Traps:* **(A)** Dropped the negative case; **(B)** Dropped the negative case; **(C)** Dropped the negative case.

Absolute-value equations & functions — Q3
Correct: **(A)**. *Traps:* **(B)** Missed the impossible equation; **(C)** Missed the impossible equation; **(D)** Missed the impossible equation.

Absolute-value equations & functions — Q4
Correct: **(A)**. *Traps:* **(B)** Vertex sign misread; **(C)** Vertex sign misread; **(D)** Vertex sign misread.

Absolute-value equations & functions — Q5
Correct: **(D)**. *Traps:* **(A)** Between and outside swapped; **(B)** Between and outside swapped; **(C)** Between and outside swapped.

Absolute-value equations with a coefficient — Q1
Correct: **(A)**. *Traps:* **(B)** Dropped the negative case; **(C)** Both signs flipped; **(D)** Coefficient ignored.

Absolute-value equations with a coefficient — Q2
Correct: **(D)**. *Traps:* **(A)** Dropped the negative case; **(B)** Both signs flipped; **(C)** Coefficient ignored.

Absolute-value equations with a coefficient — Q3
Correct: **(A)**. *Traps:* **(B)** Coefficient ignored; **(C)** Both signs flipped; **(D)** Dropped the negative case.

Absolute-value equations with a coefficient — Q4
Correct: **(A)**. *Traps:* **(B)** Both signs flipped; **(C)** Dropped the negative case; **(D)** Coefficient ignored.

Absolute-value equations with a coefficient — Q5
Correct: **(B)**. *Traps:* **(A)** Dropped the negative case; **(C)** Coefficient ignored; **(D)** Both signs flipped.

Absolute-value inequalities: less-than (between) — Q1
Correct: **(C)**. *Traps:* **(A)** Kept only one bound; **(B)** Used the outside form; **(D)** Forgot the shift.

Absolute-value inequalities: less-than (between) — Q2
Correct: **(D)**. *Traps:* **(A)** Used the outside form; **(B)** Forgot the shift; **(C)** Kept only one bound.

Absolute-value inequalities: less-than (between) — Q3
Correct: **(D)**. *Traps:* **(A)** Kept only one bound; **(B)** Forgot the shift; **(C)** Used the outside form.

Absolute-value inequalities: less-than (between) — Q4
Correct: **(B)**. *Traps:* **(A)** Forgot the shift; **(C)** Kept only one bound; **(D)** Used the outside form.

Absolute-value inequalities: less-than (between) — Q5
Correct: **(A)**. *Traps:* **(B)** Kept only one bound; **(C)** Forgot the shift; **(D)** Used the outside form.

Absolute-value inequalities: greater-than (outside) — Q1
Correct: **(A)**. *Traps:* **(B)** Forgot the shift; **(C)** Used the between form; **(D)** Kept only one piece.

Absolute-value inequalities: greater-than (outside) — Q2
Correct: **(D)**. *Traps:* **(A)** Kept only one piece; **(B)** Used the between form; **(C)** Forgot the shift.

Absolute-value inequalities: greater-than (outside) — Q3
Correct: **(C)**. *Traps:* **(A)** Kept only one piece; **(B)** Used the between form; **(D)** Forgot the shift.

Absolute-value inequalities: greater-than (outside) — Q4
Correct: **(B)**. *Traps:* **(A)** Kept only one piece; **(C)** Used the between form; **(D)** Forgot the shift.

Absolute-value inequalities: greater-than (outside) — Q5
Correct: **(C)**. *Traps:* **(A)** Used the between form; **(B)** Kept only one piece; **(D)** Forgot the shift.

Graphs of absolute-value functions (vertex form) — Q1
Correct: **(C)**. *Traps:* **(A)** Wrong sign on x; **(B)** Both signs wrong; **(D)** Wrong sign on y.

Graphs of absolute-value functions (vertex form) — Q2
Correct: **(A)**. *Traps:* **(B)** Both signs wrong; **(C)** Wrong sign on x; **(D)** Wrong sign on y.

Graphs of absolute-value functions (vertex form) — Q3
Correct: **(B)**. *Traps:* **(A)** Both signs wrong; **(C)** Wrong sign on x; **(D)** Wrong sign on y.

Graphs of absolute-value functions (vertex form) — Q4
Correct: **(C)**. *Traps:* **(A)** Wrong sign on x; **(B)** Wrong sign on y; **(D)** Both signs wrong.

Graphs of absolute-value functions (vertex form) — Q5
Correct: **(B)**. *Traps:* **(A)** Both signs wrong; **(C)** Wrong sign on y; **(D)** Wrong sign on x.

DOMAIN 2

Advanced Math

Functions, and the structure beneath them

"The book of nature is written in the language of mathematics."
— Galileo Galilei

About 35% of the section. Quadratics, polynomials, exponentials, radicals, rational expressions, nonlinear systems, function transforms, and complex numbers.

Evaluate & interpret function notation f(x)

node: sat.quadratics.Q1

The skill. The notation f(x) is the name of a function — a rule that turns an input into an output. To find f(a), put the number a in place of every x in the rule and simplify; the result is the output. Writing f(a) = b is shorthand for saying that when the input is a the output is b, so the point (a, b) sits on the graph. Two cautions: f(a) does not mean f times a, and f(x + 3) means you substitute the whole expression x + 3 for x, not add 3 to the output.

Worked example

If f(x) = 3x + 2, find f(-2).

Read. The rule is f(x) = 3x + 2. We want f(-2), the output when the input is -2, so we substitute -2 for x.

Plan.

1. Write the rule with -2 in parentheses where x was.
2. Multiply, then add, to simplify.

Solve.

$$\begin{aligned} f(-2) &= 3(-2) + 2 && \text{substitute -2 for x} \\ &= -6 + 2 && \text{multiply} \\ &= -4 && \text{add} \end{aligned}$$

Check. Substituting one number gives one number, and -4 is an output, not the input -2. So f(-2) = -4.

Answer: f(-2) = -4.

Approach Read · Plan · Solve · Check

Read. Decide what you are given and what is asked: an output from an input (find f of a number), or an input from an output (given f(a) = b, read off a or b).

Plan.

1. Write the rule with the input in parentheses in place of x.
2. Replace every x with that input.
3. Simplify to get the single output.

Solve.

1. Do the arithmetic carefully, keeping signs and parentheses.
2. Simplify to one value.

Check. Check that you substituted for x everywhere and that your answer is an output, not the input you started with. Remember f(a) is not f times a.

Common traps — every wrong answer is one of these

Reading f(x) as multiplication.
Treats f(a) as f times a, or spreads the input across the whole rule — reading f(4) as 4 times (3x + 2). The letter f is a function name, not a number, so f(4) means substitute 4 for x.

Swapping input and output.
Mixes up which number is the input and which is the output: returns the input, reads the wrong row of a table, or solves for x when the output was asked. In f(a) = b the input is a and the output is b.

Confusing f(x + h) with f(x) + h.
Computes f(x + 3) as f(x) + 3, adding to the output instead of substituting. To find f(x + 3) you replace x with the whole expression x + 3: for f(x) = 2x + 1 that gives 2(x + 3) + 1 = 2x + 7, not 2x + 4.

Practice

1. *f(x) = 3x + 2.*

What does f(4) represent?

(A) f multiplied by 4

(B) The value of the function when x = 4

(C) 4 times the whole expression, 4(3x + 2)

(D) The value of x for which f(x) = 4

2. *f(x) = 2x - 5.*

Find f(-3).

(A) 1

(B) -16

(C) -11

(D) -1

3. *A table of values: f(1) = 5, f(2) = 8, f(3) = 11.*

Find f(2).

(A) 8

(B) 11

(C) 2

(D) 5

4. *f(x) = 2x + 1.*

Find f(x + 3).

(A) 2x + 7

(B) 2x + 4

(C) 6x + 1

(D) 2x + 6

5. *C(n) gives the cost in dollars of buying n items. C(10) = 45.*

What does this mean?

(A) 10 items cost $45

(B) 45 items cost $10

(C) C times 10 equals 45

(D) Each item costs $45

Expand binomials / FOIL

node: sat.quadratics.Q2

The skill. To multiply two binomials you multiply every term in the first by every term in the second, then combine like terms. For $(x + a)(x + b)$ the result is $x^2 + (a + b)x + ab$: the middle coefficient is the SUM $a + b$ and the constant is the PRODUCT ab. Two patterns are worth memorizing: a square $(x + a)^2 = x^2 + 2ax + a^2$, where the middle term is doubled, and a difference of squares $(x + a)(x - a) = x^2 - a^2$, where the middle terms cancel.

Worked example

Expand (x - 4)(x + 3).

Read. We multiply two binomials, x - 4 and x + 3. Every term in the first must multiply every term in the second.

Plan.

1. Multiply the first terms, x and x.
2. Multiply the outer terms, x and 3, and the inner terms, -4 and x.
3. Multiply the last terms, -4 and 3.
4. Add the outer and inner terms to form the middle term.

Solve.

$$\begin{aligned} (x-4)(x+3) &= x^2 + 3x - 4x - 12 && \text{multiply each term by each term} \\ &= x^2 - x - 12 && \text{combine 3x and -4x} \end{aligned}$$

Check. There are three terms; the middle -x is the sum 3 + (-4) and the constant -12 is the product (-4)(3), matching the pattern.

Answer: $(x - 4)(x + 3) = x^2 - x - 12$.

Approach Read · Plan · Solve · Check

Read. Spot the two binomials and note the sign in front of each term.

Plan.

1. Multiply the two first terms.
2. Multiply the two outer terms, then the two inner terms.
3. Multiply the two last terms.
4. Add the outer and inner results to form the middle term, then write the three terms.

Solve.

1. Carry each sign through the multiplication.
2. Combine the like middle terms.

Check. A general product has three terms, unless the middle cancels in a difference of squares. Check that the middle term is the sum and the constant is the product, and that a square

doubled its middle term.

Common traps — every wrong answer is one of these

Forgetting the middle term.
Multiplies only the first terms and the last terms, writing $(x + a)(x + b)$ as $x^2 + ab$ or $(x + a)^2$ as $x^2 + a^2$. This skips the outer and inner products, which together make the middle term.

Sign slip in a product.
Gets a term's sign wrong, usually when a binomial has a minus sign. Each product carries the signs of both factors: in $(x - 4)(x + 2)$ the outer term is $+2x$ and the inner is $-4x$, giving a middle term of $-2x$.

Not doubling the middle of a square.
Expands $(x + a)^2$ as $x^2 + ax + a^2$, using a single ax. A square is $(x + a)(x + a)$, so the outer and inner terms are both ax and add to $2ax$ — the middle term is doubled.

Swapping the sum and the product.
Uses the product where the sum belongs, or the sum where the product belongs, so the middle coefficient and the constant get switched. For $(x + a)(x + b)$ the middle is $a + b$ and the constant is ab.

Practice

1. *Expand $(x + 3)(x + 2)$.*

Which is the result?

(A) $x^2 + 5x + 6$
(B) $x^2 + 6x + 6$
(C) $x^2 + 5x + 5$
(D) $x^2 + 6$

2. *Expand $(x - 4)(x + 2)$.*

Which is the result?

(A) $x^2 - 8$
(B) $x^2 - 2x - 8$
(C) $x^2 - 2x + 8$
(D) $x^2 + 2x - 8$

3. *Expand $(x + 5)^2$.*

Which is the result?

(A) $x^2 + 25$

(B) $x^2 + 5x + 25$

(C) $x^2 + 10x + 10$

(D) $x^2 + 10x + 25$

4. *Expand $(2x + 3)(x - 1)$.*

Which is the result?

(A) $2x^2 + x - 3$

(B) $2x^2 - x - 3$

(C) $2x^2 + x + 3$

(D) $2x^2 - 2x$

5. *Expand $(x + 6)(x - 6)$.*

Which is the result?

(A) $x^2 - 6$

(B) $x^2 + 36$

(C) $x^2 - 36$

(D) $x^2 - 12x - 36$

Factor quadratics

node: sat.quadratics.Q3

The skill. Factoring reverses expanding: you rewrite a quadratic as a product of two binomials. For $x^2 + bx + c$, look for two numbers whose PRODUCT is c and whose SUM is b; then the factors are $(x + p)(x + q)$. Always pull out any common factor first. Watch for two special shapes: a difference of squares $x^2 - a^2 = (x + a)(x - a)$, and a perfect-square trinomial $x^2 + 2ax + a^2 = (x + a)^2$. When the leading coefficient is not 1, factor by grouping. You can always check by multiplying the factors back out.

Worked example

Factor $x^2 + 2x - 15$.

Read. We want two binomials whose product is $x^2 + 2x - 15$, so we need two numbers that multiply to -15 and add to 2.

Plan.

1. List number pairs that multiply to -15.
2. Choose the pair that adds to 2.
3. Write the two factors with those numbers.

Solve.

$$x^2 + 2x - 15 = (x + 5)(x - 3) \qquad 5 \text{ and } -3 \text{ multiply to } -15 \text{ and add to } 2$$

Check. Expand to verify: $(x + 5)(x - 3) = x^2 - 3x + 5x - 15 = x^2 + 2x - 15$, the original.
Answer: $x^2 + 2x - 15 = (x + 5)(x - 3)$.

Approach Read · Plan · Solve · Check

Read. Factor out any common factor first, then note the signs of b and c to anticipate the signs in the factors.

Plan.

1. List pairs of numbers that multiply to c.
2. Pick the pair that also adds to b.
3. Write the factors $(x + p)(x + q)$ with the signs that give those numbers.
4. For a leading coefficient other than 1, use factoring by grouping.

Solve.

1. Place the numbers and their signs into the two binomials.
2. Check for a difference of squares or a perfect square.

Check. Multiply the factors back out — do you recover the original quadratic? Make sure it is factored completely, with no common factor left inside.

Common traps — every wrong answer is one of these

Right product, wrong sum.
Chooses a number pair that multiplies to c but does not add to b (or the reverse), checking only one condition. Both must hold: for $x^2 + 7x + 12$, the pair 2 and 6 multiplies to 12 but adds to 8, so it fails; 3 and 4 works.

Wrong signs in the factors.
Gets the sizes of the numbers right but the signs wrong, such as writing $(x - 2)(x - 3)$ when the middle term should be positive. The signs must reproduce both the sum b and the product c.

Mishandling the leading coefficient or a common factor.
Factors $ax^2 + bx + c$ as if a were 1, misplaces the factors, or drops a common factor that should have been pulled out first. When a is not 1 it must be accounted for by grouping.

Mis-factoring a special form.
Factors a difference of squares as a perfect square, writing $x^2 - 16$ as $(x - 4)^2$, or stops before the expression is factored completely. A difference of squares $x^2 - a^2$ is $(x + a)(x - a)$.

Practice

1. *Factor $x^2 + 7x + 12$.*

Which is correct?

(A) (x - 3)(x - 4)
(B) (x + 2)(x + 6)
(C) (x + 1)(x + 12)
(D) (x + 3)(x + 4)

2. *Factor $x^2 - x - 6$.*

Which is correct?

(A) (x + 1)(x - 6)
(B) (x + 2)(x - 3)
(C) (x + 6)(x - 1)
(D) (x - 2)(x + 3)

3. *Factor $2x^2 + 7x + 3$.*

Which is correct?

(A) (2x + 3)(x + 1)
(B) (x + 1)(x + 3)
(C) (2x - 1)(x - 3)
(D) (2x + 1)(x + 3)

4. *Factor $x^2 - 16$.*

Which is correct?

(A) $(x + 4)^2$
(B) (x - 16)(x + 1)
(C) $(x - 4)^2$
(D) (x + 4)(x - 4)

5. *Factor completely: $x^3 - 4x$.*

Which is correct?

(A) (x + 2)(x - 2) [dropped the common factor x]
(B) $x(x^2 - 4)$ [left unfactored]
(C) x(x + 2)(x - 2)
(D) $x(x - 2)^2$

Solve quadratics by factoring & the zero-product property

node: sat.quadratics.Q4

The skill. To solve a quadratic by factoring, first move every term to one side so the equation reads (quadratic) = 0. Factor the quadratic, then use the zero-product property: if a product of factors equals zero, at least one factor must be zero. Set each factor equal to zero and solve those simple linear equations; each gives a solution. This works only when one side is zero. A factor that repeats gives a single (double) root, and dividing away a variable can lose the solution x = 0.

Worked example

Solve $x^2 + 2x - 8 = 0$.

Read. The equation already equals zero. We factor the left side, then set each factor to zero.

Plan.

1. Factor $x^2 + 2x - 8$ into two binomials.
2. Set each factor equal to zero.
3. Solve each small equation.

Solve.

$x^2 + 2x - 8 = 0$	start from the equation
$(x + 4)(x - 2) = 0$	factor: 4 and -2 multiply to -8, add to 2
$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$	zero-product property
$x = -4 \quad \text{or} \quad x = 2$	solve each linear equation

Check. Substitute back: $(-4)^2 + 2(-4) - 8 = 16 - 8 - 8 = 0$, and $2^2 + 2(2) - 8 = 4 + 4 - 8 = 0$. Both work.

Answer: x = -4 or x = 2.

Approach Read · Plan · Solve · Check

Read. Check that the equation equals zero. If it does not, move all terms to one side first.

Plan.

1. Factor the quadratic completely.
2. Set each factor equal to zero.
3. Solve each of those linear equations for x.
4. Collect every distinct solution.

Solve.

1. Solve each factor equation.
2. Keep repeated and zero roots.

Check. Make sure you set the equation to zero before factoring, kept all roots including x = 0, and could substitute each solution back to get a true statement.

Common traps — every wrong answer is one of these

Using the zero-product property without a zero.

1. Sees (x - 2)(x - 3) = 2 and sets each factor equal to the right side: x - 2 = 2 and x - 3 = 2.
2. Gets x = 4 and x = 5, which do not solve the equation.
3. The zero-product property needs a product equal to zero, not 2.
4. Fix: multiply out, move the 2 over to get (quadratic) = 0, then factor and solve.

Losing a solution.
Reports only one solution when there are two, or mishandles a repeated factor. A quadratic usually has two roots; a perfect square like $(2x - 3)^2 = 0$ has one value that counts as a double root.

Dividing both sides by x.

1. Sees $x^2 = 4x$ and divides both sides by x to get x = 4.
2. This throws away the solution x = 0.
3. Dividing by x is safe only when x is known to be nonzero.
4. Fix: move everything to one side, $x^2 - 4x = 0$, factor to x(x - 4) = 0, and keep both x = 0 and x = 4.

Practice

1. *Solve $x^2 - 5x + 6 = 0$.*

What are the solutions?

(A) x = -2 or x = -3
(B) x = 1 or x = 6
(C) x = 2 or x = 3
(D) x = 2 only

2. *Solve $(x - 2)(x - 3) = 2$.*

What are the solutions?

(A) x = 2 or x = 3
(B) x = 4 or x = 5
(C) x = -1 or x = -4
(D) x = 1 or x = 4

3. *Solve $x^2 = 4x$.*

What are the solutions?

(A) x = 4 only

(B) x = 0 only

(C) x = 0 or x = -4

(D) x = 0 or x = 4

4. *Solve $4x^2$ - 12x + 9 = 0.*

What are the solutions?

(A) x = 3/2 or x = -3/2

(B) No real solution

(C) x = 3/2 (a double root)

(D) x = 2/3

5. *The graph $y = x^2$ - 7x + 10 crosses the x-axis where x^2 - 7x + 10 = 0.*

What are the x-intercepts?

(A) x = 2 and x = 5

(B) x = 7 and x = 10

(C) x = -2 and x = -5

(D) x = 2 only

Quadratic formula & discriminant (number of real solutions)

node: sat.quadratics.Q5

The skill. Every quadratic equation $ax^2 + bx + c = 0$ can be solved with the quadratic formula, x = (-b +/- $\sqrt{b^2 - 4ac}$) / (2a). The part under the square root, b^2 - 4ac, is called the discriminant, and its sign tells you how many real solutions there are: positive means two, zero means one (a double root), and negative means none. When you use the formula, remember that the entire numerator is divided by 2a and that the plus-or-minus gives two branches to keep.

Worked example

Solve x^2 - 2x - 3 = 0 using the quadratic formula.

Read. Here a = 1, b = -2, and c = -3. We substitute into x = (-b +/- $\sqrt{b^2 - 4ac}$) / (2a).

Plan.

1. Write down a, b, and c with their signs.
2. Compute the discriminant b^2 - 4ac.

3. Substitute into the formula and simplify both branches.

Solve.

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{write the formula} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} && \text{substitute a, b, c} \\
&= \frac{2 \pm \sqrt{4 + 12}}{2} && \text{square and multiply} \\
&= \frac{2 \pm \sqrt{16}}{2} && \text{add under the root} \\
&= \frac{2 \pm 4}{2} && \text{take the square root} \\
&= 3 \text{ or } -1 && \text{compute both branches}
\end{aligned}
$$

Check. The discriminant 16 is positive, so two real solutions are expected. Check x = 3: 9 - 6 - 3 = 0. Check x = -1: 1 + 2 - 3 = 0. Both work.

Answer: x = 3 or x = -1.

Approach Read · Plan · Solve · Check

Read. Make sure the equation equals zero, then read off a, b, and c with their signs.

Plan.

1. Compute the discriminant b^2 - 4ac.
2. Use its sign to count the real solutions.
3. If the actual roots are needed, substitute a, b, c into the full formula.

Solve.

1. Write -b, which is positive when b is negative.
2. Divide the whole numerator by 2a.
3. Keep both the plus and the minus branch.

Check. Does the number of solutions match the sign of the discriminant? Did you divide everything by 2a and keep both roots?

Common traps — every wrong answer is one of these

Misreading the discriminant count.
Maps the sign of b^2 - 4ac to the wrong number of solutions — claiming real solutions when it is negative, saying two when it is zero, or treating the square root of a negative as real. Positive gives two, zero gives one, negative gives none.

Sign slip substituting into the formula.
Mis-signs a value when plugging in, most often using -b as negative when b is already negative,

or mishandling 4ac when c is negative. For b = -5, -b is +5.

Breaking the structure of the formula.
Drops the plus-or-minus and keeps only one branch, or divides only part of the numerator by 2a instead of the whole thing. Both -b and the square-root term sit over 2a.

Practice

1. *Solve x^2 + 3x - 4 = 0 with the quadratic formula.*

What are the solutions?

(A) x = 2 or x = -8

(B) x = 1 or x = -4

(C) x = -1 or x = 4

(D) x = 1 only

2. *The equation x^2 + x - 6 = 0 (discriminant 25).*

How many real solutions does it have?

(A) One real solution

(B) Two real solutions

(C) Infinitely many solutions

(D) No real solutions

3. *The equation x^2 + x + 5 = 0 (discriminant -19).*

How many real solutions does it have?

(A) Two real solutions

(B) Two solutions, x = (-1 +/- $\sqrt{-19}$)/2, treated as real

(C) No real solutions

(D) One real solution

4. *Solve x^2 - 5x + 6 = 0 with the quadratic formula (a=1, b=-5, c=6).*

What are the solutions?

(A) x = 6 or x = 4

(B) x = -3 or x = -2

(C) x = 3 or x = 2

(D) x = 6 or x = -1

5. *$x^2 + kx + 9 = 0$ is to have exactly one real solution.*

For what value(s) of k?

(A) No such k exists

(B) k = 6 or k = -6

(C) k = 6 only

(D) k = 3 or k = -3

Forms of a quadratic & the parabola

node: sat.quadratics.Q6

The skill. A quadratic can be written three ways, and each shows off a different feature. Standard form $y = ax^2 + bx + c$ shows the y-intercept c and has its axis of symmetry at $x = -b/(2a)$. Vertex form $y = a(x - h)^2 + k$ shows the vertex (h, k) and the axis $x = h$. Factored form $y = a(x - r1)(x - r2)$ shows the x-intercepts r1 and r2. The sign of a sets the direction: if a is positive the parabola opens up and the vertex is the lowest point (a minimum); if a is negative it opens down and the vertex is the highest point (a maximum). The number of x-intercepts matches what the discriminant predicts.

Worked example

Find the vertex of $y = x^2 - 6x + 5$.

Read. This is standard form with a = 1, b = -6, c = 5. The vertex sits on the axis of symmetry $x = -b/(2a)$; we find that x, then its y-value.

Plan.

1. Find the axis of symmetry with $x = -b/(2a)$.
2. Substitute that x back into the equation to get y.
3. Write the vertex as an ordered pair.

Solve.

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} \qquad \text{axis-of-symmetry formula}$$

$$= \frac{6}{2} = 3 \qquad \text{simplify to the vertex x}$$

$$y = (3)^2 - 6(3) + 5 \qquad \text{substitute } x = 3$$

$$= 9 - 18 + 5 = -4 \qquad \text{simplify to the vertex y}$$

Check. Since $a = 1 > 0$ the parabola opens up, so (3, -4) is the lowest point — a minimum, as expected.

Answer: the vertex is (3, -4).

Approach Read · Plan · Solve · Check

Read. Notice which form the quadratic is written in and which feature the question wants — vertex, axis, intercepts, or direction.

Plan.

1. From vertex form, read the vertex (h, k) directly, watching the sign of h.
2. From standard form, find the axis with x = -b/(2a), then substitute to get the vertex y-value.
3. From factored form, the x-intercepts are the roots.
4. Use the sign of a for the opening direction and whether the vertex is a minimum or a maximum.

Solve.

1. Pull out the requested feature, tracking signs.
2. Convert between forms if the needed feature is hidden.

Check. In vertex form the vertex x has the opposite sign of the number inside (x - h). Check that the direction matches the sign of a and that the number of x-intercepts agrees with the discriminant.

Common traps — every wrong answer is one of these

Flipping the sign of h in vertex form.
Reads the vertex of $y = a(x - h)^2 + k$ as (-h, k). The vertex x is the value that makes the inside zero, so (x - 3) gives h = 3 and (x + 3) gives h = -3.

Wrong axis from standard form.
Computes the axis of symmetry with the wrong sign or forgets to divide by 2a. The axis is x = -b/(2a); for $y = x^2 - 6x + 5$ that is x = 6/2 = 3.

Reversing open-up and open-down.
Swaps the meaning of the sign of a — calling an upward parabola a maximum or a downward one a minimum. Positive a opens up to a minimum; negative a opens down to a maximum.

Reading the wrong feature off a form.
Pulls the wrong number from a form: treats a constant as a root, a coefficient as a coordinate, or the y-intercept as an x-intercept. Each form exposes specific features — match the feature to the form.

Practice

1. *$y = 2(x - 3)^2 + 4$.*

What is the vertex?

(A) (-3, 4)

(B) (2, 4)

(C) (4, 3)

(D) (3, 4)

2. *$y = x^2 - 6x + 5$.*

What is the axis of symmetry?

(A) x = 5

(B) x = 3

(C) x = 6

(D) x = -3

3. *$y = -2(x - 1)^2 + 8$.*

What is the maximum or minimum value?

(A) Minimum value of -8

(B) Maximum value of 8

(C) Minimum value of 8

(D) Maximum value of 1

4. *$y = (x + 2)(x - 5)$.*

What are the x-intercepts?

(A) x = -2 and x = 5

(B) x = -2 only

(C) x = 2 and x = -5

(D) (0, -10), the y-intercept

5. *The parabola $y = x^2 - 4x + 7$ (discriminant -12).*

How many times does it cross the x-axis?

(A) At x = 7

(B) It does not cross the x-axis (0 times)

(C) Once

(D) Twice

Interpret a quadratic model in context

node: sat.quadratics.Q7

The skill. When a quadratic models a real situation, each feature has a meaning. The vertex is the maximum or minimum: its y-value is the extreme amount (how high, how much profit) and its x-value is where or when that extreme happens. The zeros are moments when the output is zero, such as when a thrown object returns to the ground — and any root that makes no physical sense, like a negative time, is thrown out. The y-intercept is the starting value. Always answer the exact quantity the question asks for, and include units.

Worked example

A ball's height in feet after t seconds is $h = -16t^2 + 96t$. What is its maximum height?

Read. The height is a downward parabola ($a = -16 < 0$), so its vertex is the highest point. The maximum height is the vertex's h-value.

Plan.

1. Find the time of the vertex with $t = -b/(2a)$.
2. Substitute that time into h to get the maximum height.
3. State the answer with units.

Solve.

$$t = \frac{-b}{2a} = \frac{-96}{2(-16)} \qquad \text{time of the vertex}$$

$$= \frac{-96}{-32} = 3 \qquad \text{simplify to } t = 3 \text{ seconds}$$

$$h = -16(3)^2 + 96(3) \qquad \text{substitute } t = 3$$

$$= -144 + 288 = 144 \qquad \text{simplify to the height}$$

Check. The question asks how high, so the answer is the height 144 feet, reached at $t = 3$ seconds — not the time. The units are feet, as expected.

Answer: the maximum height is 144 feet.

Approach Read · Plan · Solve · Check

Read. Figure out what each variable and each feature means in the real situation, and what units go with it.

Plan.

1. For the largest or smallest amount, read the vertex y-value.
2. For when that extreme happens, read the vertex x-value.
3. For an event like landing, use the relevant zero.
4. For the starting amount, read the y-intercept.

Solve.

1. Read the feature and attach the correct units.
2. Discard any root that cannot happen in context.

Check. Did you answer the quantity asked — a value versus a time? Are the units right? Did you keep the physically meaningful root?

Common traps — every wrong answer is one of these

Confusing how much with when.
Mixes up the vertex coordinates: reports when the extreme happens (the x-value) when asked how large it is (the y-value), or takes the starting value for the maximum. The y-value is the amount; the x-value is the time or place.

Misreading which zero is the event.
Interprets the roots wrongly: calls the launch at t = 0 the landing, uses the vertex time as a zero, or keeps a root that cannot occur. For a thrown object, landing is the positive zero, not t = 0.

Wrong quantity or wrong units.
Does the math correctly but reports the wrong thing or the wrong units — seconds where feet are asked, or units instead of dollars. Match the answer to the quantity and its units.

Practice

1. *A ball's height (feet) after t seconds is $h = -16t^2 + 64t$, with vertex (2, 64).*

What is the maximum height?

(A) 2 feet
(B) 64 seconds
(C) 64 feet
(D) 2 seconds

2. *Same ball: $h = -16t^2 + 64t$, vertex (2, 64), with a zero at $t = 4$.*

At what time does the ball reach its maximum height?

(A) 2 seconds
(B) 2 feet
(C) 64 seconds
(D) 4 seconds

3. *Same ball: $h = -16t^2 + 64t$, with zeros at $t = 0$ and $t = 4$.*

When does the ball hit the ground?

(A) 4 seconds
(B) At 0 and 4 seconds

(C) 2 seconds

(D) 0 seconds

4. *A company's profit (dollars) is $P = -2x^2 + 40x - 50$, with vertex (10, 150).*

What is the maximum profit?

(A) \$50

(B) \$10

(C) \$150

(D) 150 units

5. *$h = -5t^2 + 20t + 25$ models the height (meters) of an object after t seconds. The vertex is (2, 45), a zero is at $t = 5$, and the initial height is 25 m.*

Which statement is correct?

(A) Maximum height 45 m at $t = 2$ s; it lands at $t = 5$ s

(B) Maximum height 2 m at $t = 45$ s

(C) The maximum height is 25 m

(D) It lands at $t = 2$ s

Completing the square

node: sat.quadratics.Q8

The skill. Completing the square rewrites a quadratic so it contains a perfect square. For $x^2 + bx$, take HALF of b, square it, and add $(b/2)^2$, because $x^2 + bx + (b/2)^2$ equals $(x + b/2)^2$. This turns standard form into vertex form $a(x - h)^2 + k$, which shows the vertex (h, k) at a glance. If the leading coefficient a is not 1, factor a out of the x^2 and x terms first. Whatever you add to build the square must be balanced — subtracted back in an expression, or added to both sides of an equation. Solving this way finishes with a plus-or-minus square root, so you get two solutions.

Worked example

Rewrite $x^2 + 8x + 10$ in vertex form.

Read. We complete the square on $x^2 + 8x$. Half of 8 is 4, and 4 squared is 16, so 16 is the constant that builds the square.

Plan.

1. Take half of the x-coefficient, then square it, to get 16.
2. Add and subtract 16 so the value does not change.
3. Group the first three terms into a perfect square.
4. Combine the leftover constants.

Solve.

$$\begin{aligned} x^2 + 8x + 10 &= x^2 + 8x + 16 - 16 + 10 && \text{add and subtract } (8/2)^2 = 16 \\ &= (x + 4)^2 - 16 + 10 && \text{group the perfect-square trinomial} \\ &= (x + 4)^2 - 6 && \text{combine -16 and 10} \end{aligned}$$

Check. Expanding $(x + 4)^2 - 6$ gives $x^2 + 8x + 16 - 6 = x^2 + 8x + 10$, the original. The vertex is (-4, -6).

Answer: $x^2 + 8x + 10 = (x + 4)^2 - 6$.

Approach Read · Plan · Solve · Check

Read. Decide the goal: rewriting into vertex form, finding a vertex, or solving an equation — and check whether the leading coefficient is 1.

Plan.

1. If a is not 1, factor a out of the x^2 and x terms.
2. Take half of b and square it to get $(b/2)^2$.
3. Add $(b/2)^2$ to form the perfect square $(x + b/2)^2$, and balance it.
4. Read the vertex, or take the square root of both sides with plus-or-minus to solve.

Solve.

1. Build the perfect-square trinomial and fold it into $(x + b/2)^2$.
2. Finish the rewrite or the solve.

Check. Did you halve b before squaring, keep the expression or equation balanced, and read the vertex x with the opposite sign of the number inside the parentheses?

Common traps — every wrong answer is one of these

Not halving before squaring.
Adds the wrong constant to complete the square — using b^2 (squared without halving), b/2 (halved without squaring), or b itself, instead of $(b/2)^2$. For $x^2 + 6x$ the correct constant is $(6/2)^2 = 9$.

Not balancing the added constant.
Adds $(b/2)^2$ to make the square but forgets to compensate — not subtracting it back in an expression, or not adding it to both sides of an equation. This throws off the constant k.

Wrong sign of h in the vertex.
Reads the vertex x straight off the parentheses, so $(x + 3)^2 + k$ is misread as vertex x = 3. Because vertex form is (x - h), x + 3 means h = -3.

Ignoring the leading coefficient.
When a is not 1, forgets to factor a out of the x^2 and x terms before completing the square, or mishandles it while doing so. For $2x^2 + 8x + 3$, factor first: $2(x^2 + 4x) + 3$.

Practice

1. *You are completing the square for $x^2 + 6x$.*

Which constant should be added to form a perfect square?

(A) 9

(B) 3

(C) 36

(D) 6

2. *Rewrite $x^2 + 6x + 5$ in vertex form.*

Which is correct?

(A) $(x + 3)^2 - 4$

(B) $(x + 3)^2 + 4$

(C) $(x - 3)^2 - 4$

(D) $(x + 3)^2 + 5$

3. *Consider the expression $(x - 2)^2 + 7$.*

What is the vertex of its graph?

(A) (-2, 7)

(B) (2, -7)

(C) (2, 7)

(D) (-2, -7)

4. *You want to complete the square for $2x^2 + 8x + 3$.*

What is the correct first step?

(A) Divide every term by 3

(B) Add $(8/2)^2 = 16$ directly to the expression

(C) Add 16 to get $2x^2 + 8x + 16 + 3$

(D) Factor 2 from the x-terms: $2(x^2 + 4x) + 3$

5. *Solve $x^2 + 4x - 1 = 0$ by completing the square.*

What are the solutions?

(A) $x = -4 \pm \sqrt{5}$

(B) $x = -2 \pm \sqrt{3}$

(C) $x = -2 \pm \sqrt{5}$

(D) $x = 2 \pm \sqrt{5}$

Composite functions & evaluating compositions

node: sat.quadratics.Q9

The skill. A composite function f(g(x)) means: first work out the INNER function g at the input, then put that result into the OUTER function f. You work from the inside out. Composition is substitution, not multiplication, so f(g(x)) is not f(x) times g(x). Order matters too: f(g(x)) is usually different from g(f(x)). When values come from a table or graph, look up the inner function first, then look up the outer function at that value.

Worked example

Let $f(x) = x^2$ and $g(x) = x - 2$. Find $f(g(5))$.

Read. In f(g(5)) the inner function is g and the outer is f. We evaluate g(5) first, then apply f to that result.

Plan.

1. Evaluate the inner function g at 5.
2. Take that output and apply the outer function f.

Solve.

$$\begin{aligned} g(5) &= 5 - 2 = 3 && \text{evaluate the inner function first} \\ f(g(5)) &= f(3) && \text{feed the result into f} \\ &= 3^2 = 9 && \text{square to finish} \end{aligned}$$

Check. Working inside-out gives 9. Doing it in the wrong order — squaring first, f(5) = 25, then subtracting 2 — would give 23, which is g(f(5)), a different question.

Answer: f(g(5)) = 9.

Approach Read · Plan · Solve · Check

Read. Identify which function is inner and which is outer, and whether you are working with a formula, single numbers, or a table or graph.

Plan.

1. Evaluate the inner function at the given input.
2. Take that output and use it as the input to the outer function.
3. For formulas, substitute the whole inner expression into the outer rule.

Solve.

1. Do the inner step first, then the outer step.
2. Never multiply the two functions together.

Check. Did you do the inner function first, substitute instead of multiply, and respect that f(g(x)) and g(f(x)) can differ? From a table, did you read the correct function at each step?

Common traps — every wrong answer is one of these

Working outside-in instead of inside-out.
Applies the outer function first, or works left to right, which computes g(f(x)) instead of f(g(x)). In f(g(x)) the inner g must be evaluated first.

Multiplying instead of substituting.
Treats f(g(x)) as the product f(x) times g(x). Composition substitutes g(x) into f in place of its input; for f(x) = x + 2 and g(x) = 3x, f(g(x)) = 3x + 2, not 3x(x + 2).

Reading the wrong value from a table.
Looks up the wrong function or the wrong entry when evaluating a composite — for example reading f at the original input instead of at the inner output. Evaluate g first, then read f at that result.

Assuming the order does not matter.
Believes f(g(x)) always equals g(f(x)). Composition is generally not commutative: for f(x) = x^2 and g(x) = x + 1, f(g(x)) = x^2 + 2x + 1 but g(f(x)) = x^2 + 1.

Practice

1. *Let $f(x) = x^2$ and $g(x) = x + 1$.*

What is f(g(3))?

(A) 9
(B) 4
(C) 16
(D) 10

2. *Let $f(x) = x + 2$ and $g(x) = 3x$.*

What is f(g(x))?

(A) (x + 2)(3x)
(B) $3x^2$ + 6x
(C) 3x + 2
(D) 3x + 6

3. *A table gives these values: $g(1) = 4$, $g(4) = 5$, $f(1) = 2$, $f(4) = 7$.*

What is f(g(1))?

(A) 5
(B) 2
(C) 7

(D) 4

4. *Let $f(x) = x^2$ and $g(x) = x + 1$.*

Is f(g(x)) equal to g(f(x))?

(A) Yes; both equal $x^2 + 1$

(B) No; $f(g(x)) = x^2 + 2x + 1$ while $g(f(x)) = x^2 + 1$

(C) Yes; both equal $x^2 + 2x + 1$

(D) Yes; composition is always commutative

5. *Let $f(x) = 2x - 1$ and $g(x) = x^2$.*

What is g(f(2))?

(A) 9

(B) 12

(C) 3

(D) 7

Domain & range of functions

node: sat.quadratics.Q10

The skill. The domain of a function is the set of allowed inputs (the x-values), and the range is the set of possible outputs (the y-values). A domain is all real numbers unless something forbids a value: a denominator cannot be zero, and the quantity under a square root (or any even root) cannot be negative. The range often comes from a key feature — for a parabola $y = a(x - h)^2 + k$ the vertex is the extreme output, so the range is $y \geq k$ when it opens up and $y \leq k$ when it opens down. From a graph, the domain is how far the curve spreads left-to-right and the range is how far it spreads up-and-down.

Worked example

Find the domain of $f(x) = \sqrt{x + 4}$.

Read. A square root is defined only when the quantity inside is zero or positive, so we need $x + 4$ to be at least 0.

Plan.

1. Set the radicand greater than or equal to zero.
2. Solve the inequality for x.

Solve.

$x + 4 \geq 0$	the radicand cannot be negative
$x \geq -4$	subtract 4 from both sides

Check. At x = -4 the inside is 0 and $\sqrt{0} = 0$ is fine; for x < -4 the inside is negative and undefined. So the domain is x >= -4.

Answer: the domain is x >= -4.

Approach Read · Plan · Solve · Check

Read. Decide whether the question asks about inputs (domain) or outputs (range), and what kind of function it is — rational, radical, parabola, or a graph.

Plan.

1. For a domain, start with all real numbers.
2. Exclude any value that makes a denominator zero.
3. Exclude any value that makes an even radicand negative.
4. For a range, read it from the vertex of a parabola or the vertical extent of a graph.

Solve.

1. Apply the relevant restriction or feature.
2. State the answer in the correct variable: x for domain, y for range.

Check. Did you give x-values for the domain and y-values for the range without swapping them? For a parabola, did you bound the range at the vertex with the correct direction?

Common traps — every wrong answer is one of these

Swapping domain and range.
Mixes up inputs and outputs — giving x-values for the range or y-values for the domain, or reading the wrong axis of a graph. Domain is the x-values; range is the y-values.

Looking for the restriction in the wrong place.
Finds the domain limit from the wrong source: using numerator zeros instead of denominator zeros, forgetting that a radicand must be nonnegative, or missing a restriction. For 1/(x - 5) the excluded value is where the denominator is zero, x = 5.

Ignoring the vertex bound on the range.
Gives a parabola's range as all real numbers, or uses the wrong direction (<= versus >=) or the wrong coordinate (h instead of k). The range is bounded at the vertex's k, with the direction set by the opening.

Misreading the extent from a graph.
Reads the domain or range off a graph incorrectly — wrong endpoints, confusing open and closed dots, or picking the wrong interval. The domain is the left-right span; the range is the up-down span.

Practice

1. *Consider f(x) = 1/(x - 5).*

What is the domain?

(A) All real numbers except x = 5

(B) All real numbers

(C) All real numbers except x = 0

(D) All real numbers except x = -5

2. *A function accepts every real number as an input, and its outputs are always at least 2.*

What is its range?

(A) x >= 2

(B) y <= 2

(C) All real numbers

(D) y >= 2

3. *Consider f(x) = (x - 3)^2 + 4.*

What is the range?

(A) y >= 3

(B) y >= 4

(C) All real numbers

(D) y <= 4

4. *Consider f(x) = $\sqrt{x - 2}$.*

What is the domain?

(A) x >= -2

(B) All real numbers

(C) x <= 2

(D) x >= 2

5. *A continuous curve is graphed; its points span x from -1 to 5 and y from 0 to 8.*

What are its domain and range?

(A) Domain -1 <= x <= 8; range 0 <= y <= 5

(B) Domain 0 <= x <= 8; range -1 <= y <= 5

(C) Domain -1 <= x <= 5; range 0 <= y <= 8

(D) Domain -1 <= y <= 5; range 0 <= x <= 8

Piecewise-defined functions

node: sat.quadratics.Q11

The skill. A piecewise function uses different rules over different intervals of the input. To evaluate it at a value, first decide which interval that value falls into, then use ONLY that interval's rule. At a boundary between pieces, the inequalities decide which piece owns the point: x >= 3 includes 3, but x < 3 does not. You never plug the input into every rule or add the pieces together — exactly one rule applies to each input.

Worked example

Let f(x) = x + 4 for x <= 1 and f(x) = 2x for x > 1. Find f(1).

Read. The input is 1, which sits exactly on the boundary. The condition x <= 1 includes 1, so the first rule owns it.

Plan.

1. Check which interval contains x = 1.
2. See that x <= 1 includes the endpoint 1.
3. Apply only the first rule, x + 4.

Solve.

$$\begin{aligned} f(1) &= 1 + 4 && \text{1 satisfies x <= 1, so use the first rule} \\ &= 5 && \text{add} \end{aligned}$$

Check. The other rule, 2x, applies only for x > 1, which excludes 1, so it is not used. Thus f(1) = 5.

Answer: f(1) = 5.

Approach Read · Plan · Solve · Check

Read. Find the input value and decide which interval contains it, paying attention if it sits exactly on a boundary.

Plan.

1. Locate the input among the intervals.
2. At a boundary, read the inequality to see which piece owns the endpoint.
3. Pick the single matching rule and evaluate it.

Solve. Apply only the selected rule and simplify.

Check. Did you use exactly one branch, let the inequality decide any boundary, and avoid evaluating all the pieces or combining them?

Common traps — every wrong answer is one of these

Using the wrong interval's rule.
Applies a rule from an interval that does not contain the input. Check which interval the input falls in first, then use only that rule.

Assigning a boundary to the wrong piece.
At a boundary value, gives the endpoint to the wrong rule by misreading <= versus <, or calls it undefined or shared by both pieces. For x >= 3, the value 3 belongs to that piece.

Evaluating every piece.
Plugs the input into all the rules and reports several answers, or adds the pieces together. Only one rule applies to a given input; pick that one.

Reading the inequality backwards.
Misreads an interval condition, such as x > 0 versus x < 0, and picks the branch by a flipped inequality. Read the direction of each inequality carefully before choosing.

Practice

1. *Let $f(x) = 2x + 1$ for $x < 3$, and $f(x) = x^2$ for $x >= 3$.*

What is f(1)?

(A) 3
(B) Both 3 and 1
(C) 1
(D) Undefined

2. *Let $f(x) = 2x + 1$ for $x < 3$, and $f(x) = x^2$ for $x >= 3$.*

What is f(4)?

(A) 16
(B) 8
(C) 9
(D) Both 9 and 16

3. *Let $f(x) = 2x + 1$ for $x < 3$, and $f(x) = x^2$ for $x >= 3$.*

What is f(3)?

(A) Undefined
(B) 9
(C) Both 7 and 9
(D) 7

4. *Let g(x) = 5 for x > 0, g(x) = -5 for x < 0, and g(x) = 0 for x = 0.*

What is g(-2)?

(A) 0

(B) Both 5 and -5

(C) 5

(D) -5

5. *Let f(x) = 2x + 1 for x < 3, and f(x) = x^2 for x >= 3.*

What is f(2) + f(5)?

(A) 29

(B) 45

(C) 30

(D) 16

Polynomial arithmetic & structure

node: sat.polynomial.P1

The skill. A polynomial is a sum of terms, each one a number times a power of x, like $3x^2$ or -5x. Its degree is the largest exponent that appears, and its leading coefficient is the number in front of that highest-power term. To add or subtract polynomials, combine only like terms — terms with the exact same power of x; a plain x^2 and an x can never be merged. When you subtract a whole polynomial, the minus sign flips the sign of every term inside the parentheses, not just the first one. To multiply, multiply each term of one polynomial by each term of the other; when two powers of x multiply, add their exponents. The degree of a product is the sum of the degrees of the two factors.

Worked example

Simplify $(2x^2 + 3x - 1) - (x^2 - 4x + 2)$.

Read. We are subtracting the second polynomial from the first. The minus sign in front of the second set of parentheses applies to all three of its terms, so we must be careful with signs.

Plan.

1. Flip the sign of every term in the second polynomial.
2. Group the like terms — the x^2 terms, the x terms, and the constants.
3. Combine each group by adding its coefficients.

Solve.

$$\begin{aligned}(2x^2+3x-1)-(x^2-4x+2) &= 2x^2+3x-1-x^2+4x-2 && \text{distribute the minus to every term}\\ &= (2x^2-x^2)+(3x+4x)+(-1-2) && \text{group like terms}\\ &= x^2+7x-3 && \text{combine each group}\end{aligned}$$

Check. The highest power is x^2, so the answer is a degree-2 polynomial, like the ones we started with. Adding $(x^2 - 4x + 2)$ back to $x^2 + 7x - 3$ recovers $2x^2 + 3x - 1$.

Answer: $x^2 + 7x - 3$.

Approach Read · Plan · Solve · Check

Read. Decide which operation you are doing — adding, subtracting, or multiplying — and note the power of x in each term so you can tell which terms are alike.

Plan.

1. To add or subtract, group the terms that share the same power of x.
2. If you are subtracting, first flip the sign of every term in the polynomial being subtracted.
3. To multiply, multiply each term of the first polynomial by each term of the second, adding exponents on matching variables.
4. Combine like terms to finish.

Solve.

1. Add or subtract the coefficients of like terms, leaving the power of x unchanged.
2. When multiplying, multiply the coefficients and add the exponents.
3. Collect everything into one simplified polynomial.

Check. Check that only like terms were combined, that a subtraction flipped every sign, and that exponents were added (not multiplied) during multiplication. The degree of a product should equal the sum of the degrees you started with.

Common traps — every wrong answer is one of these

combining unlike terms.
Combines terms with different powers of x, such as writing $3x^2 + 5x$ as $8x$, or adds the exponents when combining like terms, such as $3x^2 + 2x^2 = 5x^4$. Like terms must share the same power, and only their coefficients add: $3x^2 + 2x^2 = 5x^2$.

subtraction sign distribution.
1. Removes the parentheses in a subtraction like $(4x^2 + 3x - 2) - (x^2 - 2x + 5)$.
2. Flips the sign of only the first term, writing $-x^2 - 2x + 5$.
3. This leaves the wrong middle and last terms.
4. Fix: the minus sign flips every term inside the parentheses, giving $-x^2 + 2x - 5$.

multiplication/exponent error.
Multiplies the exponents instead of adding them when multiplying terms, such as $x^2 * x^3 = x^6$ instead of x^5; adds the coefficients instead of multiplying them; or forgets to multiply every pair of terms. When you multiply powers of the same base, add the exponents.

degree/leading-coefficient error.
Misidentifies the degree — for example counting the number of terms, or reading the exponent of the first written term instead of the highest exponent anywhere in the polynomial. The degree is the largest exponent that appears, and the leading coefficient is the number multiplying that term.

Practice

1. *Simplify $3x^2 + 5x + 2x^2 - x$.*

What is the result?

(A) $9x$
(B) $5x^3 + 4x$
(C) $5x^2 + 4x$
(D) $5x^2 + 6x$

2. *Simplify $(4x^2 + 3x - 2) - (x^2 - 2x + 5)$.*

What is the result?

(A) $3x^2 + x + 3$
(B) $5x^2 + x - 7$
(C) $3x^2 + 5x + 3$
(D) $3x^2 + 5x - 7$

3. *Multiply $2x^2 * 3x^3$.*

What is the result?

(A) $6x$
(B) $6x^6$
(C) $5x^5$
(D) $6x^5$

4. *Consider the polynomial $7x^3 - 2x^5 + 4x - 1$.*

What is its degree?

(A) 4

(B) 5

(C) 3

(D) 7

5. *Expand $(x + 2)(x^2 - 3x + 1)$.*

What is the result?

(A) $x^3 - x^2 - 5x + 2$

(B) $x^3 - x^2 + 7x + 2$

(C) $x^3 - 3x^2 + x + 2$

(D) $x^3 + x^2 - 5x + 2$

Zeros, factors & the factor theorem

node: sat.polynomial.P2

The skill. The factor theorem connects a polynomial's factors to its zeros. A zero is a value of x that makes the polynomial equal zero, and the zeros are exactly the x-intercepts of the graph. The rule is that (x - a) is a factor of P(x) exactly when P(a) = 0. To turn a factor into its zero, set the factor equal to zero and solve: (x - a) gives the zero x = a, which has the opposite sign of the number written inside. To go the other way, a zero x = a comes from the factor (x - a). A polynomial of degree n has at most n real zeros, and a repeated factor still counts as a single distinct zero.

Worked example

Find the zeros of P(x) = (x - 5)(x + 2).

Read. The polynomial is already written as a product of two factors. A zero is any x that makes the whole product equal zero, and a product is zero exactly when one of its factors is zero.

Plan.

1. Set each factor equal to zero separately.
2. Solve each small equation for x.

Solve.

$x - 5 = 0$	set the first factor to zero
$x = 5$	solve for x
$x + 2 = 0$	set the second factor to zero
$x = -2$	solve for x

Check. Each zero has the opposite sign of the number in its factor: (x - 5) gives +5 and (x + 2) gives -2. Substituting either value makes one factor zero, so P equals zero there.

Answer: the zeros are x = 5 and x = -2.

Approach Read · Plan · Solve · Check

Read. Decide which direction you are working: from factors to zeros, from zeros to factors, or using the fact that P(a) = 0 means (x - a) is a factor.

Plan.

1. To find a zero from a factor, set the factor equal to zero and solve for x.
2. To find a factor from a zero x = a, write it as (x - a).
3. When counting zeros, remember there can be at most as many as the degree.

Solve.

1. Solve each factor equal to zero, remembering the zero has the opposite sign of the constant in the factor.
2. Or assemble the factors from the given zeros.
3. Count only distinct zeros, treating a repeated factor as one.

Check. Check that each zero has the opposite sign of the number in its factor. Remember that P(a) = 0 tells you (x - a) is a factor, not that a itself is a factor, and that the number of zeros never exceeds the degree.

Common traps — every wrong answer is one of these

factor/zero sign error.
Reads the zero of (x - a) as x = -a, or writes the factor of a zero x = a as (x + a), flipping the sign between a factor and its zero. Setting x - a = 0 gives x = a, so the zero has the opposite sign of the number inside the factor.

factor-theorem meaning error.
Misreads what P(a) = 0 means — for instance thinking that a itself is a factor, or that the number a is a factor, rather than (x - a). P(a) = 0 says the input a makes the polynomial zero, so (x - a) is the factor.

number-of-zeros error.
Believes a degree-n polynomial can have more than n real zeros, or counts a repeated root several times as if it were several distinct zeros. The degree is the ceiling on the number of real zeros, and a repeated factor counts once.

Practice

1. *P(x) = (x - 4)(x + 2).*

What are the zeros?

(A) x = 4 and x = -2

(B) x = 4 and x = 2

(C) x = -4 and x = -2

(D) x = -4 and x = 2

2. *3 is a zero of the polynomial P(x).*

Which must be a factor of P(x)?

(A) 3

(B) (x + 3)

(C) (x - 3)

(D) 3x

3. *For a polynomial P(x), it is known that P(2) = 0.*

What does this tell you?

(A) (x - 2) is a factor of P(x)

(B) P(x) has no real zeros

(C) (x + 2) is a factor of P(x)

(D) 2 is a factor of P(x)

4. *Consider a polynomial of degree 4.*

What is the maximum number of real zeros it can have?

(A) 8

(B) Infinitely many

(C) 3

(D) 4

5. $P(x) = (x - 1)^2 (x + 3)$.

What are the distinct real zeros?

(A) x = -1 and x = 3

(B) x = 1, x = 1, and x = -3

(C) x = 1 and x = 3

(D) x = 1 and x = -3

Polynomial graphs: end behavior, zeros, multiplicity

node: sat.polynomial.P3

The skill. The two ends of a polynomial's graph are controlled by two things working together: whether the degree is odd or even, and whether the leading coefficient is positive or negative. An odd degree makes the two ends go in opposite directions (one up, one down); an even degree makes both ends go the same way. A positive leading coefficient pulls the right end up, a negative one pulls it down. The real zeros are the x-intercepts, found by setting each factor to zero. At a zero whose factor is repeated an even number of times, the graph just touches the x-axis and turns back; at a zero with an odd power, the graph crosses through. A degree-n polynomial has at most n real zeros — sometimes fewer, once repeated or complex roots are counted.

Worked example

Describe the far-left and far-right behavior of $f(x) = -2x^3 + x$.

Read. We only need the two ends of the graph, so the only parts that matter are the highest-power term and the sign in front of it. Here the highest-power term is $-2x^3$.

Plan.

1. Note whether the degree is odd or even.
2. Note whether the leading coefficient is positive or negative.
3. Combine the two: an odd degree gives opposite ends, and the sign tells which way the right end points.

Solve.

degree $= 3$, which is odd	read the highest power
leading coefficient $= -2$, which is negative	read the number in front of x^3
far right = falling	a negative leading coefficient makes the right end fall
far left = rising	an odd degree sends the ends in opposite directions

Check. A quick test confirms it: for a large positive x, $-2x^3$ is a large negative number (falling on the right); for a large negative x, $-2x^3$ is a large positive number (rising on the left).

Answer: the graph rises to the left and falls to the right.

Approach Read · Plan · Solve · Check

Read. Decide what the question is really about: the far-left and far-right ends (end behavior), the x-intercepts (zeros), or how the graph behaves at a repeated zero (multiplicity).

Plan.

1. For the ends, combine two facts: is the degree odd or even, and is the leading coefficient positive or negative.
2. For the zeros, set each factor equal to zero and read off the x-values.
3. For a repeated zero, check whether its power is even (touch) or odd (cross).

Solve.

1. State the end behavior from the degree's parity and the sign of the leading coefficient.

2. Solve each factor to list the x-intercepts.
3. Label each zero as a touch or a cross by its multiplicity.

Check. Make sure the end behavior used both the degree and the sign, not just one. Confirm the zeros are x-values, not y-values, and that the number of real zeros does not exceed the degree.

Common traps — every wrong answer is one of these

end-behavior error.
Gets the ends wrong by looking at only one clue — using just the sign of the leading coefficient or just the degree. For example, assuming a positive leading coefficient always sends both ends up. The ends depend on the degree's parity and the sign together.

zeros-as-x-intercepts error.
Reads the zeros straight off the factor constants with the wrong sign, or calls them y-values instead of x-values. The zeros are the x-values found by setting each factor to zero.

multiplicity error.
Assumes every zero is a simple crossing, missing that a factor raised to an even power makes the graph touch the x-axis and turn back instead of crossing.

degree-vs-zero-count error.
Assumes a degree-n polynomial must have exactly n distinct real zeros, or must cross the x-axis n times, ignoring repeated roots and complex roots. Degree n is only the maximum number of real zeros.

Practice

1. *Consider $f(x) = 2x^3 - 5x + 1$.*

How does the graph behave at its far left and far right?

(A) Rises on both ends

(B) Falls on both ends

(C) Rises to the left, falls to the right

(D) Falls to the left, rises to the right

2. *Let $f(x) = (x - 2)(x + 3)(x - 1)$.*

What are the real zeros?

(A) $y = 2$, $y = -3$, and $y = 1$

(B) $x = -2$, $x = 3$, and $x = -1$

(C) $x = 2$, $x = -3$, and $x = 1$

(D) x = 6

3. *Let $f(x) = (x - 1)^2 (x + 2)$.*

What does the graph do at x = 1?

(A) Touches the x-axis and turns back without crossing

(B) Never reaches the x-axis

(C) Has a vertical asymptote

(D) Crosses the x-axis

4. *Consider $f(x) = -x^4 + 3x^2$.*

What is the end behavior?

(A) Falls on both the left and the right

(B) Rises on both ends

(C) Rises to the left, falls to the right

(D) Falls to the left, rises to the right

5. *A polynomial has degree 4.*

Which statement must be true?

(A) Its graph must cross the x-axis 4 times

(B) It has at most 4 real zeros

(C) It has exactly 4 distinct real zeros

(D) All 4 of its zeros are real

Polynomial division & the remainder theorem

node: sat.polynomial.P4

The skill. When you divide a polynomial P(x) by (x - a), the result is a quotient plus a remainder, and that remainder is always just a constant. The remainder theorem gives a shortcut: the remainder equals P(a), so you can find it by plugging a into the polynomial instead of doing the whole division. The factor theorem is the special case where the remainder is zero: (x - a) is a factor exactly when P(a) = 0. If you do carry out long or synthetic division, put a zero placeholder in for any missing power so the columns line up, and remember the remainder always has a smaller degree than the divisor.

Worked example

Find the remainder when $P(x) = x^3 - 4x + 1$ is divided by (x - 3).

Read. We are dividing by (x - 3), so the divisor's root is a = 3. The remainder theorem says

the remainder equals P at that root, so we can just evaluate P(3) instead of dividing.

Plan.

1. Identify a from the divisor (x - a); here a = 3.
2. Substitute x = 3 into the polynomial.
3. Simplify to get the remainder.

Solve.

$$\begin{aligned} r &= P(3) && \text{the remainder theorem} \\ &= (3)^3 - 4(3) + 1 && \text{substitute x = 3} \\ &= 27 - 12 + 1 && \text{evaluate each term} \\ &= 16 && \text{add and subtract} \end{aligned}$$

Check. Because the remainder is 16, not zero, (x - 3) is not a factor of P. Using a = 3 (not -3) matches the divisor x - 3.

Answer: the remainder is 16.

Approach Read · Plan · Solve · Check

Read. Decide what the task needs: finding a remainder, testing whether something is a factor, carrying out the division, or solving for an unknown constant using a factor condition.

Plan.

1. To find the remainder of division by (x - a), evaluate P(a).
2. To test a factor, check whether P(a) equals zero.
3. To carry out long or synthetic division, fill in zero placeholders for any missing powers.
4. To find an unknown constant when (x - a) is a factor, set P(a) = 0 and solve.

Solve.

1. Read a from the divisor (x - a), keeping its sign correct.
2. Substitute a into P and simplify, or perform the division carefully.
3. Interpret the result: a zero remainder means a factor, a nonzero one does not.

Check. Check that you used a, not -a, from the divisor (x - a). Remember a zero remainder is the same as a factor, that missing powers need placeholders, and that a nonzero remainder means the divisor is not a factor.

Common traps — every wrong answer is one of these

remainder-theorem sign error.
Evaluates P(-a) instead of P(a) when dividing by (x - a), using the wrong sign for the divisor's root. For (x - a), the root is a, so the remainder is P(a).

remainder-vs-factor direction error.
Confuses the root with the remainder, or flips the direction of the factor theorem — for example taking P(a) = 0 to mean a is the remainder, or writing the factor with the wrong sign. P(a) = 0 means the remainder is zero and (x - a) is a factor.

division mechanics error.
1. Sets up long or synthetic division of a polynomial that is missing a power, such as x^3 - 8.
2. Skips the missing x^2 and x columns instead of writing zero placeholders.
3. The terms shift into the wrong columns, producing a wrong quotient.
4. Fix: write every power in, using 0 coefficients, as $x^3 + 0x^2 + 0x$ - 8.

nonzero-remainder interpretation error.
Ignores a nonzero remainder — claiming exact division or a factor when the remainder is not zero — or misreads the remainder as a root or as the quotient. Only a remainder of zero means (x - a) is a factor.

Practice

1. *Let $P(x) = x^2 + 2x + 5$.*

What is the remainder when P(x) is divided by (x - 2)?

(A) 5 (the constant term)
(B) -13
(C) 5
(D) 13

2. *For a polynomial P, it is known that P(3) = 0.*

Which statement is true?

(A) (x - 3) is a factor of P
(B) 3 is the remainder when P is divided by (x - 3)
(C) (x + 3) is a factor of P
(D) P has no real zeros

3. *Divide x^3 - 8 by (x - 2).*

What is the quotient?

(A) x^2 - 2x + 4
(B) x^2 - 8
(C) x^2 + 2x + 4
(D) x^2 + 4

4. *Dividing $P(x) = x^2 + 1$ by $(x - 1)$ leaves a remainder of 2.*

Which statement is true?

(A) (x - 1) is a factor of P

(B) (x - 1) is not a factor of P, since the remainder is not 0

(C) The remainder means x = 2 is a zero of P

(D) P(x) = (x - 1)(x + 1) exactly

5. *Let $P(x) = 2x^3 - x + k$, and suppose $(x - 1)$ is a factor of P.*

What is the value of k?

(A) k = 1

(B) k = 0

(C) k = -3

(D) k = -1

Synthetic division & the quotient

node: sat.polynomial.P5

The skill. If x = r is a root of a quadratic, then (x - r) is one of its factors, and dividing the quadratic by (x - r) leaves the other factor as the quotient. For a quadratic like $x^2 + bx + c$ that factors as (x - r)(x + s), dividing by (x - r) cancels that factor and leaves x + s. So the quotient is the partner factor — not the divisor itself, and not the middle coefficient.

Worked example

Given that x = 3 is a root, divide $x^2 + 2x - 15$ by (x - 3).

Read. Since x = 3 is a root, (x - 3) is a factor of the quadratic. Dividing by it leaves the other factor as the quotient, so the real task is to factor the quadratic.

Plan.

1. Factor $x^2 + 2x - 15$ into two factors, one being (x - 3).
2. Find two numbers that multiply to -15 and add to 2.
3. Read off the partner factor as the quotient.

Solve.

$$x^2 + 2x - 15 = (x - 3)(x + 5) \qquad \text{-3 and +5 multiply to -15 and add to 2}$$

$$\frac{x^2 + 2x - 15}{x - 3} = x + 5 \qquad \text{the (x - 3) factors cancel}$$

Check. Multiplying back, (x - 3)(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 2x - 15, the original quadratic, so the quotient x + 5 is correct.

Answer: the quotient is x + 5.

Approach Read · Plan · Solve · Check

Read. You are told x = r is a root, so (x - r) is a factor; the quotient is whatever factor is left after dividing it out.

Plan.

1. Factor the quadratic into two factors, one of which is (x - r).
2. The other factor is the quotient.

Solve.

1. Find the second number s so that (x - r)(x + s) rebuilds the quadratic.
2. Report x + s as the quotient.

Check. Multiply (x - r) by your quotient and check that it rebuilds the original quadratic, with the right sign on the constant.

Common traps — every wrong answer is one of these

used the divisor.
Reports the divisor's number back as the quotient, writing x + r instead of the partner factor that the division actually leaves.

sign error.
Writes the quotient with the wrong sign, giving x - s instead of x + s.

used the coefficient.
Puts the quadratic's middle coefficient b into the quotient instead of the partner factor's number s.

Practice

1. *Divide (x^2 + 1x - 6) by (x - 2), given that x = 2 is a root. What is the quotient?*

Which is correct?

(A) x + 2
(B) x + 3
(C) x + 1
(D) x - 3

2. *Divide (x^2 + 2x - 15) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 5
(B) x + 5

(C) x + 3

(D) x + 2

3. *Divide (x^2 + 1x - 12) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 4

(B) x + 4

(C) x + 1

(D) x + 3

4. *Divide (x^2 + 3x - 10) by (x - 2), given that x = 2 is a root. What is the quotient?*

Which is correct?

(A) x - 5

(B) x + 2

(C) x + 5

(D) x + 3

5. *Divide (x^2 + 2x - 15) by (x - 3), given that x = 3 is a root. What is the quotient?*

Which is correct?

(A) x - 5

(B) x + 5

(C) x - 3

(D) x + 2

Binomial expansion (perfect squares)

node: sat.polynomial.P6

The skill. Squaring a binomial like (x + k) does not just square each piece. Because $(x + k)^2$ means (x + k)(x + k), multiplying it out gives $x^2 + 2kx + k^2$. The middle term 2kx comes from the two cross products x*k and k*x, which add together. So a squared binomial always has three terms: the square of the first, twice the product of the two, and the square of the last. The constant is k^2, not k, and the middle term must be there.

Worked example

Expand $(x + 6)^2$.

Read. Squaring means multiplying (x + 6) by itself. We expect three terms in the answer, including a middle term.

Plan.

1. Rewrite the square as (x + 6)(x + 6).
2. Multiply all four pairs of terms.
3. Add the two middle products together.

Solve.

$$\begin{aligned}(x+6)^2 &= (x+6)(x+6) && \text{a square is the binomial times itself}\\ &= x^2+6x+6x+36 && \text{multiply all four pairs}\\ &= x^2+12x+36 && \text{add the two middle terms}\end{aligned}$$

Check. The middle term is 2 times 6 times x, which is 12x, and the constant is $6^2 = 36$. Both are present, as expected for a squared binomial.

Answer: $x^2 + 12x + 36$.

Approach Read · Plan · Solve · Check

Read. You are squaring a binomial, which means multiplying (x + k) by itself: (x + k)(x + k).

Plan.

1. Multiply the two binomials using FOIL, all four products.
2. Add the two equal middle products into a single term.

Solve.

1. The first product is x^2 and the last is k^2.
2. The two cross products are each kx, and together they make 2kx.
3. Combine into $x^2 + 2kx + k^2$.

Check. Check that the middle term 2kx is present and that the constant is k squared, not just k.

Common traps — every wrong answer is one of these

dropped the middle term.
Writes $x^2 + k^2$ and forgets the middle term. Squaring a binomial always produces the cross term 2kx as well.

did not double.
Uses kx for the middle term instead of 2kx, forgetting that the two equal cross products add together.

did not square the constant.
Leaves the constant as k instead of k^2, forgetting to square the second term.

Practice

1. *Expand $(x + 2)^2$.*

Which is correct?

(A) $x^2 + 4x + 2$
(B) $x^2 + 4$
(C) $x^2 + 2x + 4$
(D) $x^2 + 4x + 4$

2. *Expand $(x + 5)^2$.*

Which is correct?

(A) $x^2 + 25$
(B) $x^2 + 10x + 5$
(C) $x^2 + 5x + 25$
(D) $x^2 + 10x + 25$

3. *Expand $(x + 3)^2$.*

Which is correct?

(A) $x^2 + 3x + 9$
(B) $x^2 + 6x + 3$
(C) $x^2 + 9$
(D) $x^2 + 6x + 9$

4. *Expand $(x + 4)^2$.*

Which is correct?

(A) $x^2 + 8x + 4$
(B) $x^2 + 4x + 16$
(C) $x^2 + 16$
(D) $x^2 + 8x + 16$

5. *Expand $(x + 5)^2$.*

Which is correct?

(A) $x^2 + 5x + 25$
(B) $x^2 + 25$
(C) $x^2 + 10x + 25$
(D) $x^2 + 10x + 5$

Special products & factoring patterns

node: sat.polynomial.P7

The skill. Multiplying a conjugate pair (x + k)(x - k) gives a difference of squares, x^2 - k^2. When you FOIL it, the two middle terms -kx and +kx cancel, so there is no x-term at all. What remains is the first term squared minus the last term squared. Notice the sign is minus, and the constant is k^2, not k. This is different from $(x - k)^2$, which does have a middle term.

Worked example

Multiply (x + 7)(x - 7).

Read. These are a conjugate pair — the same terms with opposite signs. We expect the middle terms to cancel and leave a difference of squares.

Plan.

1. Multiply all four pairs of terms with FOIL.
2. Combine the two middle terms.

Solve.

$$\begin{aligned} (x+7)(x-7) &= x^2 - 7x + 7x - 49 && \text{multiply all four pairs} \\ &= x^2 - 49 && \text{the middle terms cancel} \end{aligned}$$

Check. The two middle terms -7x and +7x add to zero, and the constant is $-(7^2) = -49$. The result has no x-term, as expected for a difference of squares.

Answer: x^2 - 49.

Approach Read · Plan · Solve · Check

Read. You are multiplying a conjugate pair — the same two terms, once with a plus and once with a minus: (x + k)(x - k).

Plan.

1. Multiply out all four products with FOIL.
2. Notice the two middle terms are opposites and cancel.

Solve.

1. The first product is x^2 and the last is $-k^2$.
2. The middle products -kx and +kx add to zero.
3. What remains is x^2 - k^2.

Check. Check that the middle term cancelled and that the constant is minus k squared.

Common traps — every wrong answer is one of these

wrong sign.
Writes $x^2 + k^2$ instead of $x^2 - k^2$. The last product is $(+k)(-k) = -k^2$, so the sign is minus.

kept a middle term.
Treats the product like $(x - k)^2$ and keeps a $-2kx$ middle term. For a conjugate pair the middle terms cancel, so there is no x-term.

did not square k.
Leaves the constant as $-k$ instead of $-k^2$, forgetting to square the second term.

Practice

1. *Multiply $(x + 4)(x - 4)$.*

Which is correct?

(A) $x^2 - 8x + 16$
(B) $x^2 - 4$
(C) $x^2 - 16$
(D) $x^2 + 16$

2. *Multiply $(x + 5)(x - 5)$.*

Which is correct?

(A) $x^2 + 25$
(B) $x^2 - 25$
(C) $x^2 - 5$
(D) $x^2 - 10x + 25$

3. *Multiply $(x + 2)(x - 2)$.*

Which is correct?

(A) $x^2 + 4$
(B) $x^2 - 4$
(C) $x^2 - 4x + 4$
(D) $x^2 - 2$

4. *Multiply $(x + 6)(x - 6)$.*

Which is correct?

(A) $x^2 - 6$

(B) $x^2 + 36$

(C) $x^2 - 12x + 36$

(D) $x^2 - 36$

5. *Multiply (x + 6)(x - 6).*

Which is correct?

(A) $x^2 - 6$

(B) $x^2 - 36$

(C) $x^2 - 12x - 36$

(D) $x^2 + 36$

Sum & product of roots

node: sat.polynomial.P8

The skill. For a quadratic written as $x^2 + bx + c$, its two roots have a sum and a product you can read straight from the coefficients, without solving. The sum of the roots is -b, the negative of the middle coefficient, and the product of the roots is c, the constant term exactly as written. These are Vieta's formulas. The key sign to remember is that the sum flips the sign of b, while the product keeps c as it is.

Worked example

For the quadratic $x^2 + 7x + 10$, find the sum and product of its roots.

Read. The quadratic is already in the form $x^2 + bx + c$, with $b = 7$ and $c = 10$. Vieta's formulas let us read the sum and product directly.

Plan.

1. Identify $b = 7$ and $c = 10$.
2. The sum of the roots is -b.
3. The product of the roots is c.

Solve.

$$\text{sum} = -b = -7 \qquad \text{the sum is the negative of the middle coefficient}$$

$$\text{product} = c = 10 \qquad \text{the product is the constant term}$$

Check. This quadratic factors as $(x + 2)(x + 5)$, with roots -2 and -5. Their sum is -7 and their product is 10, matching the formulas.

Answer: the sum is -7 and the product is 10.

Approach Read · Plan · Solve · Check

Read. You have a quadratic in the form $x^2 + bx + c$ and want the sum and product of its roots without solving it.

Plan.

1. Read the middle coefficient b and the constant c.
2. Take the sum of the roots as -b.
3. Take the product of the roots as c.

Solve.

1. Negate b to get the sum.
2. Keep c as-is for the product.

Check. Check that you negated b for the sum and left c unchanged for the product, and that you did not swap the two.

Common traps — every wrong answer is one of these

sum sign error.
Uses +b for the sum instead of -b, forgetting to flip the sign of the middle coefficient.

product sign error.
Uses -c for the product instead of c. The product of the roots is the constant term exactly as written.

swapped sum & product.
Reports the constant as the sum and the coefficient as the product, swapping the two formulas.

Practice

1. *For the quadratic $x^2 + 3x + 2$, what are the sum and product of its roots?*

Which is correct?

(A) sum = -3, product = 2
(B) sum = 2, product = 3
(C) sum = -3, product = -2
(D) sum = 3, product = 2

2. *For the quadratic $x^2 + 5x + 6$, what are the sum and product of its roots?*

Which is correct?

(A) sum = 6, product = 5
(B) sum = -5, product = -6

(C) sum = 5, product = 6

(D) sum = -5, product = 6

3. *For the quadratic $x^2 + 7x + 10$, what are the sum and product of its roots?*

Which is correct?

(A) sum = 10, product = 7

(B) sum = -7, product = -10

(C) sum = 7, product = 10

(D) sum = -7, product = 10

4. *For the quadratic $x^2 + 7x + 2$, what are the sum and product of its roots?*

Which is correct?

(A) sum = -7, product = 2

(B) sum = 2, product = 7

(C) sum = 7, product = 2

(D) sum = -7, product = -2

5. *For the quadratic $x^2 + 5x + 6$, what are the sum and product of its roots?*

Which is correct?

(A) sum 5, product 6

(B) sum -5, product 6

(C) sum -5, product -6

(D) sum 6, product -5

Exponent rules & properties

node: sat.exponential.E1

The skill. When you multiply two powers that have the same base, keep the base and add the exponents, so x^a times x^b equals x^(a+b). When you divide same-base powers, subtract the exponents, so x^a divided by x^b equals x^(a-b). When you raise a power to another power, multiply the exponents, so $(x^a)^b$ equals x^(ab). A power on the outside of a product reaches every factor inside, so $(xy)^a$ equals x^a times y^a, and a number in front counts as a factor too — $(2x)^3$ equals $8x^3$, not $2x^3$. Three special cases come up a lot: any nonzero base to the 0 power is 1; a negative exponent means a reciprocal, so x^(-a) equals $1/x^a$ (the value stays positive); and a fractional exponent is a root, so x^(1/n) is the n-th root of x. One rule that does NOT exist: an exponent never spreads across addition, so $(x+y)^2$ is not $x^2 + y^2$.

Worked example

Simplify $(2x^2)^3$.

Read. This is a product raised to a power. The base inside is $2x^2$ and the outside exponent is 3. That outside power must reach both the coefficient 2 and the x^2.

Plan.

1. Raise the coefficient 2 to the 3rd power.
2. Raise x^2 to the 3rd power by multiplying the exponents.
3. Multiply the two results together.

Solve.

$$\begin{aligned} (2x^2)^3 &= 2^3 \cdot (x^2)^3 && \text{give the outside power to each factor} \\ &= 8 \cdot x^{2\cdot 3} && \text{compute } 2^3 \text{ and multiply the exponents} \\ &= 8x^6 && \text{finish the arithmetic} \end{aligned}$$

Check. Test with x = 2: (2 \{}cdot $2^2)^3 = 8^3 = 512$, and 8 \{}cdot $2^6 = 8$ \{}cdot 64 = 512. They agree, so $8x^6$ is right.

Answer: $8x^6$.

Approach Read · Plan · Solve · Check

Read. Look at what is happening to the powers: are same-base powers being multiplied, divided, or nested as a power of a power? Also watch for coefficients, negative exponents, and fractional exponents.

Plan.

1. To multiply same-base powers, add the exponents.
2. To divide same-base powers, subtract the exponents.
3. To raise a power to a power, multiply the exponents.
4. Give any coefficient the outside power too.
5. Rewrite a negative exponent as a reciprocal and a fractional exponent as a root.

Solve.

1. Apply the one matching rule to the exponents.
2. Raise each coefficient to the outside power.
3. Never spread an exponent across a sum.

Check. Check that you added where you should add and multiplied where you should multiply. Make sure the coefficient received the power, a negative exponent became a reciprocal (not a negative number), and any nonzero base to the 0 power is 1.

Common traps — every wrong answer is one of these

Adding and multiplying exponents mixed up.
Multiplies the exponents when the rule calls for adding them, or adds when the rule calls for multiplying. Multiplying same-base powers adds the exponents (x^3 times $x^4 = x^7$); raising a power to a power multiplies them ($(x^3)^4 = x^{12}$). Fix: multiplying means add the exponents, and a power of a power means multiply them.

Negative exponent read as a negative number.
Treats x^(-a) as $-x^a$, or drops the sign entirely. A negative exponent means the reciprocal: x^(-a) = $1/x^a$, and the value does not turn negative. Fix: move the base to the denominator and make the exponent positive.

Zero or fractional exponent mishandled.
Takes x^0 as 0 instead of 1, or reads x^(1/2) as x/2. A nonzero base to the 0 power is 1, and a fractional exponent is a root, so x^(1/2) is the square root of x. Fix: remember $b^0 = 1$ and x^(1/n) means the n-th root.

Coefficient or sum handled wrong.
Forgets to raise a coefficient to the outside power, writing $(2x)^3$ as $2x^3$ instead of $8x^3$, or spreads an exponent over a sum, writing $(x+y)^2$ as $x^2 + y^2$. Fix: the outside power hits every factor, including the number in front, and it never distributes over addition.

Practice

1. *Simplify $x^3 * x^4$.*

What is the result?

(A) x
(B) $2x^7$
(C) x^{12}
(D) x^7

2. *Simplify $(x^3)^4$.*

What is the result?

(A) x^7
(B) x^{12}
(C) x^{81}
(D) $4x^3$

3. *Rewrite x^(-2) without a negative exponent.*

What is the result?

(A) $-x^2$
(B) $-2x$
(C) $1/x^2$
(D) x^2

4. *Evaluate 5^0 + 9^(1/2).*

What is the value?

(A) 3
(B) 4
(C) 5.5
(D) 14

5. *Simplify $(2x^2)^3$.*

What is the result?

(A) $8x^6$
(B) $8x^5$
(C) $2x^6$
(D) $6x^6$

Scientific notation

node: sat.exponential.E2

The skill. Scientific notation writes a number as a times 10^n, where the coefficient a is at least 1 but less than 10 and n is a whole number (positive, negative, or zero). A number bigger than 10 uses a positive exponent, and a number between 0 and 1 uses a negative exponent; the exponent just counts how many places the decimal point moves. To multiply two numbers in this form, multiply the coefficients and add the exponents; to divide, divide the coefficients and subtract the exponents. After multiplying or dividing, fix up the coefficient so it lands back between 1 and 10, adjusting the exponent to match. To see which of two numbers is larger, compare the exponents first.

Worked example

Write the product $(6 \times 10^3)(5 \times 10^4)$ in proper scientific notation.

Read. Two numbers are given in scientific notation. Each has a coefficient (6 and 5) and a power of ten (10^3 and 10^4). We want the product back in proper form, with a coefficient between 1 and 10.

Plan.

1. Multiply the two coefficients.
2. Add the two exponents.

3. If the coefficient is 10 or more, renormalize and adjust the exponent.

Solve.

$$\begin{aligned}(6 \times 10^3)(5 \times 10^4) &= (6 \cdot 5) \times 10^{3+4} && \text{multiply coefficients, add exponents}\\ &= 30 \times 10^7 && \text{do the arithmetic}\\ &= 3 \times 10^1 \times 10^7 && \text{rewrite 30 as 3 times } 10^1\\ &= 3 \times 10^8 && \text{combine the powers of ten}\end{aligned}$$

Check. The coefficient 3 is between 1 and 10, so the form is proper. As a rough check, 6000 times 50,000 = 300,000,000, which is 3 x 10^8.

Answer: 3 x 10^8.

Approach Read · Plan · Solve · Check

Read. Decide what the task is: rewriting between standard and scientific form, multiplying or dividing, cleaning up a coefficient, or comparing sizes.

Plan.

1. To rewrite, move the decimal n places — right if n is positive, left if n is negative.
2. To multiply, multiply the coefficients and add the exponents.
3. To divide, divide the coefficients and subtract the exponents.
4. To normalize, shift the coefficient into the 1-to-10 range and change the exponent to match.
5. To compare sizes, look at the exponents first.

Solve.

1. Carry out the decimal move or the exponent arithmetic.
2. Renormalize so the coefficient is between 1 and 10.

Check. Check that the coefficient is at least 1 and under 10, that the exponent's sign fits the size of the number, and that you added exponents when multiplying rather than multiplying them.

Common traps — every wrong answer is one of these

Decimal moved the wrong number of places.
Slides the decimal point too few or too many places when converting, ending up off by one or more powers of ten. Fix: count the places from the current decimal position to just after the first nonzero digit, and let that count set the exponent.

Wrong sign on the exponent.
Uses a positive power of ten for a number less than 1, or a negative power for a large number. A number between 0 and 1 needs a negative exponent, while a number bigger than 10 needs a positive one. Fix: ask whether the number is large or small and set the sign to match.

Exponents combined the wrong way.
When multiplying, multiplies the exponents instead of adding them (or mishandles them another way). Multiplying powers of ten adds the exponents, and dividing subtracts them. Fix: when the numbers are multiplied, add the exponents; when they are divided, subtract them.

Coefficient left out of range.
Leaves the coefficient below 1 or at 10 or more, such as writing 50 x 10^6 instead of 5 x 10^7, and forgets to adjust the exponent. Fix: shift the coefficient into the 1-to-10 range and change the exponent by the number of places you moved.

Practice

1. *Consider the number 3.2 x 10^4.*

What is this in standard form?

(A) 3,200
(B) 32,000
(C) 0.00032
(D) 320

2. *Consider the number 0.00056.*

What is this in scientific notation?

(A) 5.6 x 10^{-4}
(B) 5.6 x 10^4
(C) 56 x 10^{-5}
(D) 5.6 x 10^{-3}

3. *Compute (2 x 10^3)(4 x 10^5).*

What is the product in scientific notation?

(A) 8 x 10^{15}
(B) 6 x 10^8
(C) 8 x 10^2
(D) 8 x 10^8

4. *A calculation produces the value 50 x 10^6.*

Written in proper scientific notation, this is:

(A) 5 x 10^7
(B) 5 x 10^6

(C) 0.5 x 10^8

(D) 50 x 10^6

5. *Compare 6 x 10^8 with 3 x 10^5.*

How many times larger is 6 x 10^8 than 3 x 10^5?

(A) 200

(B) 2 x 10^{40}

(C) 2,000 (that is, 2 x 10^3)

(D) 2 x 10^{13}

Exponential functions (y = a*b^x)

node: sat.exponential.E3

The skill. In the function y = a * b^x, the number a is the starting value — it is the value of y when x = 0, because $b^0 = 1$. The number b is the base, the fixed amount you multiply by for every one-step increase in x. If b is bigger than 1 the outputs grow; if b is between 0 and 1 they shrink (decay). What makes a pattern exponential is a constant ratio from one value to the next (you keep multiplying by b), which is different from a linear pattern, where you add the same amount each step. To find y for a given x, first raise b to the power x, then multiply that result by a.

Worked example

For y = 4 * 3^x, find y when x = 2.

Read. The function is y = 4 * 3^x, so a = 4 is the starting value and b = 3 is the base. We want the output when x = 2, so we raise 3 to the 2nd power and then multiply by 4.

Plan.

1. Raise the base 3 to the power 2.
2. Multiply that result by the coefficient 4.

Solve.

$$\begin{aligned} y &= 4 \cdot 3^2 && \text{substitute } x = 2 \\ &= 4 \cdot 9 && \text{compute } 3^2 = 9 \text{ first} \\ &= 36 && \text{multiply by 4} \end{aligned}$$

Check. Do the power before the multiplication: $3^2 = 9$, then 4 \{}cdot 9 = 36. Folding them together as (4 \{}cdot 3)^2 = 144 would be wrong, so the order matters.

Answer: 36.

Approach Read · Plan · Solve · Check

Read. Pick out the starting value a and the base b, and decide whether each step multiplies (exponential) or adds (linear).

Plan.

1. Read a as the value when x = 0.
2. Read b as the number multiplied at each step.
3. If b is greater than 1 the function grows; if b is between 0 and 1 it decays.
4. To evaluate, raise b to the power x first, then multiply by a.

Solve.

1. Compute b^x.
2. Multiply the result by a.

Check. Check that successive values share a constant ratio (not a constant difference), that a and b play the right roles, and that the growth-or-decay direction matches b.

Common traps — every wrong answer is one of these

Exponential pattern treated as linear.
Adds the same amount at each step instead of multiplying by the base. In a table like 3, 6, 12 the values double (multiply by 2), so the next value is 24, not 18. Fix: look for a constant ratio and multiply by it, rather than adding a constant difference.

Starting value and base swapped.
Reads the base b as the starting value, or the coefficient a as the growth factor. In $y = 5 * 2^x$ the starting value is 5 and the base is 2. Fix: a sits in front and is the value at x = 0, while b is the number being raised to the power.

Growth and decay direction reversed.
Calls a base greater than 1 decay, or a base between 0 and 1 growth. A base above 1 makes values grow, and a base between 0 and 1 makes them shrink. Fix: compare b to 1 — above 1 grows, below 1 decays.

Coefficient folded into the base.
Computes $a * b^x$ as $(a*b)^x$ or as a*b*x, either merging a into the base or treating the exponent as a multiplier. Fix: raise b to the power x by itself first, then multiply by a.

Practice

1. *An exponential function is $y = 5 * 2^x$.*

What is the initial value (the value when x = 0)?

(A) 5

(B) 2

(C) 10

(D) 7

2. *Consider $y = 3 * 2^x$, $y = 3 * (0.5)^x$, and $y = 2 * 3^x$.*

Which represents exponential decay?

(A) $y = 2 * 3^x$

(B) $y = 3 * 2^x$

(C) None of them decay

(D) $y = 3 * (0.5)^x$

3. *Let $y = 4 * 3^x$.*

What is y when x = 2?

(A) 36

(B) 144

(C) 24

(D) 10

4. *An exponential table shows $x = 0, 1, 2$ giving $y = 3, 6, 12$.*

What is y when x = 3?

(A) 36

(B) 24

(C) 18

(D) 15

5. *A population starts at 200 and doubles each year.*

Which function gives the population after x years?

(A) $y = 400^x$

(B) $y = 200 + 2x$

(C) $y = 2 * 200^x$

(D) $y = 200 * 2^x$

Exponential growth & decay in context

node: sat.exponential.E4

The skill. When a quantity grows by a rate r each period, its value after t periods is $a(1 + r)^t$; when it decays by rate r, the value is $a(1 - r)^t$. The base is 1 + r for growth or 1 - r for decay —

never the raw rate r by itself. This is compound change: every period multiplies by that base, unlike simple linear change, which would add the same amount each time as in a(1 + rt). For a half-life or doubling-time problem, the exponent is the number of periods, found by dividing the elapsed time by the length of one period, as in a(1/2)^(t/h).

Worked example

A population starts at 200 and grows 10% each year. What is the population after 2 years?

Read. The starting amount is 200, the growth rate is 10% = 0.10, and the time is t = 2 years. Because the population grows, the base is 1 + 0.10 = 1.10, and the model is $200(1.10)^t$.

Plan.

1. Convert the rate to a base by adding it to 1.
2. Raise the base to the number of years.
3. Multiply by the starting amount 200.

Solve.

$$\begin{aligned} P &= 200(1+0.10)^2 && \text{growth base is } 1 + r \\ &= 200(1.10)^2 && \text{add inside the parentheses} \\ &= 200(1.21) && \text{square } 1.10 \\ &= 242 && \text{multiply by } 200 \end{aligned}$$

Check. Year by year: 200 grows to 220 after one year (200 + 20), then 220 grows to 242 (220 + 22). This matches, and it is a little more than the 240 that a flat 10%-of-the-original addition would give.

Answer: 242 people.

Approach Read · Plan · Solve · Check

Read. Identify the starting amount, the rate, whether the quantity grows or decays, and how many periods pass.

Plan.

1. Turn the rate into a base: 1 + r for growth, 1 - r for decay.
2. Put the number of periods in the exponent, not as a multiplier.
3. For a half-life, divide the elapsed time by the period length to get the exponent.

Solve.

1. Raise the base to the number of periods.
2. Multiply by the starting amount.

Check. Check that the base is 1 plus or minus r (not the bare rate), that the direction matches growth or decay, and that the change is compounded with an exponent rather than added linearly.

Common traps — every wrong answer is one of these

Rate used as the base.
Uses the raw rate as the base instead of 1 + r or 1 - r, for example modeling 8% growth as $a(0.08)^t$ rather than $a(1.08)^t$, or miscounts the number of periods. A base of 0.08 would shrink the amount toward zero. Fix: add the rate to 1 for growth (1.08) or subtract it for decay (0.92).

Growth and decay base reversed.
Uses 1 + r for something that is shrinking, or 1 - r for something that is growing. Fix: for a decline the base is under 1 (like 0.85 for a 15% loss); for growth it is over 1 (like 1.15).

Simple interest used for compound change.
Models compound change with the linear form $a(1 + rt)$ instead of the exponential $a(1 + r)^t$. Simple change adds the same amount each period, while compounding multiplies by the base each period. Fix: put the number of periods in the exponent, not multiplied inside the parentheses.

Practice

1. *A quantity starts at 500 and grows 8% per year.*

Which models its value after t years?

(A) $500(0.92)^t$
(B) $500(1.08)^t$
(C) $500(1.8)^t$
(D) $500(0.08)^t$

2. *A car worth $20,000 loses 15% of its value each year.*

Which models its value after t years?

(A) $20000(0.85)^t$
(B) $20000(1 - 0.15t)$
(C) $20000(1.15)^t$
(D) $20000(0.15)^t$

3. *You invest $1000 at 5% interest compounded annually.*

Which gives the balance after t years?

(A) $1000(1.05t)$
(B) $1000(1.05)^t$
(C) $1000(0.05)^t$
(D) $1000(1 + 0.05t)$

4. *A population of 200 grows 10% per year.*

What is the population after 2 years?

(A) 240

(B) 220

(C) 2

(D) 242

5. *An 80 mg sample halves every 3 hours.*

How much remains after 9 hours?

(A) 10 mg

(B) 20 mg

(C) 35 mg

(D) 40 mg

Solving exponential equations by matching bases

node: sat.exponential.E5

The skill. If two powers with the same base are equal, then their exponents must be equal: $b^m = b^n$ forces m = n (as long as b is positive and not 1). So to solve an equation like $b^x = N$, rewrite N as a power of the same base b, set the exponents equal, and solve the simpler equation that remains. This usually means recognizing numbers as powers — for instance $81 = 3^4$, $8 = 2^3$, $9 = 3^2$, and 1/8 = 2^(-3). Remember that an exponential means repeated multiplication, not multiplying the base by the exponent, so $3^x = 81$ is NOT solved by dividing 81 by 3.

Worked example

Solve $4^x = 8$.

Read. Both 4 and 8 are powers of 2, so 2 is the common base. We rewrite each side as a power of 2, then match the exponents.

Plan.

1. Write 4 and 8 as powers of 2.
2. Use the power-of-a-power rule on the left side.
3. Set the exponents equal and solve for x.

Solve.

$4^x = 8$	start with the equation
$(2^2)^x = 2^3$	write $4 = 2^2$ and $8 = 2^3$
$2^{2x} = 2^3$	multiply the exponents on the left
$2x = 3$	equal bases force equal exponents
$x = \frac{3}{2}$	divide both sides by 2

Check. Substitute back: $4^{3/2} = (\{\}\text{sqrt}\{4\})^3 = 2^3 = 8$. It matches the original equation.
Answer: x = 3/2.

Approach Read · Plan · Solve · Check

Read. Look for a single base that both sides can be written as powers of.

Plan.

1. Rewrite each side as that common base raised to some exponent.
2. Set the two exponents equal to each other.
3. Solve the resulting equation for x.
4. For a fraction less than 1, use a negative exponent.

Solve.

1. Convert the numbers to powers of the common base.
2. Equate the exponents and solve.

Check. Check that you matched bases before equating exponents (and did not simply divide), that each number was rewritten as the correct power, and that a fraction target used a negative exponent.

Common traps — every wrong answer is one of these

Exponents equated without matching bases.
Sets the exponents equal before both sides share a base, sets the bases equal instead of the exponents, or just divides the two sides. For $4^x = 8$, rewrite as $2\hat{}(2x) = 2^3$ first. Fix: make the bases the same, then equate exponents.

Exponential solved as if linear.
Solves $b^x = N$ by computing N/b or N - b instead of writing N as a power of b. For $3^x = 81$, dividing gives 27, which is wrong. Fix: express 81 as 3^4, so x = 4.

Number rewritten as the wrong power.
Writes a number as an incorrect power of the base, such as $32 = 2^4$ or $81 = 3^3$, giving a wrong exponent. Fix: multiply the base out to confirm, for example $2^5 = 32$ and $3^4 = 81$.

Reciprocal or fraction exponent mishandled.
Solves $2^x = 1/8$ as x = 3 instead of x = -3, or mismanages a fractional exponent. A fraction less than 1 needs a negative exponent because 1/8 = 2^(-3). Fix: rewrite the reciprocal with a negative exponent before equating.

Practice

1. *Solve $3^x = 81$.*

What is x?

(A) 4
(B) 3
(C) 27
(D) 78

2. *Solve $2^x = 32$.*

What is x?

(A) 16
(B) 6
(C) 5
(D) 4

3. *Solve $4^x = 8$.*

What is x?

(A) 1
(B) 3/2
(C) 2
(D) 12

4. *Solve $2^x = 1/8$.*

What is x?

(A) -3
(B) -4
(C) 3
(D) 1/3

5. *Solve $9^x = 27$.*

What is x?

(A) 2/3

(B) 3

(C) 3/2

(D) 1

Graphs of exponential functions

node: sat.exponential.E6

The skill. For a function f(x) = a * b^x, the y-intercept is the point where x = 0. Since any nonzero base to the 0 power is 1, f(0) = a * b^0 = a * 1 = a, so the graph crosses the y-axis at (0, a). The starting height is the coefficient a — not the base b, and not a times b. These graphs also have a horizontal asymptote at y = 0: the curve gets closer and closer to the x-axis on one side but never touches it, so the graph does not pass through the origin.

Worked example

Find the y-intercept of f(x) = 2 * 4^x.

Read. The y-intercept happens at x = 0. Here the coefficient is a = 2 and the base is b = 4, so we evaluate f(0).

Plan.

1. Substitute x = 0.
2. Simplify using $4^0 = 1$.

Solve.

$f(0) = 2 \cdot 4^0$	substitute x = 0
$= 2 \cdot 1$	any nonzero base to the 0 power is 1
$= 2$	multiply

Check. The intercept is the coefficient 2, giving the point (0, 2). It is not the base 4, and not 2 \{}cdot 4 = 8, since $4^0 = 1$.

Answer: (0, 2).

Approach Read · Plan · Solve · Check

Read. The y-intercept is the value of the function at x = 0.

Plan.

1. Substitute x = 0 into f(x) = a * b^x.
2. Use the fact that $b^0 = 1$.

Solve.

1. Simplify a * b^0 to a * 1.
2. Read off the intercept a, giving the point (0, a).

Check. Check that you used $b^0 = 1$, so the intercept is a — not the base b and not a times b.

Common traps — every wrong answer is one of these

Base used as the intercept.
Reports the base b as the y-intercept instead of the coefficient a. For f(x) = 2 * 4^x the intercept is 2, not 4. Fix: the intercept is a * b^0 = a, the number in front.

Coefficient times base used.
Reports a * b, forgetting that $b^0 = 1$. Multiplying a by b would give f(1), not f(0). Fix: at x = 0 the base contributes $b^0 = 1$, so the intercept is just a.

Assumed the graph passes through the origin.
Assumes the curve goes through (0, 0). An exponential y = a * b^x equals a * b^0 = a at x = 0, and its horizontal asymptote is y = 0, so it never reaches (0, 0) unless a is 0. Fix: evaluate f(0) = a instead of assuming a zero intercept.

Practice

1. *The function $f(x) = 2 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 2)
(B) (0, 4)
(C) (0, 8)
(D) (0, 0)

2. *The function $f(x) = 5 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 20)
(B) (0, 5)
(C) (0, 0)
(D) (0, 4)

3. *The function $f(x) = 3 \cdot 4^x$. What is its y-intercept?*

Which is correct?

(A) (0, 12)
(B) (0, 0)
(C) (0, 4)
(D) (0, 3)

4. *The function $f(x) = 2 \cdot 3^x$. What is its y-intercept?*

Which is correct?

(A) (0, 2)
(B) (0, 3)
(C) (0, 6)
(D) (0, 0)

5. *The function $f(x) = 5 \cdot 3^x$. What is its y-intercept?*

Which is correct?

(A) (0, 0)
(B) (0, 3)
(C) (0, 5)
(D) (0, 15)

Compound interest

node: sat.exponential.E7

The skill. Compound interest follows $A = P(1 + r)^t$, where P is the starting amount, r is the interest rate written as a decimal (the percent divided by 100), and t is the number of periods. Each period the balance is multiplied by $1 + r$, so the money grows exponentially — faster than simple interest, which would just add the same amount every period. The key steps are turning the percent into a decimal and raising $1 + r$ to the power t, rather than multiplying by t.

Worked example

You invest $500 at 10% interest compounded annually. What is the balance after 3 years?

Read. The starting amount is $P = 500$, the rate is $10\% = 0.10$, and the time is $t = 3$ years. Compounding means the model is $500(1 + 0.10)^3$.

Plan.

1. Write the rate as a decimal and add it to 1.
2. Raise the base to the 3rd power.
3. Multiply by the starting amount 500.

Solve.

$$\begin{aligned} A &= 500(1+0.10)^3 && \text{compound-interest formula} \\ &= 500(1.10)^3 && \text{add inside the parentheses} \\ &= 500(1.331) && \text{cube 1.10} \\ &= 665.50 && \text{multiply by 500} \end{aligned}$$

Check. Growing one year at a time: 500 to 550, then 550 to 605, then 605 to 665.50 — each year is 1.10 times the last. Simple interest would give only 500 + 3(50) = 650, so earning a bit more from compounding makes sense.

Answer: \$665.50.

Approach Read · Plan · Solve · Check

Read. This is compound growth, so the model is $A = P(1 + r)^t$.

Plan.

1. Divide the percent by 100 to get r as a decimal.
2. Add r to 1 to form the base.
3. Raise the base to the power t, the number of periods.

Solve.

1. Form 1 + r.
2. Raise it to the power t and multiply by P.

Check. Check that the percent was divided by 100 and that t is an exponent, not a multiplier.

Common traps — every wrong answer is one of these

Percent not converted to a decimal.
Uses the whole percent as the rate, such as 1 + 10 instead of 1 + 0.10. Fix: divide the percent by 100 first, so 10% becomes 0.10 and the base is 1.10.

Simple interest used instead of compound.
Adds r*t one time, as in P(1 + rt), instead of raising (1 + r) to the power t. Compounding multiplies by 1 + r every period. Fix: put t in the exponent, giving $P(1 + r)^t$.

Dropped the 1 in the base.
Uses r^t as the base factor, leaving out the 1 +, as in $500(0.10)^3$. That would shrink the balance instead of growing it. Fix: the base must be 1 + r, so the balance keeps the original amount plus the interest.

Practice

1. *You invest \$500 at 10% interest compounded annually. Which expression gives the value after 3 years?*

Which is correct?

(A) $500(1 + 10)^3$

(B) $500(1 + (10/100)\cdot 3)$

(C) $500(1 + 10/100)^3$

(D) $500(10/100)^3$

2. *You invest \$500 at 5% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $500(1 + 5)^2$

(B) $500(1 + (5/100)\cdot 2)$

(C) $500(1 + 5/100)^2$

(D) $500(5/100)^2$

3. *You invest \$1000 at 20% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $1000(1 + 20/100)^2$

(B) $1000(1 + (20/100)\cdot 2)$

(C) $1000(20/100)^2$

(D) $1000(1 + 20)^2$

4. *You invest \$2000 at 5% interest compounded annually. Which expression gives the value after 4 years?*

Which is correct?

(A) $2000(1 + (5/100)\cdot 4)$

(B) $2000(1 + 5)^4$

(C) $2000(5/100)^4$

(D) $2000(1 + 5/100)^4$

5. *You invest \$800 at 5% interest compounded annually. Which expression gives the value after 2 years?*

Which is correct?

(A) $800(0.05)^2$
(B) $800(1 + 5)^2$
(C) $800(1 + 0.05{\cdot}2)$
(D) $800(1.05)^2$

Half-life & doubling time

node: sat.exponential.E8

The skill. In a half-life problem, the amount P left after time t is P * (1/2)^(t/h), where h is the half-life — the time it takes for half of the material to disappear. The exponent t/h is the number of half-lives that have gone by, found by dividing the elapsed time by the half-life. The base is 1/2 because the amount is cut in half each period (for a doubling problem the base would be 2 instead). The elapsed time is divided by the period length, not multiplied by it.

Worked example

A 100-gram sample has a half-life of 4 years. How much remains after 8 years?

Read. The starting amount is P = 100 grams, the half-life is h = 4 years, and the elapsed time is t = 8 years. The model is 100 * (1/2)^(t/h).

Plan.

1. Find the number of half-lives by dividing 8 by 4.
2. Raise 1/2 to that power.
3. Multiply by the starting amount 100.

Solve.

$$\begin{aligned} A &= 100\left(\frac{1}{2}\right)^{8/4} && \text{half-life model} \\ &= 100\left(\frac{1}{2}\right)^{2} && 8 \,/\, 4 = 2 \text{ half-lives} \\ &= 100 \cdot \frac{1}{4} && \text{square one-half} \\ &= 25 && \text{multiply by } 100 \end{aligned}$$

Check. Count the halvings: after 4 years 100 becomes 50, and after another 4 years 50 becomes 25. Two halvings give 25 grams.

Answer: 25 grams.

Approach Read · Plan · Solve · Check

Read. This is half-life decay, modeled by P * (1/2)^(t/h).

Plan.

1. Find the number of half-lives by dividing the elapsed time t by the half-life h.

2. Use 1/2 as the base for decay (or 2 for doubling).

Solve.

1. Form the exponent t/h.
2. Raise 1/2 to that power and multiply by P.

Check. Check that the exponent is t divided by h (not multiplied) and that the base is 1/2 for decay.

Common traps — every wrong answer is one of these

Multiplied instead of divided in the exponent.
Uses t*h as the number of half-lives instead of t/h. For t = 8 and h = 4 there are 8/4 = 2 half-lives, not 32. Fix: divide the elapsed time by the half-life.

Decay modeled linearly, not as a power.
Multiplies by t/h once, as in P * (1/2) * (t/h), instead of raising 1/2 to that power. Fix: the number of half-lives belongs in the exponent, giving (1/2)^(t/h).

Growth base used for decay.
Uses base 2, which doubles, instead of 1/2 for a shrinking amount. Fix: for decay each period halves the amount, so the base is 1/2.

Practice

1. *A sample of 100 grams decays with a half-life of 4 years. Which expression gives the amount after 8 years?*

Which is correct?

(A) 100(1/2)^(8·4)
(B) 100(1/2)·(8/4)
(C) 100(1/2)^(8/4)
(D) 100(2)^(8/4)

2. *A sample of 80 grams decays with a half-life of 2 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 80(1/2)·(20/2)
(B) 80(1/2)^(20/2)
(C) 80(1/2)^(20·2)
(D) 80(2)^(20/2)

3. *A sample of 80 grams decays with a half-life of 4 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 80(1/2)^(20·4)

(B) 80(2)^(20/4)

(C) 80(1/2)·(20/4)

(D) 80(1/2)^(20/4)

4. *A sample of 240 grams decays with a half-life of 4 years. Which expression gives the amount after 20 years?*

Which is correct?

(A) 240(1/2)^(20/4)

(B) 240(1/2)^(20·4)

(C) 240(2)^(20/4)

(D) 240(1/2)·(20/4)

5. *A 200-gram sample decays with a half-life of 5 years. Which expression gives the amount after 15 years?*

Which is correct?

(A) 200·2^(15/5)

(B) 200·(1/2)·3

(C) 200·(1/2)^(15/5)

(D) 200·(1/2)^(15·5)

Radicals & rational exponents (simplify & convert)

node: sat.radical.N1

The skill. A rational (fractional) exponent packs together a root and a power. In x^(m/n), the bottom number n is the root and the top number m is the power, so x^(m/n) is the n-th root of x, raised to the m power. That makes x^(1/2) the square root of x and x^(2/3) the cube root of x squared. Radicals multiply and divide across factors: $\sqrt{a*b} = \sqrt{a}*\sqrt{b}$ and $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$, so you can simplify by pulling out any perfect-square (or perfect n-th-power) factor. But a root never splits over a sum — $\sqrt{a+b}$ is not $\sqrt{a}+\sqrt{b}$. Finally, a negative fractional exponent means a reciprocal, so x^(-m/n) = 1/x^(m/n).

Worked example

Evaluate 8^(2/3).

Read. The exponent 2/3 means a root and a power. The bottom number 3 calls for the cube root, and the top number 2 calls for squaring. Taking the root first keeps the numbers small.

Plan.

1. Take the cube root of 8 (the bottom of the fraction).
2. Raise that result to the 2nd power (the top of the fraction).

Solve.

$$\begin{aligned} 8^{2/3} &= \left(\sqrt[3]{8}\right)^2 && \text{denominator is the root, numerator is the power} \\ &= 2^2 && \text{the cube root of 8 is 2} \\ &= 4 && \text{square the 2} \end{aligned}$$

Check. Check the other order: $8^2 = 64$, and the cube root of 64 is 4 — the same answer, which confirms 4.

Answer: 4.

Approach Read · Plan · Solve · Check

Read. Decide the task: convert between radical and exponent form, simplify a radical, or evaluate. Watch for a sum under the root or a negative exponent.

Plan.

1. Convert using bottom number = root, top number = power.
2. Simplify by factoring out a perfect power and taking its root.
3. Add or subtract inside a root before taking it; never split a root over a sum.
4. Turn a negative exponent into a reciprocal first.

Solve.

1. Apply the root and the power in their correct roles.
2. Or extract the perfect-power factor from under the radical.

Check. Check that the bottom number was the root, that no root was split over addition, that a negative exponent became a reciprocal, and that any perfect-power factor was fully pulled out.

Common traps — every wrong answer is one of these

Root and power swapped.
In x^(m/n), uses the top number as the root or the bottom as the power, or treats the fraction as a coefficient like (2/3)x. The bottom is the root and the top is the power, so x^(2/3) is the cube root of x squared. Fix: the denominator names the root and the numerator names the

power.

Root split over a sum.
Treats $\sqrt{a+b}$ as $\sqrt{a}+\sqrt{b}$, or handles a sum under a root as if it were a product. For example $\sqrt{9+16} = \sqrt{25} = 5$, not 3 + 4 = 7. Fix: combine what is under the root first, then take the root.

Radical simplified incorrectly.
Fails to take the root of a perfect-square factor, leaves a radical that can still be simplified, or pulls out the wrong factor. For $\sqrt{50}$, factor out 25 to get 5*$\sqrt{2}$. Fix: pull out the largest perfect square and take its root.

Negative exponent's reciprocal dropped.
Treats x^(-1/2) as $\sqrt{x}$, or as the negative of $\sqrt{x}$. A negative exponent gives a reciprocal, so x^(-1/2) = $1/\sqrt{x}$. Fix: move the base to the denominator and make the exponent positive.

Practice

1. *Consider the expression x^(2/3), where x is positive.*

Which is equivalent?

(A) two-thirds of x

(B) the square root of x cubed

(C) the cube root of x squared

(D) x times two-thirds

2. *Evaluate the square root of (9 + 16).*

What is the value?

(A) 7

(B) 12

(C) 5

(D) 25

3. *Simplify the square root of 50.*

What is the simplified form?

(A) 25 times the square root of 2

(B) 5 times the square root of 2

(C) the square root of 50 (already simplest)

(D) 2 times the square root of 5

4. *Consider x^(-1/2), where x is positive.*

Which is equivalent?

(A) 1 divided by x squared

(B) 1 divided by the square root of x

(C) the negative of the square root of x

(D) the square root of x

5. *Evaluate 8^(2/3).*

What is the value?

(A) 1/4

(B) 64

(C) 4

(D) 16/3

Radical & rational equations (solving with extraneous checks)

node: sat.radical.N2

The skill. To solve an equation with a square root, first get the radical by itself on one side, then square both sides to remove it. Squaring can introduce EXTRANEOUS solutions — answers that satisfy the squared equation but not the original — so you must substitute every candidate back into the original equation and throw out any that do not work. To solve a rational equation (one with variables in denominators), multiply through by the least common denominator to clear the fractions, then solve; but any value that makes a denominator zero is not allowed, even if it satisfies the cleared equation. And when you square a sum, use FOIL: $(\sqrt{x} + 3)^2 = x + 6*\sqrt{x} + 9$, not x + 9.

Worked example

Solve $\sqrt{x+2} = x$.

Read. The radical is already alone on the left. We square both sides to remove it, solve the quadratic, then check each answer in the original equation, because squaring can create false solutions.

Plan.

1. Square both sides to remove the square root.
2. Rearrange into a quadratic equal to 0 and factor.
3. Check each candidate in the original equation.

Solve.

$$\begin{aligned} \sqrt{x+2} &= x && \text{the radical is already isolated} \\ x+2 &= x^2 && \text{square both sides} \\ 0 &= x^2 - x - 2 && \text{move all terms to one side} \\ 0 &= (x-2)(x+1) && \text{factor the quadratic} \\ x &= 2 \quad \text{or} \quad x = -1 && \text{set each factor equal to } 0 \end{aligned}$$

Check. Test both in the original. x = 2 gives \{}sqrt{4} = 2, which is true. x = -1 gives \{}sqrt{1} = 1, but 1 does not equal -1, so x = -1 is extraneous and is discarded. Only x = 2 works.

Answer: x = 2.

Approach Read · Plan · Solve · Check

Read. Decide whether the equation has a radical or a fraction, and note any denominators or squared sums that need care.

Plan.

1. For a radical equation, isolate the radical, then square both sides.
2. Plan to check every candidate for extraneous roots.
3. For a rational equation, multiply by the least common denominator to clear fractions.
4. Mark any value that makes a denominator zero as excluded.

Solve.

1. Solve the equation that remains (often a quadratic).
2. Substitute each candidate into the original and keep only the ones that work.

Check. Check that the radical was isolated before squaring, that a squared sum kept its middle term, that each candidate was tested in the original, and that no excluded value was reported.

Common traps — every wrong answer is one of these

Extraneous root not checked. 1. Squares both sides and solves for the candidate values.

2. Reports every candidate without testing them.
3. Keeps a value that fails the original equation, or discards a valid one.
4. Fix: substitute each candidate into the ORIGINAL equation and keep only the ones that hold.

Squaring or expansion done wrong.
Squares before the radical is alone, squares only one side, or expands $(\sqrt{x}+3)^2$ as x + 9 instead of x + 6*$\sqrt{x}$ + 9. Fix: isolate the radical first, square both whole sides, and FOIL a squared sum so the middle term survives.

Denominator-zero value kept.
Reports a value that makes a denominator zero, or mishandles the least common denominator. In x/(x-2) = 2/(x-2), x = 2 makes the denominator zero, so there is no solution. Fix: exclude any value that zeroes a denominator, even if the cleared equation allows it.

Practice

1. *Solve the equation: the square root of (x + 2) equals x.*

What is the solution set?

(A) No solution
(B) x = 2
(C) x = 2 or x = -1
(D) x = -1

2. *Solve: the square root of x, plus 3, equals 7.*

What is x?

(A) 4
(B) 40
(C) 46
(D) 16

3. *Solve the rational equation: x / (x - 2) = 2 / (x - 2).*

What is the solution?

(A) x = 2 and x = -2
(B) x = 2
(C) No solution
(D) All real numbers

4. *Solve: the square root of (2x + 3) equals x.*

What is the solution set?

(A) x = 1 or x = 3
(B) x = -1
(C) x = 3 or x = -1
(D) x = 3

5. *Solve the rational equation: $x + 6/x = 5$.*

What is the solution set?

(A) x = 2 or x = 3

(B) x = 0, 2, or 3

(C) x = 5

(D) x = 2 only

Simplify radicals (perfect-square factors)

node: sat.radical.N3

The skill. To simplify a square root, break the number under the root into a perfect-square factor times whatever is left, then take the root of the perfect-square part and move it outside. In symbols, $\sqrt{k^2 * m}$ = k * $\sqrt{m}$: only the perfect square comes out (as its square root k), while the rest stays under the radical. For example $\sqrt{50} = \sqrt{25 * 2} = 5*\sqrt{2}$. Use the LARGEST perfect-square factor so that nothing left under the root can still be simplified.

Worked example

Simplify $\sqrt{72}$.

Read. We want to pull out the largest perfect-square factor of 72. The perfect squares that divide 72 are 4, 9, and 36; the largest is 36, and 72 = 36 * 2.

Plan.

1. Write 72 as 36 times 2.
2. Split the root across the two factors.
3. Take the square root of 36 and leave 2 inside.

Solve.

$$\begin{aligned} \sqrt{72} &= \sqrt{36 \cdot 2} && \text{36 is the largest perfect-square factor} \\ &= \sqrt{36} \cdot \sqrt{2} && \text{a root spreads over a product} \\ &= 6\sqrt{2} && \text{the square root of 36 is 6} \end{aligned}$$

Check. The leftover 2 has no perfect-square factors, so it is fully simplified. Estimate: $6*\sqrt{2}$ is about 6 \{}cdot 1.414 = 8.49, and $\sqrt{72}$ is also about 8.49 — they agree.

Answer: $6*\sqrt{2}$.

Approach Read · Plan · Solve · Check

Read. Find the largest perfect square that divides the number under the root.

Plan.

1. Write the radicand as (perfect square) times (leftover).

2. Take the square root of the perfect-square factor.
3. Leave the leftover under the radical.

Solve.

1. Pull out the square root of the perfect square as k.
2. Write k in front of $\sqrt{leftover}$.

Check. Check that the leftover under the root has no perfect-square factors remaining, and that only the square root k (not k^2) came outside.

Common traps — every wrong answer is one of these

Radical dropped or square left outside.
Removes the root entirely, like $\sqrt{50} = 10$, or places the perfect square k^2 outside instead of its root k, like writing $9*\sqrt{5}$ for $\sqrt{45}$. Fix: only the square root of the perfect square comes out, so $\sqrt{45} = \sqrt{9 * 5} = 3*\sqrt{5}$.

Simplification left unfinished.
Pulls a number out front but keeps the whole original number under the root, like $\sqrt{45} = 3*\sqrt{45}$. Fix: after taking out the factor, the radicand must shrink to just the leftover, so $\sqrt{45} = 3*\sqrt{5}$.

Perfect square not rooted.
Writes $\sqrt{k^2 * m}$ as $k^2 * \sqrt{m}$, failing to take the square root of the perfect-square factor. Fix: $\sqrt{k^2}$ is k, not k^2, so $\sqrt{72} = \sqrt{36 * 2} = 6*\sqrt{2}$.

Practice

1. *Simplify* $\sqrt{45}$.

Which is correct?

(A) 15
(B) $3\sqrt{5}$
(C) $3\sqrt{45}$
(D) $9\sqrt{5}$

2. *Simplify* $\sqrt{80}$.

Which is correct?

(A) $16\sqrt{5}$
(B) $4\sqrt{80}$
(C) $4\sqrt{5}$

(D) 20

3. *Simplify* $\sqrt{20}$.

Which is correct?

(A) $2\sqrt{20}$
(B) 10
(C) $2\sqrt{5}$
(D) $4\sqrt{5}$

4. *Simplify* $\sqrt{54}$.

Which is correct?

(A) $3\sqrt{54}$
(B) $9\sqrt{6}$
(C) 18
(D) $3\sqrt{6}$

5. *Simplify* $\sqrt{72}$.

Which is correct?

(A) $2\sqrt{18}$
(B) 12
(C) $6\sqrt{2}$
(D) $36\sqrt{2}$

Add & subtract like radicals

node: sat.radical.N4

The skill. Radicals that have the same index and the same number under the root are called like radicals, and they combine just like like terms: a*$\sqrt{k}$ + b*$\sqrt{k}$ = (a + b)*$\sqrt{k}$. You add the coefficients out front and keep the same radical — the number under the root does not change, and the coefficients are added, not multiplied. Radicals that are not alike (different numbers under the root) cannot be combined at all.

Worked example

Simplify 3*$\sqrt{5}$ + 2*$\sqrt{5}$.

Read. Both terms have the same radical, $\sqrt{5}$, so they are like radicals and can be combined. The coefficients are 3 and 2.

Plan.

1. Confirm both terms share $\sqrt{5}$.
2. Add the coefficients 3 and 2.
3. Keep $\sqrt{5}$ unchanged.

Solve.

$$\begin{aligned} 3\sqrt{5} + 2\sqrt{5} &= (3+2)\sqrt{5} && \text{like radicals combine like like terms} \\ &= 5\sqrt{5} && \text{add the coefficients} \end{aligned}$$

Check. Think of $\sqrt{5}$ as a common unit: 3 of them plus 2 of them makes 5 of them, so 5*$\sqrt{5}$. The number under the root stays 5, not 10.

Answer: 5*$\sqrt{5}$.

Approach Read · Plan · Solve · Check

Read. Check whether the terms have the same radical $\sqrt{k}$.

Plan.

1. Confirm the numbers under the root match.
2. Add the coefficients in front.
3. Keep the same radical $\sqrt{k}$.

Solve.

1. Add the coefficients to get (a + b).
2. Attach $\sqrt{k}$ unchanged.

Check. Check that the number under the root stayed the same and that the coefficients were added, not multiplied.

Common traps — every wrong answer is one of these

Numbers under the root added.
Adds the radicands, turning $\sqrt{k} + \sqrt{k}$ into $\sqrt{2k}$ instead of 2*$\sqrt{k}$. Fix: the radicand stays fixed; only the coefficients out front combine.

Coefficients multiplied instead of added.
Multiplies the coefficients rather than adding them, like 3*$\sqrt{5}$ + 2*$\sqrt{5}$ = 6*$\sqrt{5}$ instead of 5*$\sqrt{5}$. Fix: like radicals combine like like terms, so add the coefficients: 3 + 2 = 5.

Like radicals treated as uncombinable.
Claims like radicals cannot be combined even though their radicands match. Fix: if the number under the root is the same, add the coefficients and keep the radical.

Practice

1. *Simplify* $3\sqrt{5} + 2\sqrt{5}$.

Which is correct?

(A) Cannot be combined
(B) $6\sqrt{5}$
(C) $5\sqrt{5}$
(D) $5\sqrt{10}$

2. *Simplify* $3\sqrt{5} + 3\sqrt{5}$.

Which is correct?

(A) $9\sqrt{5}$
(B) Cannot be combined
(C) $6\sqrt{5}$
(D) $6\sqrt{10}$

3. *Simplify* $4\sqrt{3} + 5\sqrt{3}$.

Which is correct?

(A) $9\sqrt{3}$
(B) Cannot be combined
(C) $9\sqrt{6}$
(D) $20\sqrt{3}$

4. *Simplify* $4\sqrt{2} + 2\sqrt{2}$.

Which is correct?

(A) $6\sqrt{4}$
(B) Cannot be combined
(C) $6\sqrt{2}$
(D) $8\sqrt{2}$

5. *Simplify* $4\sqrt{3} + 3\sqrt{3}$.

Which is correct?

(A) $7\sqrt{6}$
(B) cannot be combined
(C) $12\sqrt{3}$
(D) $7\sqrt{3}$

Multiply radicals & rationalize denominators

node: sat.radical.N5

The skill. Square roots multiply across factors: $\sqrt{a}*\sqrt{b} = \sqrt{a*b}$. To rationalize a denominator that contains a square root, such as n/$\sqrt{k}$, multiply the top and bottom by $\sqrt{k}$. Because $\sqrt{k}*\sqrt{k} =$ k, this turns the denominator into the plain number k and moves the radical up into the numerator: n/$\sqrt{k}$ = n*$\sqrt{k}$/k. Multiplying by $\sqrt{k}/\sqrt{k}$ does not change the value, since that fraction equals 1.

Worked example

Rationalize the denominator of 3/$\sqrt{5}$.

Read. There is a square root, $\sqrt{5}$, in the denominator. Multiplying the top and bottom by $\sqrt{5}$ will clear it, since $\sqrt{5}*\sqrt{5} = 5$.

Plan.

1. Multiply the numerator and denominator by $\sqrt{5}$.
2. Simplify the denominator using $\sqrt{5}*\sqrt{5} = 5$.

Solve.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \qquad \text{multiply by } \sqrt{5}/\sqrt{5}, \text{ which equals 1}$$

$$= \frac{3\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} \qquad \text{multiply straight across}$$

$$= \frac{3\sqrt{5}}{5} \qquad \sqrt{5} \text{ times } \sqrt{5} \text{ is } 5$$

Check. The denominator 5 is now rational and $\sqrt{5}$ is on top. As a decimal check, 3/$\sqrt{5}$ is about 3/2.236 = 1.342, and 3*$\sqrt{5}$/5 is about 6.708/5 = 1.342 — the same value.

Answer: 3*$\sqrt{5}$/5.

Approach Read · Plan · Solve · Check

Read. Check whether a square root sits in the denominator.

Plan.

1. Multiply the numerator and denominator by $\sqrt{k}$.
2. Use $\sqrt{k}*\sqrt{k} =$ k to clear the denominator.

Solve.

1. The numerator becomes n*$\sqrt{k}$.
2. The denominator becomes k.

Check. Check that the denominator is now a plain number and that $\sqrt{k}$ ended up in the numerator.

Common traps — every wrong answer is one of these

Radical lost or numerator dropped.
Removes the radical without placing $\sqrt{k}$ in the numerator, or loses the original numerator n. Fix: multiplying by $\sqrt{k}/\sqrt{k}$ keeps n and puts $\sqrt{k}$ on top, so n/$\sqrt{k}$ = n*$\sqrt{k}$/k.

Rational denominator dropped.
Puts $\sqrt{k}$ on top but forgets the denominator k, writing something like n*$\sqrt{k}$ with no denominator. Fix: $\sqrt{k}$*$\sqrt{k}$ = k, so the denominator becomes k and must stay.

Radical left in the denominator.
Leaves n/$\sqrt{k}$ unchanged, or rationalizes only the numerator, so the denominator stays irrational. Fix: multiply top and bottom by $\sqrt{k}$ so the denominator becomes the rational number k.

Practice

1. *Rationalize the denominator: 1/$\sqrt{2}$.*

Which is correct?

(A) $\sqrt{2}/2$
(B) $1\sqrt{2}$
(C) $1\sqrt{2}/2$
(D) $1/2$

2. *Rationalize the denominator: 1/$\sqrt{7}$.*

Which is correct?

(A) $\sqrt{7}/7$
(B) $1\sqrt{7}/7$
(C) $1/7$
(D) $1\sqrt{7}$

3. *Rationalize the denominator: 3/$\sqrt{5}$.*

Which is correct?

(A) $3\sqrt{5}/5$
(B) $3/5$
(C) $3\sqrt{5}$
(D) $\sqrt{5}/5$

4. *Rationalize the denominator: 1/$\sqrt{3}$.*

Which is correct?

(A) $1/3$

(B) $\sqrt{3}/3$

(C) $1\sqrt{3}$

(D) $1\sqrt{3}/3$

5. *Rationalize the denominator: $3/\sqrt{5}$.*

Which is correct?

(A) $3/\sqrt{5}$

(B) $3\sqrt{5}$

(C) $3/5$

(D) $3\sqrt{5}/5$

nth roots & cube roots

node: sat.radical.N6

The skill. The cube root undoes cubing: the cube root of c^3 is c, because c is the number that, multiplied by itself three times, gives the radicand. More generally, the n-th root of x^n is x. Taking a cube root is not the same as squaring, not the same as dividing by the index 3, and not the same as leaving the number unchanged. To find the cube root of a number, ask which value cubed produces it — for instance the cube root of 64 is 4, because $4^3 = 64$.

Worked example

Simplify the cube root of 64.

Read. We need the number whose cube is 64. Cubing a number means multiplying it by itself three times.

Plan.

1. Test small whole numbers cubed.
2. Find the one whose cube equals 64.

Solve.

$\sqrt[3]{64} = c \;\text{ where }\; c^3 = 64$	the cube root undoes cubing
$4^3 = 4 \cdot 4 \cdot 4 = 64$	test c = 4
$\sqrt[3]{64} = 4$	4 cubed is 64

Check. Confirm: 4 \{}cdot 4 \{}cdot 4 = 64. Note it is not 64/3 and not $4^2 = 16$.
Answer: 4.

Approach Read · Plan · Solve · Check

Read. Ask which number, when cubed, gives the radicand.

Plan.

1. Find c such that c^3 equals the number under the root.
2. That value c is the cube root.

Solve.

1. Test small values: c^3 for c = 2, 3, 4, and so on.
2. Stop when c^3 matches the radicand.

Check. Check that c^3 really equals the radicand — not c^2, and not the radicand divided by 3.

Common traps — every wrong answer is one of these

Wrong operation for the root.
Squares the number, leaves the radicand unchanged, or otherwise does not take the cube root. Fix: the cube root of 64 is the c with $c^3 = 64$, which is 4 — not 16 and not 64.

Multiplied by the index.
Multiplies the radicand by the index 3, or the root by 3, instead of taking the root, like calling the cube root of 64 equal to 12. Fix: find the number whose cube is the radicand, so the cube root of 64 is 4.

Divided by the index.
Computes the n-th root as the radicand divided by n, like calling the cube root of 27 equal to 9. Fix: the cube root of 27 is 3 because $3^3 = 27$, not 27/3.

Practice

1. *Simplify the cube root of 64.*

Which is correct?

(A) 64
(B) 16
(C) 12
(D) 4

2. *Simplify the cube root of 125.*

Which is correct?

(A) 15
(B) 25

(C) 125

(D) 5

3. *Simplify the cube root of 8.*

Which is correct?

(A) 2

(B) 4

(C) 8

(D) 6

4. *Simplify the cube root of 27.*

Which is correct?

(A) 81

(B) 3

(C) 9

(D) 729

5. *Simplify the cube root of 125.*

Which is correct?

(A) 375

(B) about 41.7

(C) 5

(D) 15625

Operations with rational exponents

node: sat.radical.N7

The skill. A rational exponent m/n tells you to do two things: take the n-th root (from the bottom of the fraction) and raise to the m power (from the top). So base^(m/n) equals the n-th root of the base, raised to the m power. Both actions must happen — the root and the power. Taking the root first usually keeps the numbers small, as in 27^(2/3) = (cube root of 27)^2 = 3^2 = 9.

Worked example

Evaluate 27^(2/3).

Read. The exponent 2/3 means take the cube root (bottom = 3) and raise to the 2nd power (top = 2). Doing the root first keeps the numbers manageable.

Plan.

1. Take the cube root of 27.
2. Raise that result to the 2nd power.

Solve.

$$\begin{aligned} 27^{2/3} &= \left(\sqrt[3]{27}\right)^2 && \text{denominator is the root, numerator is the power} \\ &= 3^2 && \text{the cube root of 27 is 3} \\ &= 9 && \text{square the 3} \end{aligned}$$

Check. The other order agrees: $27^2 = 729$, and the cube root of 729 is 9. Both routes give 9.
Answer: 9.

Approach Read · Plan · Solve · Check

Read. Read the fraction: the bottom is the root and the top is the power.

Plan.

1. Take the n-th root of the base, using the denominator.
2. Raise that result to the m power, using the numerator.

Solve.

1. Compute the root first.
2. Raise the result to the power.

Check. Check that BOTH the root and the power were applied, not just one.

Common traps — every wrong answer is one of these

Root skipped.
Raises the base to the power but never takes the root, like calling 27^(2/3) equal to $27^2 = 729$. Fix: take the cube root first, then square, so (cube root of 27)$^2 = 3^2 = 9$.

Power skipped.
Takes the root but never raises to the power, like calling 27^(2/3) equal to just the cube root 3. Fix: after the root, apply the top number as a power, so $3^2 = 9$.

Exponent used as a plain multiplier.
Multiplies the base by m/n, or uses only m or only n, instead of splitting into a root and a power. Fix: the fraction is not a multiplier — the denominator is a root and the numerator is a power.

Practice

1. *Evaluate 27^(2/3).*

Which is correct?

(A) 3
(B) 9
(C) 729
(D) 27

2. *Evaluate 4^(3/2).*

Which is correct?

(A) 64
(B) 4
(C) 8
(D) 2

3. *Evaluate 9^(3/2).*

Which is correct?

(A) 27
(B) 9
(C) 3
(D) 729

4. *Evaluate 8^(2/3).*

Which is correct?

(A) 8
(B) 2
(C) 4
(D) 64

5. *Evaluate 16^(3/4).*

Which is correct?

(A) 4096
(B) 12
(C) 2
(D) 8

Radical functions: domain & graphs

node: sat.radical.N8

The skill. For a function like f(x) = $\sqrt{x - a}$, whatever is inside the square root must be zero or greater, because the square root of a negative number is not a real number. Setting the inside to be at least 0 gives x - a >= 0, which solves to x >= a. So the domain — the set of allowed x-values — is x >= a. Because you are only adding a to both sides, the inequality keeps its direction (it does not flip), and the boundary value comes straight from where the inside equals 0.

Worked example

Find the domain of f(x) = $\sqrt{x - 3}$.

Read. The inside of the square root, x - 3, cannot be negative. To find the allowed x-values, we require x - 3 to be at least 0.

Plan.

1. Set the inside x - 3 greater than or equal to 0.
2. Solve for x by adding 3 to both sides.

Solve.

$x - 3 \geq 0$	the inside of a square root must be 0 or greater
$x \geq 3$	add 3 to both sides

Check. Test a value in the domain and one outside it. x = 4 gives \{}sqrt{1} = 1, which is fine; x = 2 gives \{}sqrt{-1}, which is not real. So x >= 3 is correct.

Answer: x >= 3.

Approach Read · Plan · Solve · Check

Read. The expression inside the square root must be zero or greater.

Plan.

1. Set the inside expression greater than or equal to 0.
2. Solve the inequality for x.

Solve.

1. Add a to both sides of x - a >= 0.
2. Read off x >= a.

Check. Check that the inequality points the right way and that the boundary value has the correct sign.

Common traps — every wrong answer is one of these

Inequality direction flipped.
Reports x <= a instead of x >= a. Adding a to both sides does not flip the inequality. Fix: solve x - a >= 0 by adding a, which gives x >= a.

Sign error on the constant.
Writes x >= -a instead of x >= a. From x - a >= 0 you add a, not subtract it. Fix: for $\sqrt{x-3}$ the domain is x >= 3, and for $\sqrt{x+2}$ it is x >= -2.

Restriction ignored.
Claims the domain is all real numbers, forgetting that the inside of a square root cannot be negative. Fix: set the inside greater than or equal to 0 and solve for x.

Practice

1. *What is the domain of* $f(x) = \sqrt{x-3}$*?*

Which is correct?

(A) all real numbers
(B) $x \geq 3$
(C) $x \geq -3$
(D) $x \leq 3$

2. *What is the domain of* $f(x) = \sqrt{x-4}$*?*

Which is correct?

(A) $x \geq 4$
(B) all real numbers
(C) $x \geq -4$
(D) $x \leq 4$

3. *What is the domain of* $f(x) = \sqrt{x-5}$*?*

Which is correct?

(A) all real numbers
(B) $x \leq 5$
(C) $x \geq 5$
(D) $x \geq -5$

4. *What is the domain of* $f(x) = \sqrt{x-2}$*?*

Which is correct?

(A) $x \geq -2$
(B) all real numbers
(C) $x \geq 2$

(D) x ≤ 2

5. *What is the domain of* $f(x) = \sqrt{x+2}$*?*

Which is correct?

(A) x ≤ -2
(B) x ≥ -2
(C) all real numbers
(D) x ≥ 2

Rational expressions: simplify & operate

node: sat.rational.F1

The skill. A rational expression is one polynomial divided by another — a fraction whose top and bottom are both pieces of algebra. To simplify one, factor the top and the bottom completely, then cancel any factor that appears in both. A factor is a whole piece that multiplies, not a single term that is added. To multiply two of them, factor everything, cancel across the two fractions, then multiply the tops together and the bottoms together. To divide, keep the first fraction and multiply by the flip (reciprocal) of the second. To add or subtract, first rewrite both fractions over the same denominator. Finally, a rational expression is undefined wherever a denominator would be zero, so those x-values are excluded from the domain — even a value that cancels out later was still never allowed in the original.

Worked example

Simplify $(x^2 - x - 6)/(x^2 - 9)$ and state the excluded values.

Read. The top and bottom are both quadratics. We want a simpler equivalent fraction, plus the x-values that were never allowed because they make the original bottom zero.

Plan.

1. Factor the numerator.
2. Factor the denominator.
3. Cancel any factor shared by both.
4. Read the excluded values from the original denominator.

Solve.

$$\frac{x^2 - x - 6}{x^2 - 9} = \frac{(x-3)(x+2)}{(x-3)(x+3)} \qquad \text{factor the top and the bottom}$$

$$= \frac{x+2}{x+3} \qquad \text{cancel the common factor x-3}$$

Check. The denominator $x^2 - 9$ is zero at x = 3 and x = -3, so both are excluded — even though (x-3) canceled, x = 3 was never allowed. Testing x = 0 in the original gives -6/-9 = 2/3, and the simplified form also gives 2/3.

Answer: (x+2)/(x+3), with x not equal to 3 and x not equal to -3.

Approach Read · Plan · Solve · Check

Read. Decide which operation you are doing — simplify, multiply, divide, add, or subtract — and note every denominator, since those set the excluded values.

Plan.

1. Factor the top and bottom of each expression completely.
2. To simplify, cancel any factor that appears on both the top and the bottom.
3. To divide, flip the second fraction and multiply instead.
4. To add or subtract, rewrite everything over a common denominator first.
5. List the excluded values from the original denominators.

Solve.

1. Carry out the factoring and the canceling.
2. Multiply or combine the remaining pieces.
3. Keep track of the excluded values.

Check. Check that you canceled whole factors and not single terms, used a common denominator before adding, flipped the correct fraction when dividing, and excluded every value that makes an original denominator zero.

Common traps — every wrong answer is one of these

Canceling terms instead of factors.
Cancels pieces that are added instead of factors that multiply, such as turning (3x+6)/3 into x+6 by crossing out the 3 in 3x. Fix: factor first — (3x+6)/3 = 3(x+2)/3 = x+2 — and cancel only a factor shared by the whole top and the whole bottom.

Combining straight across when adding.
Adds fractions by adding the tops and adding the bottoms, treating a/b + c/d as (a+c)/(b+d). Fix: rewrite both fractions over a common denominator first, then add only the numerators.

Dividing without flipping.
Divides by multiplying straight across, or flips the wrong fraction. Fix: when dividing, keep the first fraction and multiply by the reciprocal of the second.

Missing or wrong excluded values.
Leaves out values that make a denominator zero, uses the numerator's zeros instead, or forgets a restriction from a factor that cancels. Fix: set every original denominator equal to zero and exclude all those x-values, even ones that cancel later.

Practice

1. *Simplify (3x + 6) / 3.*

What is the simplified form?

(A) x + 6

(B) x + 2

(C) x

(D) 3x + 2

2. *Simplify 1/x + 1/y as a single fraction.*

What is the result?

(A) x + y

(B) 2 / (x + y)

(C) 1 / (x + y)

(D) (x + y) / (xy)

3. *Simplify (a/b) divided by (c/d).*

What is the result?

(A) bc / ad

(B) ac / bd

(C) ad / bc

(D) bd / ac

4. *Consider the expression (x + 1) / (x^2 - 4).*

Which values of x are excluded from the domain?

(A) x = 4

(B) None; all real numbers are allowed

(C) x = 2 and x = -2

(D) x = -1

5. *Simplify (x^2 - x - 6) / (x^2 - 9) and state the domain restriction.*

Which is correct?

(A) (x + 2)/(x + 3), with no restriction

(B) (x + 2)/(x + 3), with x not equal to -3 only

(C) (-x - 6)/(-9), by canceling the x^2 terms

(D) (x + 2)/(x + 3), with x not equal to 3 and x not equal to -3

Multiply & divide rational expressions

node: sat.rational.F2

The skill. To divide one rational expression by another, keep the first fraction and multiply it by the reciprocal of the second — that is, flip only the divisor (the fraction you are dividing by) upside down and change the division into multiplication. Then cancel any factor that appears on both a top and a bottom, and multiply what is left. Dividing is not the same as multiplying straight across, and only the second fraction gets flipped, never the first and never the final answer.

Worked example

Simplify (5/x) divided by (3/x).

Read. This is one fraction divided by another, and both have the same denominator x. We want a single simplified value.

Plan.

1. Flip the second fraction to turn the division into multiplication.
2. Cancel the common factor x.
3. Multiply what remains.

Solve.

$$\frac{5}{x} \div \frac{3}{x} = \frac{5}{x} \times \frac{x}{3} \qquad \text{multiply by the reciprocal of the divisor}$$

$$= \frac{5x}{3x} \qquad \text{multiply straight across}$$

$$= \frac{5}{3} \qquad \text{cancel the common factor x}$$

Check. The x's match, so they cancel and the answer is just 5/3. Since 5/x is larger than 3/x, their quotient should be greater than 1, and 5/3 is about 1.67.

Answer: 5/3.

Approach Read · Plan · Solve · Check

Read. You are dividing one fraction by another, so plan to flip the divisor and multiply.

Plan.

1. Leave the first fraction as it is.
2. Flip the second fraction (the divisor) upside down.
3. Change the division sign to multiplication.
4. Cancel factors shared by a top and a bottom, then multiply.

Solve.

1. Rewrite (a/x) divided by (b/x) as (a/x) times (x/b).
2. Cancel the x on top with the x on the bottom.
3. Multiply the remaining numbers to get a/b.

Check. Check that you flipped only the second fraction, that the variable canceled, and that the answer is a/b rather than its reciprocal.

Common traps — every wrong answer is one of these

Multiplied without flipping.
Multiplies the two fractions straight across without taking the reciprocal, so (5/x) divided by (3/x) becomes $15/x^2$. Fix: dividing means multiply by the flip of the second fraction.

Flipped the final answer.
Solves correctly but reports the reciprocal of the result, giving 3/5 instead of 5/3. Fix: flip only the divisor before multiplying, never the answer at the end.

Canceled, then multiplied the leftovers.
Cancels the variable correctly but then multiplies the two coefficients instead of dividing them, giving 15 instead of 5/3. Fix: after the flip, 5/x times x/3 leaves 5/3, not 5 times 3.

Practice

1. *Divide: (5/x) ÷ (3/x). Simplify.*

Which is correct?

(A) 15
(B) 3/5
(C) $15/x^2$
(D) 5/3

2. *Divide: (2/x) ÷ (3/x). Simplify.*

Which is correct?

(A) 2/3
(B) 3/2
(C) 6
(D) $6/x^2$

3. *Divide: (3/x) ÷ (5/x). Simplify.*

Which is correct?

(A) $15/x^2$
(B) 5/3
(C) 3/5

(D) 15

4. *Divide: (3/x) ÷ (7/x). Simplify.*

Which is correct?

(A) $21/x^2$
(B) 21
(C) 7/3
(D) 3/7

5. *Divide: (8/x) ÷ (2/x). Simplify.*

Which is correct?

(A) 4
(B) $16/x^2$
(C) 1/4
(D) 16

Add & subtract rational expressions

node: sat.rational.F3

The skill. When two fractions already share the same denominator, add them by adding just the numerators and keeping that common denominator: a/x + b/x = (a+b)/x. The denominator stays the same — you do not add the two denominators together and you do not multiply them. Only the tops combine.

Worked example

Add 4/x + 5/x.

Read. Two fractions are being added, and they already have the same denominator, x, so they are ready to combine.

Plan.

1. Add the numerators 4 and 5.
2. Keep the common denominator x.

Solve.

$$\frac{4}{x}+\frac{5}{x}=\frac{4+5}{x} \qquad \text{add the numerators over the common denominator}$$
$$=\frac{9}{x} \qquad \text{add on top}$$

Check. The denominator stays x. Picking x = 1 gives 4 + 5 = 9, and 9/1 = 9, which matches.
Answer: 9/x.

Approach Read · Plan · Solve · Check

Read. Check that the two fractions have the same denominator, so they are ready to combine.

Plan.

1. Add the numerators together.
2. Keep the shared denominator unchanged.
3. Simplify the result if possible.

Solve. Add the two numerators and write the sum over the same denominator, giving (a+b)/x.

Check. Check that the denominator is still x, and that you added the numerators rather than multiplying them.

Common traps — every wrong answer is one of these

Added the denominators.
Adds the bottoms as well as the tops, turning 4/x + 5/x into 9/(2x). Fix: with a common denominator the bottom stays x — only the numerators are added.

Multiplied the numerators.
Multiplies the tops instead of adding them, giving 20/x from 4/x + 5/x. Fix: adding fractions means add the numerators, so 4 + 5 = 9.

Multiplied the denominators.
Multiplies the two denominators, turning 4/x + 5/x into $9/x^2$. Fix: the shared denominator x is kept as is, not multiplied by itself.

Practice

1. *Add: 4/x + 5/x. Simplify.*

Which is correct?

(A) 9/(2x)
(B) 20/x
(C) 9/x
(D) $9/x^2$

2. *Add: 3/x + 5/x. Simplify.*

Which is correct?

(A) 8/x
(B) 8/(2x)
(C) 15/x

(D) $8/x^2$

3. *Add: 3/x + 1/x. Simplify.*

Which is correct?

(A) $4/(2x)$
(B) $4/x$
(C) $3/x$
(D) $4/x^2$

4. *Add: 2/x + 5/x. Simplify.*

Which is correct?

(A) $7/x^2$
(B) $7/x$
(C) $10/x$
(D) $7/(2x)$

5. *Add: 3/x + 7/x. Simplify.*

Which is correct?

(A) $10/x^2$
(B) $21/x$
(C) $10/(2x)$
(D) $10/x$

Complex (compound) fractions

node: sat.rational.F4

The skill. A complex fraction is a fraction divided by another fraction, like (1/a) divided by (1/b). To simplify it, flip the bottom fraction and multiply: (1/a) divided by (1/b) equals (1/a) times (b/1), which is b/a. Dividing by 1/b is the same as multiplying by b. Flip only the bottom fraction, and remember that dividing by a small fraction makes the result larger.

Worked example

Simplify the complex fraction (1/4) divided by (1/5).

Read. The whole thing is one fraction, 1/4, divided by another fraction, 1/5. We want a single simplified value.

Plan.

1. Flip the bottom fraction 1/5 to get 5/1.

2. Multiply 1/4 by 5/1.
3. Simplify.

Solve.

$$\frac{1/4}{1/5} = \frac{1}{4} \times \frac{5}{1} \qquad \text{flip the bottom fraction and multiply}$$

$$= \frac{5}{4} \qquad \text{multiply straight across}$$

Check. Dividing by 1/5 should make the result 5 times bigger, and 1/4 times 5 is 5/4, which is larger than 1/4.

Answer: 5/4.

Approach Read · Plan · Solve · Check

Read. See the expression as a top fraction divided by a bottom fraction.

Plan.

1. Flip the bottom fraction upside down.
2. Change the division into multiplication.
3. Multiply the two fractions and simplify.

Solve. For (1/a) divided by (1/b), multiply (1/a) times (b/1) to get b/a.

Check. Check that you flipped only the bottom fraction, and that dividing by a fraction less than 1 made the answer bigger.

Common traps — every wrong answer is one of these

Gave the reciprocal of the answer.
Reports a/b instead of b/a, flipping the wrong fraction. Fix: it is the bottom fraction that gets flipped, so (1/a) divided by (1/b) is b/a.

Multiplied the denominators.
Multiplies straight across the bottoms and gets 1/(ab) instead of dividing. Fix: change to multiplication by the reciprocal first, do not multiply as written.

Multiplied straight across.
Multiplies tops and bottoms as written without flipping, treating division like multiplication. Fix: dividing by a fraction means multiply by its reciprocal.

Practice

1. *Simplify the complex fraction: (1/4) / (1/5).*

Which is correct?

(A) 4/5
(B) 5/4
(C) 1/20
(D) 20

2. *Simplify the complex fraction: (1/4) / (1/3).*

Which is correct?

(A) 4/3
(B) 3/4
(C) 12
(D) 1/12

3. *Simplify the complex fraction: (1/2) / (1/5).*

Which is correct?

(A) 10
(B) 2/5
(C) 5/2
(D) 1/10

4. *Simplify the complex fraction: (1/3) / (1/5).*

Which is correct?

(A) 15
(B) 1/15
(C) 5/3
(D) 3/5

5. *Simplify the complex fraction: (3/4) / (2/5).*

Which is correct?

(A) 15/8
(B) 8/15
(C) 3/20
(D) 3/10

Solve rational equations

node: sat.rational.F5

The skill. To solve an equation like a/x = b, first clear the fraction by multiplying both sides by x, which gives a = bx. Then divide both sides by b to get x by itself: x = a/b. Multiplying and dividing are different steps and must not be swapped, and both sides of the equation change at each step. A quick check: put your answer back in and make sure a/x really equals b.

Worked example

Solve 12/x = 3 for x.

Read. The unknown x sits in the denominator. We want the value of x that makes twelve divided by x equal to 3.

Plan.

1. Multiply both sides by x to clear the fraction.
2. Divide both sides by 3 to isolate x.

Solve.

$$\frac{12}{x} = 3 \qquad \text{start with the equation}$$
$$12 = 3x \qquad \text{multiply both sides by x}$$
$$4 = x \qquad \text{divide both sides by 3}$$

Check. Put x = 4 back in: 12/4 = 3, which matches the equation.
Answer: x = 4.

Approach Read · Plan · Solve · Check

Read. The unknown x is stuck in a denominator, so plan to clear the fraction first.

Plan.

1. Multiply both sides by x to clear the fraction, giving a = bx.
2. Divide both sides by b to isolate x.
3. Write x = a/b.

Solve. Divide a by b to get the value of x.

Check. Check that you divided (not multiplied) by b, and put x back into a/x to confirm it equals b.

Common traps — every wrong answer is one of these

Multiplied instead of dividing.
Multiplies a and b to get x, turning 12/x = 3 into x = 36. Fix: after a = bx, you divide by b, so x = 12/3 = 4.

Dropped the coefficient.
Reports x = a, ignoring b entirely, so 12/x = 3 gives x = 12. Fix: divide a by b — the 3 cannot just disappear.

Inverted the answer.
Reports b/a instead of a/b, giving x = 3/12. Fix: solving a = bx for x divides a by b, so x = a/b = 12/3.

Practice

1. *Solve 12/x = 3 for x.*

Which is correct?

(A) x = 36
(B) x = 4
(C) x = 3/12
(D) x = 12

2. *Solve 9/x = 3 for x.*

Which is correct?

(A) x = 27
(B) x = 9
(C) x = 3
(D) x = 3/9

3. *Solve 8/x = 2 for x.*

Which is correct?

(A) x = 2/8
(B) x = 16
(C) x = 8
(D) x = 4

4. *Solve 6/x = 2 for x.*

Which is correct?

(A) x = 2/6
(B) x = 6
(C) x = 3
(D) x = 12

5. *Solve 20/x = 5 for x.*

Which is correct?

(A) 20

(B) 1/4

(C) 100

(D) 4

Excluded values & domain

node: sat.rational.F6

The skill. A rational expression is undefined wherever its denominator equals zero, because dividing by zero has no meaning. To find the excluded value, ignore the numerator, set the denominator equal to zero, and solve. For n/(x - a) this gives x - a = 0, so x = a. Watch the sign: n/(x + a) is undefined at x = -a. The excluded values always come from the denominator, never the numerator.

Worked example

Find the value of x that is excluded from the domain of 3/(x - 4).

Read. The expression is a fraction with x in the denominator. It is undefined wherever the denominator equals zero, and that x-value is the one we exclude.

Plan.

1. Set the denominator x - 4 equal to zero.
2. Solve for x.

Solve.

$$x - 4 = 0 \qquad \text{set the denominator to zero}$$
$$x = 4 \qquad \text{add 4 to both sides}$$

Check. At x = 4 the denominator is 4 - 4 = 0, so the expression is undefined there. Every other x is fine.

Answer: x = 4.

Approach Read · Plan · Solve · Check

Read. Ask which x-value would make the denominator zero.

Plan.

1. Set the denominator equal to zero.
2. Solve that little equation for x.
3. State that x cannot equal that value.

Solve. Solve x - a = 0 to get x = a, or x + a = 0 to get x = -a.

Check. Check that you used the denominator, not the numerator, and that the sign of the answer is right.

Common traps — every wrong answer is one of these

Sign error on the excluded value.
Solves the denominator but flips the sign, reporting x = -a when it should be x = a (or the reverse). Fix: solve x - 4 = 0 carefully to get x = 4, and x + 6 = 0 to get x = -6.

Used the numerator.
Sets the numerator to zero instead of the denominator, so 3/(x-4) gives x = 3. Fix: only the denominator makes the expression undefined, so set x - 4 = 0.

Missed the exclusion.
Claims every x is allowed and there is no excluded value. Fix: the denominator can be zero, so there is always at least one excluded value to find.

Practice

1. *For the expression 3/(x - 4), what value of x is excluded from the domain?*

Which is correct?

(A) x = -4
(B) x = 4
(C) none
(D) x = 3

2. *For the expression 2/(x - 3), what value of x is excluded from the domain?*

Which is correct?

(A) x = 3
(B) x = -3
(C) x = 2
(D) none

3. *For the expression 1/(x - 5), what value of x is excluded from the domain?*

Which is correct?

(A) x = 1
(B) none
(C) x = -5
(D) x = 5

4. *For the expression 2/(x - 5), what value of x is excluded from the domain?*

Which is correct?

(A) x = 5

(B) none

(C) x = 2

(D) x = -5

5. *For the expression 5/(x + 6), what value of x is excluded from the domain?*

Which is correct?

(A) x = 5

(B) none

(C) x = -6

(D) x = 6

Rational functions: asymptotes & holes

node: sat.rational.F7

The skill. A vertical asymptote is a vertical line that the graph shoots up or down along but never crosses. For a simple rational function like f(x) = n/(x - a), it occurs where the denominator is zero but the numerator is not: set x - a = 0 to get the vertical asymptote x = a. Watch the sign — n/(x + a) has its vertical asymptote at x = -a. Do not confuse this with the horizontal asymptote, which for this form is the line y = 0.

Worked example

Find the vertical asymptote of f(x) = 1/(x - 3).

Read. This is a simple rational function. Its graph has a vertical asymptote wherever the denominator equals zero, since the function blows up there.

Plan.

1. Set the denominator x - 3 equal to zero.
2. Solve for x.
3. Write the asymptote as x = that value.

Solve.

$x - 3 = 0$	set the denominator to zero
$x = 3$	add 3 to both sides

Check. As x gets close to 3, the denominator gets close to 0 and the function grows without bound, so x = 3 is the vertical asymptote. The horizontal asymptote here would be y = 0, a different line.

Answer: x = 3.

Approach Read · Plan · Solve · Check

Read. A vertical asymptote sits where the denominator is zero.

Plan.

1. Set the denominator equal to zero.
2. Solve for x.
3. Write the asymptote as the vertical line x = that value.

Solve. Solve x - a = 0 to get x = a, or x + a = 0 to get x = -a.

Check. Check that you gave a vertical line x = something (not the horizontal y = 0), and that the sign is right.

Common traps — every wrong answer is one of these

Sign error.
Flips the sign, giving x = -a when the denominator x - a is zero at x = a. Fix: solve x - 3 = 0 to get x = 3, and x + 2 = 0 to get x = -2.

Horizontal and vertical mixed up.
Reports the horizontal asymptote y = 0 instead of the vertical one. Fix: the vertical asymptote is an x = line found from the denominator; y = 0 is the separate horizontal asymptote.

Used the numerator.
Uses the numerator's value instead of the denominator's zero. Fix: a vertical asymptote comes from setting the denominator to zero, not the top.

Practice

1. *The function f(x) = 1/(x - 3) has a vertical asymptote at:*

Which is correct?

(A) x = 1
(B) x = 3
(C) x = -3
(D) y = 0

2. *The function f(x) = 2/(x - 4) has a vertical asymptote at:*

Which is correct?

(A) x = 4
(B) x = -4
(C) x = 2

(D) y = 0

3. *The function f(x) = 3/(x - 4) has a vertical asymptote at:*

Which is correct?

(A) x = 3
(B) x = -4
(C) x = 4
(D) y = 0

4. *The function f(x) = 2/(x - 3) has a vertical asymptote at:*

Which is correct?

(A) x = -3
(B) x = 2
(C) x = 3
(D) y = 0

5. *The function f(x) = 1/(x + 2) has a vertical asymptote at:*

Which is correct?

(A) y = -2
(B) x = 2
(C) x = 1
(D) x = -2

Direct & inverse variation

node: sat.rational.F8

The skill. In direct variation, y = kx, two quantities keep a constant ratio k = y/x, so when one grows the other grows by the same factor. To use it, first find k by dividing a known y by its x. Then predict a new y by multiplying k by the new x. Direct variation scales by multiplying, not by adding — if x is larger, y grows in proportion; it does not just go up by the change in x.

Worked example

y varies directly with x. When x = 2, y = 8. Find y when x = 5.

Read. Direct variation means y = kx for some constant k. We know one pair, x = 2 with y = 8, and want y when x = 5.

Plan.

1. Find k by dividing the known y by its x.

2. Multiply k by the new x, 5.

Solve.

$$\begin{aligned} k = \frac{y}{x} = \frac{8}{2} &\qquad \text{find the constant from the known pair} \\ = 4 &\qquad \text{divide} \\ y = 4 \times 5 &\qquad \text{scale to the new x = 5} \\ = 20 &\qquad \text{multiply} \end{aligned}$$

Check. The ratio y/x stays 4: at x = 5, y = 20 gives 20/5 = 4, matching the original 8/2 = 4.

Answer: y = 20.

Approach Read · Plan · Solve · Check

Read. The relationship is y = kx, so find the constant k first.

Plan.

1. Find k by dividing the known y by its x.
2. Multiply k by the new x to predict the new y.

Solve. Compute k = y1/x1, then y = k times x2.

Check. Check that you found k before predicting, and that you scaled by multiplying rather than adding the change in x.

Common traps — every wrong answer is one of these

Reused the given y.
Reports the original y instead of scaling to the new x, so y stays 8. Fix: find k, then multiply by the new x to get a new y.

Added instead of scaling.
Adds the change in x to y rather than multiplying by k, treating a proportion as additive. Fix: direct variation multiplies — y = k times the new x.

Scaled without finding k.
Multiplies the given y by the new x without first dividing by the old x, so 8 times 5 = 40. Fix: divide first to get k = 4, then multiply k by the new x.

Practice

1. *y varies directly with x. When x = 2, y = 8. Find y when x = 5.*

Which is correct?

(A) 8
(B) 11
(C) 20
(D) 40

2. *y varies directly with x. When $x = 2$, $y = 6$. Find y when $x = 4$.*

Which is correct?

(A) 12
(B) 8
(C) 24
(D) 6

3. *y varies directly with x. When $x = 2$, $y = 6$. Find y when $x = 6$.*

Which is correct?

(A) 6
(B) 36
(C) 18
(D) 10

4. *y varies directly with x. When $x = 3$, $y = 12$. Find y when $x = 5$.*

Which is correct?

(A) 60
(B) 14
(C) 12
(D) 20

5. *y varies directly with x. When $x = 3$, $y = 12$. Find y when $x = 7$.*

Which is correct?

(A) 28
(B) 84
(C) 16
(D) 12

Nonlinear systems: line & parabola

node: sat.nlsystems.X1

The skill. A system with one straight-line equation and one quadratic (parabola) equation is solved by substitution: solve the linear equation for one variable and put that expression into the quadratic. This collapses the system into a single quadratic equation, which you move all to one side to set equal to zero. The number of real solutions matches the number of points where the line and parabola cross: two if the line cuts through, one if it just touches (tangent), and zero if they miss. Each solution is a point (x, y), so once you find an x, put it back into either equation to get its y. Do not add or subtract the two equations the way you would with two straight lines — that trick only works when both are linear.

Worked example

Solve the system $y = x^2$ and $y = x + 6$.

Read. One equation is a parabola, $y = x^2$, and the other is a line, $y = x + 6$. We want the points where they cross. Since both are already solved for y, we can set the two expressions for y equal.

Plan.

1. Set the two expressions for y equal to each other.
2. Move everything to one side to get a quadratic equal to zero.
3. Factor and solve for x.
4. Substitute each x back into $y = x^2$ to find y.

Solve.

$$\begin{aligned} x^2 &= x + 6 && \text{set the two expressions for y equal} \\ x^2 - x - 6 &= 0 && \text{move every term to one side} \\ (x - 3)(x + 2) &= 0 && \text{factor the quadratic} \\ x = 3 \text{ or } x &= -2 && \text{set each factor to zero} \end{aligned}$$

Check. At $x = 3$, $y = 3^2 = 9$ and the line gives $3 + 6 = 9$, so (3, 9) works. At $x = -2$, $y = 4$ and the line gives $-2 + 6 = 4$, so (-2, 4) works too.

Answer: the points (3, 9) and (-2, 4).

Approach Read · Plan · Solve · Check

Read. One equation is a line and one is a parabola; the solutions are the points where they meet.

Plan.

1. Solve the linear equation for one variable.
2. Substitute that expression into the quadratic equation.
3. Move every term to one side so the quadratic equals zero.
4. Solve the quadratic, keeping all roots.
5. Back-substitute each x to find its y.

Solve.

1. Solve the single quadratic for x, keeping every root.
2. Substitute each x back to get the matching y.
3. Write each solution as an ordered pair (x, y).

Check. Check that you substituted rather than added the equations, kept every root, reported ordered pairs and not bare x-values, and that the number of points matches the discriminant.

Common traps — every wrong answer is one of these

Miscounting the intersections.
Assumes a line-and-parabola system always has two solutions, or misreads the discriminant of the substituted quadratic. Fix: the count comes from the discriminant — positive gives two points, zero gives one, negative gives none.

Adding instead of substituting.
Adds or subtracts the two equations as if both were linear, or substitutes but mishandles a term. Fix: solve the line for one variable and substitute it into the quadratic — elimination does not work here.

Giving x instead of the point.
Solves for x but stops there, reporting bare x-values, a single coordinate, or swapped coordinates instead of full points. Fix: back-substitute each x to get y and write the ordered pair (x, y).

Dropping the second solution.
Finds one crossing point but keeps only one root of the quadratic, losing the other. Fix: a quadratic can have two roots — report both points unless the discriminant says otherwise.

Practice

1. *Consider the system $y = x^2 + 1$ and $y = 2x + 4$.*

Substituting to solve, which single equation results?

(A) $x^2 + 1 = 2x$
(B) $x^2 + 1 + 2x + 4 = 0$
(C) $x^2 + 1 = 2x + 4$
(D) $x^2 = 2x + 4$

2. *The system $y = x^2$ and $y = x + 6$.*

What are the intersection points?

(A) (9, 3) and (4, -2)
(B) (3, 3) and (-2, -2)

(C) (3, 9) and (-2, 4)

(D) x = 3 and x = -2

3. *The system $y = x^2 + 4$ and $y = 2x$.*

How many real solutions does it have?

(A) One

(B) Infinitely many

(C) Zero

(D) Two

4. *The parabola $y = x^2 - 4$ and the line $y = 0$ (the x-axis).*

What are all solution points?

(A) (2, 0) only

(B) (-2, 0) only

(C) (2, 0), since a line meets a parabola at one point

(D) (2, 0) and (-2, 0)

5. *Solve the system $y = x^2 - 2x$ and $y = 3$.*

What are all solution points?

(A) (3, 3) only

(B) (3, 3) and (-1, 3)

(C) No real solution

(D) x = 3 and x = -1

Solve line & parabola by substitution

node: sat.nlsystems.X2

The skill. To solve a line-and-parabola system by substitution, set the two expressions for y equal, then bring every term to one side so you have a quadratic equal to zero. For $y = x^2$ and $y = mx - c$, this becomes $x^2 - mx + c = 0$. Factor that quadratic; its two roots are the x-values where the line and parabola meet — usually two of them. Each root is a real crossing point, so do not stop after finding just one.

Worked example

Solve the system $y = x^2$ and $y = 5x - 4$. Find the x-values where the curves meet.

Read. A parabola $y = x^2$ and a line $y = 5x - 4$. Setting the two expressions for y equal gives a single quadratic to solve for x.

Plan.

1. Set x^2 equal to 5x - 4.
2. Move all terms to one side.
3. Factor and solve for x.

Solve.

$$\begin{aligned} x^2 &= 5x - 4 && \text{set the expressions for y equal} \\ x^2 - 5x + 4 &= 0 && \text{move every term to one side} \\ (x-1)(x-4) &= 0 && \text{factor the quadratic} \\ x = 1 &\text{ or } x = 4 && \text{set each factor to zero} \end{aligned}$$

Check. At x = 1 the line gives 5(1) - 4 = 1 and the parabola gives 1, so they meet. At x = 4 the line gives 5(4) - 4 = 16 and the parabola gives 16 too.

Answer: x = 1 or x = 4.

Approach Read · Plan · Solve · Check

Read. Set the line equal to the parabola and build a quadratic that equals zero.

Plan.

1. Set x^2 equal to mx - c.
2. Move all terms to one side to get x^2 - mx + c = 0.
3. Factor the quadratic.
4. Set each factor to zero and solve.

Solve.

1. Factor into (x - r1)(x - r2) = 0.
2. Read off the two roots r1 and r2.

Check. Check that you found two roots and that each one satisfies both the line and the parabola.

Common traps — every wrong answer is one of these

Reading off the coefficients.
Reports the numbers m and c from the line as the solutions instead of solving the quadratic. Fix: substitute and factor — for y = 5x - 4 the roots are 1 and 4, not 5 and 4.

Sign error on the roots.
Factors but takes the wrong signs, giving the negatives of the true roots. Fix: (x - 1)(x - 4) = 0 gives x = 1 and x = 4, both positive.

Stopping at one solution.
Reports only one intersection when there are two. Fix: a factored quadratic usually gives two

roots — report both.

Practice

1. *Solve the system $y = x^2$ and $y = 5x - 4$. What are the x-values of the solutions?*

Which is correct?

(A) x = -1 and x = -4
(B) x = 1 and x = 4
(C) x = 1 only
(D) x = 5 and x = 4

2. *Solve the system $y = x^2$ and $y = 4x - 3$. What are the x-values of the solutions?*

Which is correct?

(A) x = 4 and x = 3
(B) x = 1 only
(C) x = 1 and x = 3
(D) x = -1 and x = -3

3. *Solve the system $y = x^2$ and $y = 6x - 8$. What are the x-values of the solutions?*

Which is correct?

(A) x = 6 and x = 8
(B) x = 2 only
(C) x = -2 and x = -4
(D) x = 2 and x = 4

4. *Solve the system $y = x^2$ and $y = 6x - 5$. What are the x-values of the solutions?*

Which is correct?

(A) x = 1 and x = 5
(B) x = -1 and x = -5
(C) x = 1 only
(D) x = 6 and x = 5

5. *Solve the system $y = x^2$ and $y = 6x - 8$. What are the x-values of the solutions?*

Which is correct?

(A) x = 2 and x = 4

(B) x = -2 and x = -4

(C) x = 2

(D) x = 6 and x = -8

Line & circle systems

node: sat.nlsystems.X3

The skill. To solve a line-and-circle system, substitute the line into the circle equation. For a circle $x^2 + y^2 = R^2$ and a horizontal line y = b, replacing y gives $x^2 + b^2 = R^2$, so $x^2 = R^2 - b^2$. Taking the square root gives two x-values, $x = +\sqrt{R^2 - b^2}$ and $x = -\sqrt{R^2 - b^2}$, because a horizontal line usually cuts a circle at two points. Remember the plus-or-minus, and subtract b^2 before taking the root.

Worked example

The circle $x^2 + y^2 = 52$ meets the line y = 6. Find the x-coordinates of the intersection points.

Read. A circle with radius-squared 52 and a horizontal line y = 6. Substituting y = 6 into the circle equation leaves an equation in x alone.

Plan.

1. Substitute y = 6 into $x^2 + y^2 = 52$.
2. Solve for x^2.
3. Take the square root, keeping both signs.

Solve.

$x^2 + 6^2 = 52$	substitute y = 6
$x^2 + 36 = 52$	square the 6
$x^2 = 16$	subtract 36 from both sides
$x = \pm 4$	take the square root of both sides

Check. Both points must lie on the circle: $4^2 + 6^2 = 16 + 36 = 52$, and $(-4)^2 + 6^2 = 52$ as well. Two points check out.

Answer: x = 4 and x = -4.

Approach Read · Plan · Solve · Check

Read. Substitute the line's y-value into the circle equation.

Plan.

1. Replace y in the circle equation with the line's value b.
2. Rearrange to $x^2 = R^2 - b^2$.
3. Take the square root of both sides, keeping plus and minus.

Solve. Compute R^2 - b^2, then x = plus-or-minus its square root.

Check. Check that you subtracted b^2 correctly and kept both the positive and negative square root.

Common traps — every wrong answer is one of these

Using the line's value.
Reports x = plus-or-minus b, reusing the line's y-value instead of solving for x. Fix: substitute y = b, then solve $x^2 = R^2 - b^2$ for x.

Keeping only one point.
Gives a single x instead of both, forgetting the negative root. Fix: a square root gives two values, so x = +sqrt and x = -sqrt.

Using R-squared as x.
Reports R^2 itself (like 52) as an x-value without subtracting or taking a root. Fix: solve $x^2 = R^2 - b^2$ and then take the square root.

Practice

1. *The circle $x^2 + y^2 = 52$ meets the line $y = 6$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 4 and x = -4

(B) x = 6 and x = -6

(C) x = 52

(D) x = 4

2. *The circle $x^2 + y^2 = 25$ meets the line $y = 3$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 4

(B) x = 25

(C) x = 3 and x = -3

(D) x = 4 and x = -4

3. *The circle $x^2 + y^2 = 34$ meets the line $y = 3$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 5 and x = -5

(B) x = 3 and x = -3

(C) x = 5

(D) x = 34

4. *The circle $x^2 + y^2 = 61$ meets the line $y = 6$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 61

(B) x = 6 and x = -6

(C) x = 5

(D) x = 5 and x = -5

5. *The circle $x^2 + y^2 = 25$ meets the line $y = 3$. What are the x-coordinates of the intersection points?*

Which is correct?

(A) x = 3 and x = -3

(B) x = 4 and x = -4

(C) x = 5 and x = -5

(D) x = 4

Number of intersections (discriminant)

node: sat.nlsystems.X4

The skill. Once a system has been reduced to a single quadratic $ax^2 + bx + c = 0$, the discriminant b^2 - 4ac tells how many real intersection points there are: a positive value means two, zero means exactly one (the line just touches), and a negative value means none. To compute it, square b, then subtract 4 times a times c — it is a subtraction, and the 4 and the c both have to be there.

Worked example

A line-parabola system reduces to x^2 - 4x + 2 = 0. Find the discriminant b^2 - 4ac.

Read. The quadratic is in standard form $ax^2 + bx + c = 0$ with a = 1, b = -4, and c = 2. The discriminant will tell how many real intersections there are.

Plan.

1. Identify a = 1, b = -4, and c = 2.
2. Square b.
3. Subtract 4 times a times c.

Solve.

$$\begin{aligned} b^2 - 4ac &= (-4)^2 - 4(1)(2) && \text{substitute a, b, and c} \\ &= 16 - 8 && \text{square -4 and multiply out 4ac} \\ &= 8 && \text{subtract} \end{aligned}$$

Check. The result 8 is positive, so the system has two real intersection points, which fits a line cutting through a parabola.

Answer: the discriminant is 8.

Approach Read · Plan · Solve · Check

Read. Read a, b, and c off the reduced quadratic $ax^2 + bx + c = 0$.

Plan.

1. Identify a, b, and c.
2. Square b.
3. Subtract 4 times a times c.

Solve. Compute b^2 - 4ac; when a = 1 this is b^2 - 4c.

Check. Check that you subtracted (not added) and that you included both the 4 and the c.

Common traps — every wrong answer is one of these

Adding instead of subtracting.
Computes b^2 + 4ac instead of b^2 - 4ac. Fix: the discriminant subtracts 4ac, so for x^2 - 4x + 2 it is 16 - 8 = 8, not 16 + 8.

Dropping the c.
Uses b^2 - 4 instead of b^2 - 4c, leaving the constant out. Fix: multiply the 4 by a and by c — for x^2 - 4x + 2 that is 4(1)(2) = 8.

Dropping the 4.
Uses b^2 - c instead of b^2 - 4c, leaving the 4 out. Fix: the term is 4ac, so include the 4: b^2 - 4(1)(2) = 16 - 8 = 8.

Practice

1. *A line-parabola system reduces to x^2 - 4x + 2 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 12

(B) 8

(C) 24

(D) 14

2. *A line-parabola system reduces to x^2 - 6x + 3 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 32
(B) 33
(C) 24
(D) 48

3. *A line-parabola system reduces to x^2 - 4x + 3 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 28
(B) 12
(C) 13
(D) 4

4. *A line-parabola system reduces to x^2 - 5x + 2 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 33
(B) 17
(C) 23
(D) 21

5. *A line-parabola system reduces to x^2 - 6x + 5 = 0. What is the discriminant b^2 - 4ac?*

Which is correct?

(A) 31
(B) 56
(C) 16
(D) 36

Report intersection points (x and y)

node: sat.nlsystems.X5

The skill. A solution to a system is a whole point (x, y), not just an x-value. After you find each x, substitute it back into one of the equations — here $y = x^2$ — to get the matching y, then write the pair as (x, y) with x first and y second. Two x-values give two points. Do not report bare x-values, do not reuse x as if it were y, and do not swap the order of the coordinates.

Worked example

A line-parabola system has solutions $x = 1$ and $x = 4$. Using $y = x^2$, write the intersection points.

Read. We already have the two x-values. Each intersection is a point (x, y), and the parabola $y = x^2$ gives the y that goes with each x.

Plan.

1. Substitute $x = 1$ into $y = x^2$.
2. Substitute $x = 4$ into $y = x^2$.
3. Write each result as an ordered pair (x, y).

Solve.

$$y = (1)^2 = 1 \qquad \text{find y when } x = 1$$
$$y = (4)^2 = 16 \qquad \text{find y when } x = 4$$

Check. Both points lie on $y = x^2$: $1 = 1^2$ and $16 = 4^2$. Writing x first gives (1, 1) and (4, 16).
Answer: (1, 1) and (4, 16).

Approach Read · Plan · Solve · Check

Read. Each solution is a point (x, y), so every x needs its own y.

Plan.

1. Take each x-value you found.
2. Substitute it into $y = x^2$ to get y.
3. Write the pair as (x, y).

Solve. For each x, compute $y = x^2$ and pair them as (x, x^2).

Check. Check that you actually computed y for each x and wrote the coordinates in (x, y) order.

Common traps — every wrong answer is one of these

Reusing x as y.
Pairs (x, x) without computing y, so $x = 4$ becomes (4, 4). Fix: substitute into $y = x^2$ — $x = 4$ gives $y = 16$, so (4, 16).

Swapping the coordinates.
Writes the point as (y, x) instead of (x, y), like (16, 4) instead of (4, 16). Fix: x always comes first, then y.

Giving only the x-values.
Stops at $x = 1$ and $x = 4$ without turning them into points. Fix: each x needs its y, giving full points (1, 1) and (4, 16).

Practice

1. *A line-parabola system has solutions $x = 1$ and $x = 4$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (1, 1) and (16, 4)
(B) (1, 1) and (4, 16)
(C) (1, 1) and (4, 4)
(D) x = 1 and x = 4

2. *A line-parabola system has solutions $x = 2$ and $x = 4$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) x = 2 and x = 4
(B) (2, 4) and (4, 16)
(C) (4, 2) and (16, 4)
(D) (2, 2) and (4, 4)

3. *A line-parabola system has solutions $x = 1$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (1, 1) and (3, 9)
(B) (1, 1) and (3, 3)
(C) x = 1 and x = 3
(D) (1, 1) and (9, 3)

4. *A line-parabola system has solutions $x = 2$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (4, 2) and (9, 3)
(B) (2, 2) and (3, 3)
(C) x = 2 and x = 3
(D) (2, 4) and (3, 9)

5. *A line-parabola system has solutions $x = 2$ and $x = 3$. Using $y = x^2$, what are the intersection points?*

Which is correct?

(A) (2, 4) and (3, 9)

(B) (2, 2) and (3, 3)

(C) (4, 2) and (9, 3)

(D) x = 2 and x = 3

Nonlinear systems in context

node: sat.nlsystems.X6

The skill. If two numbers have a known sum S and a known product P, they are the two roots of the quadratic x^2 - Sx + P = 0. The sum comes in with a minus sign on the x-term, because a factored form (x - r1)(x - r2) expands to x^2 - (r1 + r2)x + r1 r2. The product becomes the constant term with a plus sign. So a sum of 7 and a product of 12 give x^2 - 7x + 12 = 0.

Worked example

Two numbers add to 7 and multiply to 12. Write the quadratic equation they satisfy.

Read. We know the sum, 7, and the product, 12, of two unknown numbers. Numbers with a given sum and product are the roots of x^2 - (sum)x + (product) = 0.

Plan.

1. Start with x^2.
2. Subtract the sum times x.
3. Add the product as the constant.

Solve.

$x^2 - Sx + P = 0$	the pattern for a known sum and product
$x^2 - 7x + 12 = 0$	put in S = 7 and P = 12

Check. This factors as (x - 3)(x - 4) = 0, and 3 + 4 = 7 with 3 times 4 = 12, matching the given sum and product.

Answer: x^2 - 7x + 12 = 0.

Approach Read · Plan · Solve · Check

Read. Turn the given sum and product into a quadratic whose roots are the two numbers.

Plan.

1. Write x^2 as the first term.
2. Put minus the sum on the x-term.
3. Put the product as the constant.
4. Set the whole thing equal to zero.

Solve. Assemble $x^2 - Sx + P = 0$ from the sum S and the product P.

Check. Check that the sum is negated on the x-term and the product is the plus constant.

Common traps — every wrong answer is one of these

Wrong sign on the sum.
Uses $+Sx$ instead of $-Sx$, writing $x^2 + 7x + 12 = 0$. Fix: the middle term is minus the sum, so $x^2 - 7x + 12 = 0$.

Swapping sum and product.
Puts the product on the x-term and the sum as the constant, like $x^2 - 12x + 7 = 0$. Fix: the sum goes on the x-term (negated) and the product is the constant.

Wrong sign on the product.
Writes $-P$ instead of $+P$, giving $x^2 - 7x - 12 = 0$. Fix: for two numbers with a positive product the constant is $+12$.

Practice

1. *Two numbers add to 7 and multiply to 12. Which equation do they satisfy?*

Which is correct?

(A) $x^2 - 12x + 7 = 0$
(B) $x^2 + 7x + 12 = 0$
(C) $x^2 - 7x + 12 = 0$
(D) $x^2 - 7x - 12 = 0$

2. *Two numbers add to 6 and multiply to 8. Which equation do they satisfy?*

Which is correct?

(A) $x^2 - 6x - 8 = 0$
(B) $x^2 - 6x + 8 = 0$
(C) $x^2 - 8x + 6 = 0$
(D) $x^2 + 6x + 8 = 0$

3. *Two numbers add to 8 and multiply to 15. Which equation do they satisfy?*

Which is correct?

(A) $x^2 + 8x + 15 = 0$
(B) $x^2 - 8x - 15 = 0$
(C) $x^2 - 15x + 8 = 0$

(D) x^2 - 8x + 15 = 0

4. *Two numbers add to 9 and multiply to 18. Which equation do they satisfy?*

Which is correct?

(A) x^2 - 9x - 18 = 0
(B) x^2 - 9x + 18 = 0
(C) x^2 + 9x + 18 = 0
(D) x^2 - 18x + 9 = 0

5. *Two numbers add to 9 and multiply to 20. Which equation do they satisfy?*

Which is correct?

(A) x^2 + 9x + 20 = 0
(B) x^2 - 9x - 20 = 0
(C) x^2 - 20x + 9 = 0
(D) x^2 - 9x + 20 = 0

Function transformations

node: sat.transform.T1

The skill. Every transformation starts from y = f(x). A change written outside the function, like f(x) + k or a*f(x), affects the output (the y-values) and does exactly what it looks like. A change written inside the parentheses, like f(x - h) or f(-x), affects the input (the x-values) and does the opposite of what the sign suggests. So f(x) + k slides the graph up by k, while f(x - h) slides it right by h — right, even though the sign is minus, because the input has to grow to make up for it. A minus outside, -f(x), flips the graph top-to-bottom across the x-axis; a minus inside, f(-x), flips it left-to-right across the y-axis. Multiplying the output, a*f(x), stretches the graph taller when a is bigger than 1 and squashes it flatter when a is between 0 and 1.

Worked example

The point (1, 4) lies on y = f(x). Find the corresponding point on g(x) = f(x + 2) - 3.

Read. We know (1, 4) is on f, so f(1) = 4. The rule has an inside + 2 and an outside - 3. The inside change moves the graph sideways, and the outside change moves it up and down. We want the matching point on g.

Plan.

1. The inside + 2 is a horizontal change and works backwards, so it shifts the graph left 2.
2. The outside - 3 is a vertical change and works as written, so it shifts the graph down 3.
3. Apply both to the point (1, 4).

Solve.

$$\begin{aligned} x_{\text{new}} &= 1 - 2 && \text{inside + 2 shifts the graph left 2} \\ &= -1 && \text{simplify} \\ y_{\text{new}} &= 4 - 3 && \text{outside - 3 shifts the graph down 3} \\ &= 1 && \text{simplify} \end{aligned}$$

Check. So (1, 4) moves to (-1, 1). Left 2 matches the inside + 2 running backwards, and down 3 matches the outside - 3 running as written.

Answer: the point is (-1, 1).

Approach Read · Plan · Solve · Check

Read. Decide whether the change is inside the parentheses (it acts on x, moves the graph sideways, and works backwards) or outside (it acts on y, moves the graph up and down, and works as written). Then decide whether it is a shift, a reflection, or a stretch.

Plan.

1. An outside + k slides the graph up by k, or down if k is negative.
2. An inside (x - h) slides the graph right by h; an inside (x + h) slides it left by h.
3. A minus outside, -f(x), reflects the graph across the x-axis.
4. A minus inside, f(-x), reflects the graph across the y-axis.
5. An outside multiplier a*f(x) stretches vertically when a is bigger than 1 and compresses when a is between 0 and 1.

Solve. For a combined rule, handle the inside change and the outside change one at a time.

Check. Check each piece: did the sideways move go opposite to the inside sign, did you keep inside (x) and outside (y) separate, is the reflection across the right axis, and does stretch versus compress match the size of a?

Common traps — every wrong answer is one of these

Horizontal shift goes the wrong way.
Reads f(x - h) as a shift left by h because of the minus sign, instead of a shift right. An inside change moves the graph opposite to its sign, so f(x - h) shifts right. Fix: for an inside change, flip the direction the sign suggests.

Inside and outside mixed up.
Confuses which change affects which variable — for example treating an outside + k as a sideways shift, or an inside change as an up-and-down shift, or mixing adding with scaling. Fix: outside changes act on y and move the graph vertically; inside changes act on x and move it horizontally.

Reflection across the wrong axis.
Swaps the axes, thinking -f(x) flips across the y-axis and f(-x) flips across the x-axis, or mistakes a reflection for an inverse. Fix: a minus outside flips across the x-axis; a minus inside flips across the y-axis.

Stretch and compression reversed.
Reverses stretch and compression for a*f(x) — for instance calling a bigger than 1 a compression — or confuses vertical scaling with horizontal scaling. Fix: with the multiplier outside, a bigger than 1 stretches vertically and a between 0 and 1 compresses.

Practice

1. *Let g(x) = f(x) + 3.*

The graph of g is the graph of f:

(A) stretched vertically by a factor of 3
(B) shifted up 3 units
(C) shifted left 3 units
(D) shifted right 3 units

2. *Let g(x) = f(x - 2).*

The graph of g is the graph of f:

(A) shifted right 2 units
(B) shifted down 2 units
(C) shifted left 2 units
(D) shifted up 2 units

3. *Let g(x) = -f(x).*

The graph of g is the graph of f reflected across:

(A) the origin
(B) the x-axis
(C) the line y = x
(D) the y-axis

4. *Let g(x) = 3*f(x).*

The graph of g is the graph of f:

(A) shifted up 3 units
(B) stretched vertically by a factor of 3

(C) stretched horizontally by a factor of 3

(D) compressed vertically by a factor of 3

5. *Let g(x) = f(x + 1) - 4.*

The graph of g is the graph of f:

(A) shifted left 1 unit and down 4 units

(B) shifted right 1 unit and down 4 units

(C) shifted left 1 unit and up 4 units

(D) shifted left 4 units and down 1 unit

Horizontal shifts

node: sat.transform.T2

The skill. The rule g(x) = f(x - h) changes the input, x, so it slides the graph sideways rather than up or down. Even though the sign is minus, the graph moves right by h, because x has to be h larger to feed the same number into f. In the same way, g(x) = f(x + h) slides the graph left by h. Inside changes are always horizontal and always move opposite to the sign; the changes that move the graph up or down are the ones written outside.

Worked example

The point (2, 5) lies on y = f(x). Find the corresponding point on g(x) = f(x - 6).

Read. The point (2, 5) on f means f(2) = 5. The - 6 is inside the parentheses, so it moves the graph sideways. We want the matching point on g.

Plan.

1. The change is inside the parentheses, so the graph moves sideways.
2. Minus inside means shift right, so add 6 to the x-coordinate.
3. The y-coordinate stays the same.

Solve.

$$\begin{aligned} x_{\text{new}} &= 2 + 6 && \text{minus inside shifts right 6} \\ &= 8 && \text{simplify} \\ y_{\text{new}} &= 5 && \text{a horizontal shift leaves y unchanged} \end{aligned}$$

Check. Check: g(8) = f(8 - 6) = f(2) = 5, so (8, 5) is on g. The graph moved right 6, matching a minus inside.

Answer: the point is (8, 5).

Approach Read · Plan · Solve · Check

Read. Confirm the change is inside the parentheses, so it moves the graph sideways.

Plan.

1. f(x - h) shifts the graph right by h.
2. f(x + h) shifts the graph left by h.

Solve. Move opposite to the sign inside the parentheses.

Check. Did you move opposite to the inside sign, and did you shift sideways rather than up or down?

Common traps — every wrong answer is one of these

Horizontal shift goes the wrong way.
Shifts left for f(x - h) instead of right. An inside change moves opposite to its sign. Fix: minus inside means shift right, plus inside means shift left.

Inside change treated as vertical.
Reads an inside change f(x - h) as an up-or-down shift instead of a sideways one. Fix: changes inside the parentheses always move the graph horizontally.

Inside change applied to the output.
Shifts the output up or down for what is really a horizontal, inside change. Fix: an inside change never moves the graph vertically.

Practice

1. *Let $g(x) = f(x - 3)$. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 3 units

(B) shifted up 3 units

(C) shifted down 3 units

(D) shifted left 3 units

2. *Let $g(x) = f(x - 4)$. The graph of g is the graph of f:*

Which is correct?

(A) shifted down 4 units

(B) shifted right 4 units

(C) shifted up 4 units

(D) shifted left 4 units

3. *Let g(x) = f(x - 5). The graph of g is the graph of f:*

Which is correct?

(A) shifted left 5 units
(B) shifted right 5 units
(C) shifted up 5 units
(D) shifted down 5 units

4. *Let g(x) = f(x - 2). The graph of g is the graph of f:*

Which is correct?

(A) shifted up 2 units
(B) shifted down 2 units
(C) shifted right 2 units
(D) shifted left 2 units

5. *Let g(x) = f(x - 5). The graph of g is the graph of f:*

Which is correct?

(A) shifted left 5 units
(B) shifted up 5 units
(C) shifted right 5 units
(D) shifted down 5 units

Reflection over the x-axis

node: sat.transform.T3

The skill. The rule g(x) = -f(x) puts a minus sign outside the function, so it changes the sign of the output. Every point keeps its x-coordinate but flips the sign of its y-coordinate: a point (a, b) becomes (a, -b). This turns the graph upside down across the x-axis. Compare it with g(x) = f(-x), where the minus is inside and flips the graph left-to-right across the y-axis instead. A minus on the outside always means a flip across the x-axis.

Worked example

The point (3, 5) lies on y = f(x). Find the corresponding point on g(x) = -f(x).

Read. The point (3, 5) on f means f(3) = 5. The minus sign is outside the function, so it acts on the output. We want the matching point on g.

Plan.

1. The minus is outside, so it acts on the output y.
2. Keep the x-coordinate 3.

3. Change the sign of the y-coordinate 5.

Solve.

$$x_{\text{new}} = 3 \qquad \text{an outside change leaves x unchanged}$$
$$y_{\text{new}} = -5 \qquad \text{negate the output}$$

Check. Check: g(3) = -f(3) = -5, so (3, -5) is on g. Only the y-coordinate flipped, which is a reflection across the x-axis.

Answer: the point is (3, -5).

Approach Read · Plan · Solve · Check

Read. Check that the minus sign is outside the function, so it acts on the output.

Plan.

1. Keep the x-coordinate the same.
2. Change the sign of the y-coordinate.
3. A point (a, b) becomes (a, -b).

Solve. Negate the y-coordinate of each point.

Check. Did you flip the y-coordinate and leave x alone?

Common traps — every wrong answer is one of these

Reflected across the wrong axis.
Flips across the y-axis by negating x, when -f(x) should flip across the x-axis by negating y. Fix: a minus outside changes the sign of y, not x.

Wrong coordinate changed.
Leaves the point unchanged or changes the x-coordinate instead of the y-coordinate. Fix: for -f(x), only the y-coordinate changes sign.

No reflection applied.
Leaves the point exactly as it was, forgetting to negate the output. Fix: -f(x) must flip the sign of the y-coordinate.

Practice

1. *The point (2, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (2, 2)
(B) (-2, -2)
(C) (2, -2)

(D) (-2, 2)

2. *The point (1, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (1, -2)
(B) (-1, 2)
(C) (1, 2)
(D) (-1, -2)

3. *The point (1, 3) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (-1, -3)
(B) (1, -3)
(C) (-1, 3)
(D) (1, 3)

4. *The point (3, 2) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (3, 2)
(B) (-3, -2)
(C) (3, -2)
(D) (-3, 2)

5. *The point (5, -3) lies on y = f(x). Under g(x) = -f(x), the corresponding point on g is:*

Which is correct?

(A) (-5, 3)
(B) (5, -3)
(C) (-5, -3)
(D) (5, 3)

Reflection over the y-axis

node: sat.transform.T4

The skill. The rule g(x) = f(-x) puts a minus sign inside the function, so it changes the sign of the input. Every point keeps its y-coordinate but flips the sign of its x-coordinate: a point (a, b) becomes (-a, b). This flips the graph left-to-right across the y-axis. Compare it with g(x) = -f(x),

where the minus is outside and flips the graph upside down across the x-axis. A minus on the inside always means a flip across the y-axis.

Worked example

The point (4, 2) lies on y = f(x). Find the corresponding point on g(x) = f(-x).

Read. The point (4, 2) on f means f(4) = 2. The minus sign is inside the function, so it acts on the input. We want the matching point on g.

Plan.

1. The minus is inside, so it acts on the input x.
2. Change the sign of the x-coordinate 4.
3. Keep the y-coordinate 2.

Solve.

$$x_{\text{new}} = -4 \qquad \text{negate the input}$$
$$y_{\text{new}} = 2 \qquad \text{an inside change leaves y unchanged}$$

Check. Check: g(-4) = f(-(-4)) = f(4) = 2, so (-4, 2) is on g. Only the x-coordinate flipped, a reflection across the y-axis.

Answer: the point is (-4, 2).

Approach Read · Plan · Solve · Check

Read. Check that the minus sign is inside the function, so it acts on the input.

Plan.

1. Keep the y-coordinate the same.
2. Change the sign of the x-coordinate.
3. A point (a, b) becomes (-a, b).

Solve. Negate the x-coordinate of each point.

Check. Did you flip the x-coordinate and leave y alone?

Common traps — every wrong answer is one of these

Reflected across the wrong axis.
Flips across the x-axis by negating y, when f(-x) should flip across the y-axis by negating x. Fix: a minus inside changes the sign of x, not y.

Wrong coordinate changed.
Leaves the point unchanged or changes the y-coordinate instead of the x-coordinate. Fix: for f(-x), only the x-coordinate changes sign.

Output negated instead of input.
Changes the sign of the output y instead of the input x, flipping across the wrong axis. Fix:

f(-x) negates the input, so the x-coordinate is the one that flips.

Practice

1. *The point (3, 2) lies on $y = f(x)$. Under $g(x) = f(-x)$, the corresponding point on g is:*

Which is correct?

(A) (3, 2)
(B) (3, -2)
(C) (-3, -2)
(D) (-3, 2)

2. *The point (1, 3) lies on $y = f(x)$. Under $g(x) = f(-x)$, the corresponding point on g is:*

Which is correct?

(A) (-1, -3)
(B) (-1, 3)
(C) (1, 3)
(D) (1, -3)

3. *The point (2, 3) lies on $y = f(x)$. Under $g(x) = f(-x)$, the corresponding point on g is:*

Which is correct?

(A) (-2, 3)
(B) (2, 3)
(C) (2, -3)
(D) (-2, -3)

4. *The point (3, 3) lies on $y = f(x)$. Under $g(x) = f(-x)$, the corresponding point on g is:*

Which is correct?

(A) (3, 3)
(B) (-3, -3)
(C) (3, -3)
(D) (-3, 3)

5. *The point (4, 5) lies on $y = f(x)$. Under $g(x) = f(-x)$, the corresponding point on g is:*

Which is correct?

(A) (-4, -5)

(B) (4, 5)

(C) (-4, 5)

(D) (4, -5)

Vertical stretch & compression

node: sat.transform.T5

The skill. The rule g(x) = a*f(x) multiplies the output by a, which stretches or squashes the graph in the up-and-down direction. When a is bigger than 1, every y-value gets larger, so the graph stretches taller by a factor of a. When a is between 0 and 1, every y-value gets smaller, so the graph is squashed flatter. Because the multiplier is outside the function, it always scales the graph vertically, never sideways.

Worked example

The point (2, 3) lies on y = f(x). Find the corresponding point on g(x) = 4f(x).

Read. The point (2, 3) on f means f(2) = 3. The multiplier 4 is outside the function, so it scales the output. We want the matching point on g.

Plan.

1. The multiplier 4 is outside, so it scales the output.
2. Keep the x-coordinate 2.
3. Multiply the y-coordinate 3 by 4.

Solve.

$x_{\text{new}} = 2$	an outside multiplier leaves x unchanged
$y_{\text{new}} = 4 \cdot 3$	multiply the output by 4
$= 12$	simplify

Check. Check: g(2) = 4f(2) = 4(3) = 12, so (2, 12) is on g. The point moved farther from the x-axis, a vertical stretch by 4.

Answer: the point is (2, 12).

Approach Read · Plan · Solve · Check

Read. Check that the multiplier is outside the function, so it scales the output.

Plan.

1. If a is bigger than 1, stretch the graph vertically by a factor of a.
2. If a is between 0 and 1, compress the graph vertically.
3. Multiply each y-value by a.

Solve. Multiply the output by a.

Check. Is the scaling vertical, and does a bigger than 1 give a stretch while a between 0 and 1 gives a compression?

Common traps — every wrong answer is one of these

Stretch and compression reversed.
Calls a bigger than 1 a compression, or treats the change as horizontal. Fix: with the multiplier outside, a bigger than 1 stretches the graph taller.

Vertical scaling treated as horizontal.
Reads the outside multiplier as a sideways change. Fix: an outside multiplier scales the graph vertically.

Scaling turned into a shift.
Shifts the graph up or down by a instead of multiplying the output by a. Fix: a*f(x) multiplies every y-value, it does not add a.

Practice

1. *Let g(x) = 3*f(x), where 3 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed vertically by a factor of 3
(B) stretched vertically by a factor of 3
(C) stretched horizontally by a factor of 3
(D) shifted up 3 units

2. *Let g(x) = 2*f(x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 2
(B) stretched horizontally by a factor of 2
(C) compressed vertically by a factor of 2
(D) shifted up 2 units

3. *Let g(x) = 4*f(x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 4
(B) compressed vertically by a factor of 4
(C) shifted up 4 units

(D) stretched horizontally by a factor of 4

4. *Let g(x) = 4*f(x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) stretched horizontally by a factor of 4

(B) stretched vertically by a factor of 4

(C) shifted up 4 units

(D) compressed vertically by a factor of 4

5. *Let g(x) = (1/2)*f(x). The graph of g is the graph of f:*

Which is correct?

(A) stretched vertically by a factor of 2

(B) compressed horizontally

(C) shifted down 2 units

(D) compressed vertically by a factor of 1/2

Horizontal stretch & compression

node: sat.transform.T6

The skill. The rule g(x) = f(b*x) multiplies the input by b, which stretches or squashes the graph in the sideways direction — and it works backwards. When b is bigger than 1, the graph is squeezed horizontally to 1/b of its width, because the input reaches each value b times faster. When b is between 0 and 1, the graph stretches wider. Because the multiplier is inside the function, it always scales the graph horizontally and in the opposite way from what you might expect.

Worked example

The point (6, 4) lies on y = f(x). Find the corresponding point on g(x) = f(3x).

Read. The point (6, 4) on f means f(6) = 4. The multiplier 3 is inside the function, so it scales the input and works backwards. We want the matching point on g.

Plan.

1. The multiplier 3 is inside, so it scales the input and works backwards.
2. Multiply the x-coordinate 6 by 1/3.
3. Keep the y-coordinate 4.

Solve.

$$\begin{aligned} x_{\text{new}} &= \frac{1}{3} \cdot 6 && \text{an inside multiplier scales x by } 1/b \\ &= 2 && \text{simplify} \\ y_{\text{new}} &= 4 && \text{a horizontal scaling leaves y unchanged} \end{aligned}$$

Check. Check: g(2) = f(3 \{}cdot 2) = f(6) = 4, so (2, 4) is on g. The point moved closer to the y-axis, a horizontal compression by 1/3.

Answer: the point is (2, 4).

Approach Read · Plan · Solve · Check

Read. Check that the multiplier is inside the function, so it scales the input.

Plan.

1. If b is bigger than 1, compress the graph horizontally by a factor of 1/b.
2. If b is between 0 and 1, stretch the graph horizontally.
3. The sideways scaling works backwards from the size of b.

Solve. Scale each x-coordinate by 1/b.

Check. Is the scaling horizontal, and does b bigger than 1 give a compression rather than a stretch?

Common traps — every wrong answer is one of these

Stretch and compression reversed.
Calls f(bx) with b bigger than 1 a horizontal stretch instead of a compression by 1/b. Fix: an inside multiplier bigger than 1 squeezes the graph horizontally.

Horizontal scaling treated as vertical.
Reads the inside multiplier as an up-and-down change. Fix: an inside multiplier scales the graph horizontally.

Scaling confused with a shift.
Treats the inside multiplier as a sideways shift instead of a stretch or compression. Fix: multiplying the input scales the graph, it does not slide it.

Practice

1. *Let g(x) = f(4x), where 4 > 1. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 4 units

(B) compressed horizontally by a factor of 1/4

(C) stretched horizontally by a factor of 4

(D) compressed vertically by a factor of 4

2. *Let g(x) = f(2x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 2 units

(B) stretched horizontally by a factor of 2

(C) compressed vertically by a factor of 2

(D) compressed horizontally by a factor of 1/2

3. *Let g(x) = f(3x), where 3 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed horizontally by a factor of 1/3

(B) stretched horizontally by a factor of 3

(C) shifted right 3 units

(D) compressed vertically by a factor of 3

4. *Let g(x) = f(2x), where 2 > 1. The graph of g is the graph of f:*

Which is correct?

(A) compressed vertically by a factor of 2

(B) shifted right 2 units

(C) stretched horizontally by a factor of 2

(D) compressed horizontally by a factor of 1/2

5. *Let g(x) = f((1/2)x). The graph of g is the graph of f:*

Which is correct?

(A) compressed horizontally by a factor of 1/2

(B) stretched vertically by a factor of 2

(C) shifted left 2 units

(D) stretched horizontally by a factor of 2

Combined transformations

node: sat.transform.T7

The skill. The rule g(x) = f(x - h) + k combines two shifts at once. The (x - h) is inside the function, so it moves the graph sideways — right by h, opposite to the minus sign. The + k is

outside the function, so it moves the graph up by k, exactly as written. Handle the inside and outside changes separately: the inside change controls the left-right motion and runs backwards, while the outside change controls the up-down motion and runs as written.

Worked example

The point (1, 2) lies on y = f(x). Find the corresponding point on g(x) = f(x - 4) + 3.

Read. The point (1, 2) on f means f(1) = 2. The inside (x - 4) moves the graph sideways and the outside + 3 moves it up and down. We want the matching point on g.

Plan.

1. The inside (x - 4) shifts the graph right 4, so add 4 to the x-coordinate.
2. The outside + 3 shifts the graph up 3, so add 3 to the y-coordinate.
3. Apply both to the point (1, 2).

Solve.

$$\begin{aligned} x_{\text{new}} &= 1 + 4 && \text{inside } (x - 4) \text{ shifts right } 4 \\ &= 5 && \text{simplify} \\ y_{\text{new}} &= 2 + 3 && \text{outside } + 3 \text{ shifts up } 3 \\ &= 5 && \text{simplify} \end{aligned}$$

Check. Check: g(5) = f(5 - 4) + 3 = f(1) + 3 = 2 + 3 = 5, so (5, 5) is on g. Right 4 matches the inside minus running backwards, and up 3 matches the outside plus running as written.

Answer: the point is (5, 5).

Approach Read · Plan · Solve · Check

Read. Split the rule into the inside change (sideways) and the outside change (up and down).

Plan.

1. The inside (x - h) shifts the graph right by h.
2. The outside + k shifts the graph up by k.
3. Combine the two shifts.

Solve. Apply the horizontal shift and the vertical shift one at a time.

Check. Did the inside shift run opposite to its sign, and did the outside shift run as written?

Common traps — every wrong answer is one of these

Horizontal shift goes the wrong way.
Shifts left instead of right for the inside (x - h). Fix: minus inside means shift right.

Horizontal and vertical mixed up.
Swaps which change is sideways and which is up-and-down, or reverses their directions. Fix:

the inside change is horizontal, the outside change is vertical.

Vertical shift goes the wrong way.
Moves the graph down for + k, or up for - k, reversing the outside shift. Fix: an outside + k shifts up and an outside - k shifts down.

Practice

1. *Let g(x) = f(x - 3) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 3 and up 2
(B) shifted left 3 and down 2
(C) shifted right 3 and down 2
(D) shifted left 3 and up 2

2. *Let g(x) = f(x - 2) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 2 and up 2
(B) shifted right 2 and down 2
(C) shifted left 2 and down 2
(D) shifted left 2 and up 2

3. *Let g(x) = f(x - 4) + 2. The graph of g is the graph of f:*

Which is correct?

(A) shifted right 4 and down 2
(B) shifted right 4 and up 2
(C) shifted left 4 and down 2
(D) shifted left 4 and up 2

4. *Let g(x) = f(x - 2) + 5. The graph of g is the graph of f:*

Which is correct?

(A) shifted left 2 and up 5
(B) shifted left 2 and down 5
(C) shifted right 2 and up 5
(D) shifted right 2 and down 5

5. *Let g(x) = f(x - 2) + 5. The graph of g is the graph of f:*

Which is correct?

(A) shifted left 2 and up 5

(B) shifted right 2 and up 5

(C) shifted up 2 and right 5

(D) shifted right 2 and down 5

Evaluating transformed functions

node: sat.transform.T8

The skill. To find the value of a transformed function at a point, start from the known value of f and then apply the change written in the rule. For g(x) = f(x) + k, you find g(a) by taking f(a) and adding k: g(a) = f(a) + k. The key is to use the given output f(a), not the input a, and to apply the outside operation to that output.

Worked example

If f(4) = 7 and g(x) = f(x) + 5, what is g(4)?

Read. We are told f(4) = 7 and that g adds 5 to the output of f. We want g(4).

Plan.

1. Use the given value f(4) = 7.
2. The rule adds 5 to the output.
3. Add 5 to 7.

Solve.

$$\begin{aligned} g(4) &= f(4) + 5 && \text{apply the rule at } x = 4 \\ &= 7 + 5 && \text{substitute } f(4) = 7 \\ &= 12 && \text{add} \end{aligned}$$

Check. Starting from the output 7 and adding the outside 5 gives 12; the input 4 was only used to look up f(4).

Answer: g(4) = 12.

Approach Read · Plan · Solve · Check

Read. Identify the rule for g in terms of f, and the value of f that is given.

Plan.

1. Look up the given value of f(a).
2. Apply the outside operation, such as adding k.
3. That result is g(a).

Solve. Substitute f(a) and carry out the outside operation.

Check. Did you start from f(a) rather than a, and apply the + k to that output?

Common traps — every wrong answer is one of these

Forgot to apply the change.
Reports f(a) by itself without adding the + k. Fix: g(a) = f(a) + k, so include the k.

Wrong operation or sign.
Subtracts k instead of adding, or applies it to the input instead of the output. Fix: add k to the output f(a).

Used the input instead of the output.
Uses the input a in place of f(a), computing a + k instead of f(a) + k. Fix: start from the given output f(a), not from a.

Practice

1. *If f(3) = 6 and g(x) = f(x) + 2, what is g(3)?*

Which is correct?

(A) 5
(B) 8
(C) 6
(D) 4

2. *If f(1) = 6 and g(x) = f(x) + 2, what is g(1)?*

Which is correct?

(A) 6
(B) 3
(C) 4
(D) 8

3. *If f(3) = 5 and g(x) = f(x) + 3, what is g(3)?*

Which is correct?

(A) 2
(B) 6
(C) 8
(D) 5

4. *If f(3) = 8 and g(x) = f(x) + 2, what is g(3)?*

Which is correct?

(A) 10
(B) 6
(C) 8
(D) 5

5. *If f(5) = 10 and g(x) = f(x) - 3, what is g(5)?*

Which is correct?

(A) 7
(B) 2
(C) 13
(D) 10

Complex numbers: arithmetic with i

node: sat.complex.C1

The skill. The imaginary unit i is the number whose square is -1, so $i^2 = -1$ and i is the square root of -1. This lets you take square roots of negatives: the square root of -9 is 3i, because $(3i)^2 = 9 * (-1) = -9$. A complex number a + bi has a real part a and an imaginary part b, and the two parts stay separate. To add or subtract, combine real parts with real parts and imaginary parts with imaginary parts. To multiply, distribute every term times every term (FOIL), then replace each i^2 with -1 and recombine. The real and imaginary parts never merge into a single number.

Worked example

Multiply (2 + i)(3 + i).

Read. We multiply two complex numbers. After distributing all four products, one term will contain i^2, which we replace with -1.

Plan.

1. Multiply all four pairs of terms with FOIL.
2. Replace i^2 with -1.
3. Combine the real parts and the imaginary parts separately.

Solve.

$$\begin{aligned} (2+i)(3+i) &= 6 + 2i + 3i + i^2 && \text{multiply all four pairs} \\ &= 6 + 5i + (-1) && \text{add the middle terms and replace } i^2 \text{ with } -1 \\ &= 5 + 5i && \text{combine the real parts} \end{aligned}$$

Check. The real part is 6 - 1 = 5 and the imaginary part is 2 + 3 = 5, giving 5 + 5i. The i^2 term correctly lowered the real part by 1.

Answer: 5 + 5i.

Approach Read · Plan · Solve · Check

Read. Decide which kind of problem this is: adding or subtracting (combine like parts), multiplying (distribute, then use $i^2 = -1$), or a square root of a negative (rewrite it using i).

Plan.

1. To add or subtract, combine the real parts together and the imaginary parts together.
2. To multiply, distribute all the products, then replace every i^2 with -1.
3. For the square root of a negative, factor out i as the square root of -1.

Solve.

1. Carry out the operation term by term.
2. Wherever an i^2 appears, replace it with -1.
3. Recombine into a single real part and a single imaginary part.

Check. Check that every i^2 became -1, that real and imaginary parts stayed separate, that all four products were included when multiplying, and that a square root of a negative was written with i.

Common traps — every wrong answer is one of these

i-squared sign error.
Fails to use $i^2 = -1$: treats i^2 as +1, as 0, or leaves it as i, which throws off the real part of the answer. Every i^2 must become -1.

combine real/imaginary error.
Merges the real and imaginary parts as if they were like terms, for example writing 3 + 2i as 5i or as 5. The two parts stay separate.

multiplication/FOIL error.
Mishandles the distribution when multiplying, dropping one of the cross terms or a product. Every term of the first number must multiply every term of the second.

square-root-of-negative error.
Mishandles the square root of a negative — returning a negative real, forgetting to take the root, or calling it undefined — instead of writing it as a multiple of i. The square root of -9 is 3i.

Practice

1. *Recall that i is the imaginary unit.*

What is i^2?

(A) -i

(B) 1

(C) -1

(D) i

2. *Add the complex numbers (3 + 2i) and (1 + 4i).*

What is the sum?

(A) 6 + 4i

(B) 10i

(C) 4 + 6i

(D) 10

3. *Multiply (2 + i)(3 + i).*

What is the product?

(A) 5

(B) 7 + 5i

(C) 6 + 5i

(D) 5 + 5i

4. *Simplify $\sqrt{-9}$.*

What is the result?

(A) undefined

(B) 9i

(C) 3i

(D) -3

5. *Expand $(1 + i)^2$.*

What is the result?

(A) 2 + 2i

(B) 0

(C) 1 + 2i

(D) 2i

Powers of i & complex division

node: sat.complex.C2

The skill. The powers of i repeat in a cycle of four: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, and then the pattern starts over. To find i^n, divide n by 4 and use only the remainder: remainder 1 gives i, 2 gives -1, 3 gives -i, and 0 gives 1. The conjugate of a + bi is a - bi — you flip the sign of just the imaginary part. Multiplying a number by its conjugate gives a real result, since $(a + bi)(a - bi) = a^2 + b^2$. This is the trick for dividing complex numbers: multiply the top and bottom by the conjugate of the bottom, which turns the denominator into a real number you can divide by.

Worked example

Write (1 + 2i) / (1 - i) in the form a + bi.

Read. To divide complex numbers we clear the i out of the denominator. The conjugate of the denominator 1 - i is 1 + i, so we multiply top and bottom by 1 + i.

Plan.

1. Multiply the numerator and denominator by the conjugate of the denominator, 1 + i.
2. Expand both products, replacing i^2 with -1.
3. Divide each part by the real denominator.

Solve.

$$\begin{aligned} \frac{1+2i}{1-i} &= \frac{(1+2i)(1+i)}{(1-i)(1+i)} && \text{multiply by the conjugate over itself} \\ &= \frac{1+3i+2i^2}{1-i^2} && \text{expand both products} \\ &= \frac{-1+3i}{2} && \text{replace } i^2 \text{ with } -1 \\ &= -\frac{1}{2}+\frac{3}{2}i && \text{divide each part by 2} \end{aligned}$$

Check. The denominator (1 - i)(1 + i) = 1 + 1 = 2 is real, as intended. Splitting -1 + 3i over 2 gives the real part -1/2 and the imaginary part 3/2.

Answer: -1/2 + (3/2)i.

Approach Read · Plan · Solve · Check

Read. Decide which kind of problem this is: a power of i (use the four-step cycle), a conjugate (flip the imaginary sign), or a division (multiply by the conjugate of the bottom).

Plan.

1. For a power of i, divide the exponent by 4 and keep the remainder.
2. Match the remainder to its value: 1 gives i, 2 gives -1, 3 gives -i, 0 gives 1.
3. For a conjugate, change only the sign of the imaginary part.
4. For a division, multiply the numerator and denominator by the denominator's conjugate.

Solve.

1. Apply the cycle, form the conjugate, or carry out the conjugate multiplication.
2. Replace any i^2 with -1 while simplifying.
3. Divide through by the real denominator to reach the form a + bi.

Check. Check that you used the remainder after dividing by 4, that the conjugate flipped only the imaginary part, and that you multiplied by the conjugate — not the denominator itself — when dividing.

Common traps — every wrong answer is one of these

power-cycle error.
Maps i^n to the wrong value, such as $i^3 = i$ or $i^4 = -1$, by mishandling the four-step cycle or its remainder. The cycle is i, -1, -i, 1.

conjugate error.
Forms the wrong conjugate — flipping the real part, or both parts — or mishandles the conjugate product so an i term is left where the result should be real. The conjugate flips only the imaginary part.

division-method error.
Divides without multiplying by the denominator's conjugate, or multiplies by the denominator itself, or forgets to divide through after simplifying. Multiplying top and bottom by the conjugate is what makes the denominator real.

Practice

1. *Recall the powers of the imaginary unit.*

What is i^3?

(A) -1
(B) i
(C) 1
(D) -i

2. *Use the period-4 cycle of powers of i.*

What is i^{10}?

(A) -1
(B) i
(C) 1
(D) -i

3. *Consider the complex number 3 - 5i.*

What is its conjugate?

(A) -3 + 5i
(B) -3 - 5i
(C) 5 - 3i
(D) 3 + 5i

4. *Multiply (2 + 3i)(2 - 3i).*

What is the product?

(A) -5
(B) 4 - 6i
(C) 13
(D) 4 + 9i

5. *Simplify (1 + 2i) / (1 - i).*

What is the result in the form a + bi?

(A) (1 + 2i)/2
(B) -1 + 3i
(C) -1/2 + (3/2)i
(D) It cannot be simplified

Add & subtract complex numbers

node: sat.complex.C3

The skill. Complex numbers add and subtract one piece at a time: (a + bi) + (c + di) = (a + c) + (b + d)i. You combine the real parts with the real parts and the imaginary parts with the imaginary parts, like keeping two separate columns. The result keeps both a real part and an imaginary part — they never collapse into a single number, and the i-parts are added, not multiplied.

Worked example

Simplify (7 + 2i) + (1 + 5i).

Read. We add two complex numbers. The real parts are 7 and 1; the imaginary parts are 2 and 5. Each pair combines on its own.

Plan.

1. Add the real parts, 7 and 1.
2. Add the imaginary parts, 2 and 5.
3. Write the two results as one complex number.

Solve.

$$(7+2i)+(1+5i) = (7+1)+(2+5)i \quad \text{group real with real and imaginary with imaginary}$$
$$= 8+7i \quad \text{add each group}$$

Check. The real part 8 and the imaginary part 7 stay separate, and the imaginary coefficients were added ($2+5=7$), not multiplied.

Answer: 8 + 7i.

Approach Read · Plan · Solve · Check

Read. Pick out the real parts and the imaginary parts of the two numbers so you can combine matching pieces.

Plan.

1. Add the real part of one to the real part of the other.
2. Add the imaginary part of one to the imaginary part of the other.

Solve.

1. Combine the reals into a + c.
2. Combine the imaginaries into (b + d)i.
3. Write the result as (a + c) + (b + d)i.

Check. Check that the real and imaginary parts stayed separate and that you added the i-parts rather than multiplying them.

Common traps — every wrong answer is one of these

merged real & imaginary.
Adds all four numbers into a single real value, collapsing the real and imaginary parts together. They must stay separate.

swapped the parts.
Puts the imaginary sum in the real slot and the real sum in the imaginary slot, swapping the two parts.

multiplied the i-parts.
Multiplies the imaginary coefficients instead of adding them. For a sum, the imaginary parts add.

Practice

1. *Simplify (3 + 1i) + (5 + 4i).*

Which is correct?

(A) 8 + 5i
(B) 5 + 8i
(C) 13
(D) 8 + 4i

2. *Simplify (3 + 2i) + (5 + 4i).*

Which is correct?

(A) 14
(B) 8 + 6i
(C) 8 + 8i
(D) 6 + 8i

3. *Simplify (4 + 3i) + (2 + 2i).*

Which is correct?

(A) 6 + 5i
(B) 11
(C) 5 + 6i
(D) 6 + 6i

4. *Simplify (4 + 1i) + (1 + 3i).*

Which is correct?

(A) 5 + 3i
(B) 9
(C) 4 + 5i
(D) 5 + 4i

5. *Simplify (7 + 2i) + (1 + 5i).*

Which is correct?

(A) 8 + 10i
(B) 7 + 8i
(C) 8 + 7i
(D) 15

Multiply complex numbers (FOIL with i)

node: sat.complex.C4

The skill. To multiply two complex numbers, use FOIL to get all four products, then replace i^2 with -1: (a + bi)(c + di) = ac + adi + bci + bd*i^2 = (ac - bd) + (ad + bc)i. Because the last product carries $i^2 = -1$, that term flips sign, so the real part is ac - bd, not ac + bd. The imaginary part collects both cross terms, ad + bc.

Worked example

Multiply (1 + 2i)(4 + 3i).

Read. We multiply two complex numbers with FOIL. The final product 2i times 3i contains i^2, which we replace with -1.

Plan.

1. Multiply all four pairs of terms.
2. Replace i^2 with -1.
3. Combine the real parts and the imaginary parts.

Solve.

$$\begin{aligned} (1+2i)(4+3i) &= 4+3i+8i+6i^2 && \text{multiply all four pairs} \\ &= 4+11i+6(-1) && \text{add the cross terms and replace } i^2 \\ &= -2+11i && \text{combine the real parts} \end{aligned}$$

Check. The real part is 4 - 6 = -2 and the imaginary part is 3 + 8 = 11. Both cross terms were used, and i^2 correctly subtracted 6.

Answer: -2 + 11i.

Approach Read · Plan · Solve · Check

Read. You are multiplying two complex binomials, so use FOIL and watch the term that contains i^2.

Plan.

1. Multiply all four pairs of terms.
2. Replace i^2 with -1 in the last product.
3. Collect the real parts and the imaginary parts separately.

Solve.

1. The real part is ac - bd, since the i^2 term became negative.
2. The imaginary part is ad + bc, adding both cross terms.

Check. Check that the i^2 term turned into -1 and that both middle (cross) terms were included in the imaginary part.

Common traps — every wrong answer is one of these

i-squared as +1.
Leaves the last term positive, giving a real part of ac + bd instead of ac - bd. The i^2 makes that term subtract.

straight-across product.
Multiplies only real times real and i times i, skipping the two cross terms, so the imaginary part goes missing.

one cross term.
Uses only one of the two middle products for the imaginary part, leaving out the other cross term. The imaginary part is ad + bc.

Practice

1. *Simplify (2 + 3i)(3 + 1i).*

Which is correct?

(A) 3 + 2i
(B) 3 + 11i
(C) 6 + 3i
(D) 9 + 11i

2. *Simplify (3 + 3i)(3 + 1i).*

Which is correct?

(A) 6 + 3i
(B) 6 + 12i
(C) 12 + 12i
(D) 9 + 3i

3. *Simplify (3 + 1i)(1 + 1i).*

Which is correct?

(A) 2 + 4i
(B) 3 + 1i
(C) 2 + 3i
(D) 4 + 4i

4. *Simplify (1 + 2i)(1 + 1i).*

Which is correct?

(A) -1 + 1i

(B) -1 + 3i

(C) 3 + 3i

(D) 1 + 2i

5. *Simplify (1 + 2i)(4 + 3i).*

Which is correct?

(A) 10 + 11i

(B) 4 + 6i

(C) -2 + 11i

(D) -2 + 3i

Complex conjugates & division

node: sat.complex.C5

The skill. Multiplying a complex number by its conjugate always gives a real number. Writing (a + bi)(a - bi) as a difference of squares gives a^2 - $(bi)^2$, and since $i^2 = -1$, the term $-(bi)^2$ becomes $+b^2$. So $(a + bi)(a - bi) = a^2 + b^2$, with the imaginary parts cancelling. This real result is exactly what makes the conjugate the right thing to multiply by when dividing complex numbers.

Worked example

Multiply (5 + 2i)(5 - 2i).

Read. These are a conjugate pair — the same number with the sign of the imaginary part flipped. Their product should come out to a real number.

Plan.

1. Multiply as a difference of squares, a^2 - $(bi)^2$.
2. Replace i^2 with -1.

Solve.

$$\begin{aligned} (5+2i)(5-2i) &= 5^2 - (2i)^2 && \text{multiply the conjugate pair as a difference of squares} \\ &= 25 - 4i^2 && \text{square each term} \\ &= 25 + 4 && \text{replace } i^2 \text{ with } -1 \\ &= 29 && \text{add} \end{aligned}$$

Check. The imaginary parts cancelled and $i^2 = -1$ turned $-4i^2$ into $+4$, giving the real number $a^2 + b^2 = 25 + 4 = 29$.

Answer: 29.

Approach Read · Plan · Solve · Check

Read. You are multiplying a conjugate pair, (a + bi)(a - bi), the same number with the imaginary sign flipped.

Plan.

1. Multiply it out as a difference of squares.
2. Simplify the i^2 term using $i^2 = -1$.

Solve.

1. The middle terms cancel, leaving $a^2 - b^2\, i^2$.
2. Since $i^2 = -1$, this becomes $a^2 + b^2$.

Check. Check that the $-b^2\, i^2$ term turned into $+b^2$ and that the result is a real number with no i left.

Common traps — every wrong answer is one of these

i-squared as +1.
Computes $a^2 - b^2$ instead of $a^2 + b^2$, forgetting that $i^2 = -1$ turns the last term positive.

dropped the b^2 term.
Reports only a^2, losing the $+b^2$ that comes from the conjugate product.

left an imaginary unit.
Keeps an i on the answer, even though a conjugate product is always a real number.

Practice

1. *What is the product (4 + 3i)(4 - 3i)?*

Which is correct?

(A) 16
(B) 7
(C) 25i
(D) 25

2. *What is the product (4 + 1i)(4 - 1i)?*

Which is correct?

(A) 16
(B) 17i
(C) 15
(D) 17

3. *What is the product (2 + 3i)(2 - 3i)?*

Which is correct?

(A) -5
(B) 13
(C) 13i
(D) 4

4. *What is the product (3 + 1i)(3 - 1i)?*

Which is correct?

(A) 10
(B) 8
(C) 9
(D) 10i

5. *What is the product (5 + 2i)(5 - 2i)?*

Which is correct?

(A) 21
(B) 25 - 4i
(C) 25
(D) 29

Complex solutions of quadratics

node: sat.complex.C6

The skill. When a quadratic leads to the square root of a negative number, its solutions are complex rather than real. For an equation like $x^2 + k = 0$, move the constant to get $x^2 = -k$, then take the square root of both sides: x = plus or minus the square root of -k, which equals plus or minus the square root of k times i. A quadratic that has 'no real solution' still has these two complex solutions. The key steps are taking the square root of k (not leaving k as it is) and attaching the i.

Worked example

Solve $x^2 + 9 = 0$, allowing complex solutions.

Read. Solving this will set x^2 equal to a negative number, so the answers will be complex. We take a square root and attach i.

Plan.

1. Subtract 9 from both sides to isolate x^2.
2. Take the square root of both sides, keeping plus-or-minus.

3. Rewrite the square root of -9 using i.

Solve.

$$\begin{aligned} x^2 + 9 &= 0 && \text{the original equation} \\ x^2 &= -9 && \text{subtract 9 from both sides} \\ x &= \pm\sqrt{-9} && \text{take the square root of both sides} \\ &= \pm 3i && \text{the square root of -9 is 3i} \end{aligned}$$

Check. Squaring 3i gives $(3i)^2 = 9i^2 = -9$, and adding 9 returns 0. Both x = 3i and x = -3i work.

Answer: x = 3i or x = -3i.

Approach Read · Plan · Solve · Check

Read. Check whether solving the equation sets x^2 equal to a negative number, which signals complex solutions.

Plan.

1. Move the constant to the other side to get x^2 by itself.
2. Take the square root of both sides, keeping the plus-or-minus.
3. Write the square root of the negative as the square root of the positive times i.

Solve.

1. Take the square root of k for the size of the answer.
2. Attach i to make it imaginary.
3. Keep both the plus and the minus root.

Check. Check that you took the square root of k rather than using k itself, that you attached i, and that you kept both roots.

Common traps — every wrong answer is one of these

dropped the imaginary unit.
Solves $x^2 = k$ instead of $x^2 = -k$, giving real answers plus or minus the square root of k with no i. The negative under the root is what makes the answer imaginary.

stopped at 'no solution'.
Stops at 'no real solution' instead of continuing to find the complex roots. A negative under the square root still has complex solutions.

forgot the square root.
Uses k itself as the coefficient of i instead of the square root of k. For $x^2 + 9 = 0$ the answer is 3i, not 9i.

Practice

1. *Solve $x^2 + 25 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 5i or x = -5i

(B) x = 5 or x = -5

(C) x = 25i or x = -25i

(D) No real or complex solution

2. *Solve $x^2 + 16 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 4i or x = -4i

(B) No real or complex solution

(C) x = 16i or x = -16i

(D) x = 4 or x = -4

3. *Solve $x^2 + 9 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 9i or x = -9i

(B) x = 3i or x = -3i

(C) x = 3 or x = -3

(D) No real or complex solution

4. *Solve $x^2 + 4 = 0$ (allow complex solutions).*

Which is correct?

(A) x = 2 or x = -2

(B) x = 2i or x = -2i

(C) x = 4i or x = -4i

(D) No real or complex solution

5. *Solve $x^2 + 9 = 0$ (allow complex solutions).*

Which is correct?

(A) x = ±3

(B) x = ±3i

(C) x = ±9i

(D) no real solution

Answer Key — Advanced Math

Evaluate & interpret function notation f(x) — Q1
Correct: **(B)**. *Traps:* **(A)** Reading f(x) as multiplication; **(C)** Reading f(x) as multiplication; **(D)** Swapping input and output.

Evaluate & interpret function notation f(x) — Q2
Correct: **(C)**. *Traps:* **(A)** Moving a term without flipping its sign; **(B)** Reading f(x) as multiplication; **(D)** Moving a term without flipping its sign.

Evaluate & interpret function notation f(x) — Q3
Correct: **(A)**. *Traps:* **(B)** Swapping input and output; **(C)** Swapping input and output; **(D)** Swapping input and output.

Evaluate & interpret function notation f(x) — Q4
Correct: **(A)**. *Traps:* **(B)** Confusing f(x + h) with f(x) + h; **(C)** Reading f(x) as multiplication; **(D)** Forgetting to distribute to every term.

Evaluate & interpret function notation f(x) — Q5
Correct: **(A)**. *Traps:* **(B)** Swapping input and output; **(C)** Reading f(x) as multiplication; **(D)** Swapping input and output.

Expand binomials / FOIL — Q1
Correct: **(A)**. *Traps:* **(B)** Swapping the sum and the product; **(C)** Swapping the sum and the product; **(D)** Forgetting the middle term.

Expand binomials / FOIL — Q2
Correct: **(B)**. *Traps:* **(A)** Forgetting the middle term; **(C)** Sign slip in a product; **(D)** Sign slip in a product.

Expand binomials / FOIL — Q3
Correct: **(D)**. *Traps:* **(A)** Forgetting the middle term; **(B)** Not doubling the middle of a square; **(C)** Swapping the sum and the product.

Expand binomials / FOIL — Q4
Correct: **(A)**. *Traps:* **(B)** Sign slip in a product; **(C)** Sign slip in a product; **(D)** Forgetting to distribute to every term.

Expand binomials / FOIL — Q5
Correct: **(C)**. *Traps:* **(A)** Not doubling the middle of a square; **(B)** Sign slip in a product; **(D)** Sign slip in a product.

Factor quadratics — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong signs in the factors; **(B)** Right product, wrong sum; **(C)** Right product, wrong sum.

Factor quadratics — Q2
Correct: **(B)**. *Traps:* **(A)** Right product, wrong sum; **(C)** Right product, wrong sum; **(D)** Wrong signs in the factors.

Factor quadratics — Q3
Correct: **(D)**. *Traps:* **(A)** Mishandling the leading coefficient or a common factor; **(B)** Mishandling the leading coefficient or a common factor; **(C)** Wrong signs in the factors.

Factor quadratics — Q4
Correct: **(D)**. *Traps:* **(A)** Mis-factoring a special form; **(B)** Right product, wrong sum; **(C)** Mis-factoring a special form.

Factor quadratics — Q5
Correct: **(C)**. *Traps:* **(A)** Mishandling the leading coefficient or a common factor; **(B)** Mis-factoring a special form; **(D)** Mis-factoring a special form.

Solve quadratics by factoring & the zero-product property — Q1
Correct: **(C)**. *Traps:* **(A)** Moving a term without flipping its sign; **(B)** Right product, wrong sum; **(D)** Losing a solution.

Solve quadratics by factoring & the zero-product property — Q2
Correct: **(D)**. *Traps:* **(A)** Using the zero-product property without a zero; **(B)** Using the zero-product property without a zero; **(C)** Moving a term without flipping its sign.

Solve quadratics by factoring & the zero-product property — Q3
Correct: **(D)**. *Traps:* **(A)** Dividing both sides by x; **(B)** Losing a solution; **(C)** Moving a term without flipping its sign.

Solve quadratics by factoring & the zero-product property — Q4
Correct: **(C)**. *Traps:* **(A)** Moving a term without flipping its sign; **(B)** Losing a solution; **(D)** Dividing the wrong way.

Solve quadratics by factoring & the zero-product property — Q5
Correct: **(A)**. *Traps:* **(B)** Right product, wrong sum; **(C)** Moving a term without flipping its sign; **(D)** Losing a solution.

Quadratic formula & discriminant (number of real solutions) — Q1
Correct: **(B)**. *Traps:* **(A)** Breaking the structure of the formula; **(C)** Sign slip substituting into the formula; **(D)** Breaking the structure of the formula.

Quadratic formula & discriminant (number of real solutions) — Q2
Correct: **(B)**. *Traps:* **(A)** Misreading the discriminant count; **(C)** Misreading the discriminant count; **(D)** Misreading the discriminant count.

Quadratic formula & discriminant (number of real solutions) — Q3
Correct: **(C)**. *Traps:* **(A)** Misreading the discriminant count; **(B)** Misreading the discriminant count; **(D)** Misreading the discriminant count.

Quadratic formula & discriminant (number of real solutions) — Q4
Correct: **(C)**. *Traps:* **(A)** Breaking the structure of the formula; **(B)** Sign slip substituting into the formula; **(D)** Sign slip substituting into the formula.

Quadratic formula & discriminant (number of real solutions) — Q5
Correct: **(B)**. *Traps:* **(A)** Misreading the discriminant count; **(C)** Breaking the structure of the formula; **(D)** Dividing the wrong way.

Forms of a quadratic & the parabola — Q1
Correct: **(D)**. *Traps:* **(A)** Flipping the sign of h in vertex form; **(B)** Reading the wrong feature off a form; **(C)** Reading the wrong feature off a form.

Forms of a quadratic & the parabola — Q2
Correct: **(B)**. *Traps:* **(A)** Reading the wrong feature off a form; **(C)** Wrong axis from standard form; **(D)** Wrong axis from standard form.

Forms of a quadratic & the parabola — Q3
Correct: **(B)**. *Traps:* **(A)** Reversing open-up and open-down; **(C)** Reversing open-up and open-down; **(D)** Reading the wrong feature off a form.

Forms of a quadratic & the parabola — Q4
Correct: **(A)**. *Traps:* **(B)** Losing a solution; **(C)** Moving a term without flipping its sign; **(D)** Reading the wrong feature off a form.

Forms of a quadratic & the parabola — Q5
Correct: **(B)**. *Traps:* **(A)** Reading the wrong feature off a form; **(C)** Misreading the discriminant count; **(D)** Misreading the discriminant count.

Interpret a quadratic model in context — Q1
Correct: **(C)**. *Traps:* **(A)** Confusing how much with when; **(B)** Wrong quantity or wrong units; **(D)** Confusing how much with when.

Interpret a quadratic model in context — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong quantity or wrong units; **(C)** Confusing how much with when; **(D)** Misreading which zero is the event.

Interpret a quadratic model in context — Q3
Correct: **(A)**. *Traps:* **(B)** Misreading which zero is the event; **(C)** Misreading which zero is the event; **(D)** Misreading which zero is the event.

Interpret a quadratic model in context — Q4
Correct: **(C)**. *Traps:* **(A)** Reading the wrong feature off a form; **(B)** Confusing how much with when; **(D)** Wrong quantity or wrong units.

Interpret a quadratic model in context — Q5
Correct: **(A)**. *Traps:* **(B)** Confusing how much with when; **(C)** Confusing how much with when; **(D)** Misreading which zero is the event.

Completing the square — Q1
Correct: **(A)**. *Traps:* **(B)** Not halving before squaring; **(C)** Not halving before squaring; **(D)** Not halving before squaring.

Completing the square — Q2
Correct: **(A)**. *Traps:* **(B)** Not balancing the added constant; **(C)** Wrong sign of h in the vertex; **(D)** Not balancing the added constant.

Completing the square — Q3
Correct: **(C)**. *Traps:* **(A)** Wrong sign of h in the vertex; **(B)** Wrong sign of h in the vertex; **(D)** Wrong sign of h in the vertex.

Completing the square — Q4
Correct: **(D)**. *Traps:* **(A)** Ignoring the leading coefficient; **(B)** Ignoring the leading coefficient; **(C)** Ignoring the leading coefficient.

Completing the square — Q5
Correct: **(C)**. *Traps:* **(A)** Not halving before squaring; **(B)** Not balancing the added constant; **(D)** Wrong sign of h in the vertex.

Composite functions & evaluating compositions — Q1
Correct: **(C)**. *Traps:* **(A)** Reading the wrong value from a table; **(B)** Reading the wrong value from a table; **(D)** Working outside-in instead of inside-out.

Composite functions & evaluating compositions — Q2
Correct: **(C)**. *Traps:* **(A)** Multiplying instead of substituting; **(B)** Multiplying instead of substituting; **(D)** Working outside-in instead of inside-out.

Composite functions & evaluating compositions — Q3
Correct: **(C)**. *Traps:* **(A)** Reading the wrong value from a table; **(B)** Reading the wrong value from a table; **(D)** Reading the wrong value from a table.

Composite functions & evaluating compositions — Q4
Correct: **(B)**. *Traps:* **(A)** Assuming the order does not matter; **(C)** Assuming the order does not matter; **(D)** Assuming the order does not matter.

Composite functions & evaluating compositions — Q5
Correct: **(A)**. *Traps:* **(B)** Multiplying instead of substituting; **(C)** Reading the wrong value from a table; **(D)** Working outside-in instead of inside-out.

Domain & range of functions — Q1
Correct: **(A)**. *Traps:* **(B)** Looking for the restriction in the wrong place; **(C)** Looking for the restriction in the wrong place; **(D)** Looking for the restriction in the wrong place.

Domain & range of functions — Q2
Correct: **(D)**. *Traps:* **(A)** Swapping domain and range; **(B)** Ignoring the vertex bound on the range; **(C)** Swapping domain and range.

Domain & range of functions — Q3
Correct: **(B)**. *Traps:* **(A)** Ignoring the vertex bound on the range; **(C)** Ignoring the vertex bound on the range; **(D)** Ignoring the vertex bound on the range.

Domain & range of functions — Q4
Correct: **(D)**. *Traps:* **(A)** Looking for the restriction in the wrong place; **(B)** Looking for the restriction in the wrong place; **(C)** Looking for the restriction in the wrong place.

Domain & range of functions — Q5
Correct: **(C)**. *Traps:* **(A)** Misreading the extent from a graph; **(B)** Swapping domain and range; **(D)** Swapping domain and range.

Piecewise-defined functions — Q1
Correct: **(A)**. *Traps:* **(B)** Evaluating every piece; **(C)** Using the wrong interval's rule; **(D)** Assigning a boundary to the wrong piece.

Piecewise-defined functions — Q2
Correct: **(A)**. *Traps:* **(B)** Using the wrong interval's rule; **(C)** Using the wrong interval's rule; **(D)** Evaluating every piece.

Piecewise-defined functions — Q3
Correct: **(B)**. *Traps:* **(A)** Assigning a boundary to the wrong piece; **(C)** Assigning a boundary to the wrong piece; **(D)** Assigning a boundary to the wrong piece.

Piecewise-defined functions — Q4
Correct: **(D)**. *Traps:* **(A)** Reading the inequality backwards; **(B)** Evaluating every piece; **(C)** Reading the inequality backwards.

Piecewise-defined functions — Q5
Correct: **(C)**. *Traps:* **(A)** Using the wrong interval's rule; **(B)** Evaluating every piece; **(D)** Using the wrong interval's rule.

Polynomial arithmetic & structure — Q1
Correct: **(C)**. *Traps:* **(A)** combining unlike terms; **(B)** combining unlike terms; **(D)** subtraction sign distribution.

Polynomial arithmetic & structure — Q2
Correct: **(D)**. *Traps:* **(A)** subtraction sign distribution; **(B)** subtraction sign distribution; **(C)** subtraction sign distribution.

Polynomial arithmetic & structure — Q3
Correct: **(D)**. *Traps:* **(A)** multiplication/exponent error; **(B)** multiplication/exponent error; **(C)** multiplication/exponent error.

Polynomial arithmetic & structure — Q4
Correct: **(B)**. *Traps:* **(A)** degree/leading-coefficient error; **(C)** degree/leading-coefficient error; **(D)** degree/leading-coefficient error.

Polynomial arithmetic & structure — Q5
Correct: **(A)**. *Traps:* **(B)** subtraction sign distribution; **(C)** multiplication/exponent error; **(D)** combining unlike terms.

Zeros, factors & the factor theorem — Q1
Correct: **(A)**. *Traps:* **(B)** factor/zero sign error; **(C)** factor/zero sign error; **(D)** factor/zero sign error.

Zeros, factors & the factor theorem — Q2
Correct: **(C)**. *Traps:* **(A)** factor-theorem meaning error; **(B)** factor/zero sign error; **(D)** factor-theorem meaning error.

Zeros, factors & the factor theorem — Q3
Correct: **(A)**. *Traps:* **(B)** factor-theorem meaning error; **(C)** factor/zero sign error; **(D)** factor-theorem meaning error.

Zeros, factors & the factor theorem — Q4
Correct: **(D)**. *Traps:* **(A)** number-of-zeros error; **(B)** number-of-zeros error; **(C)** number-of-zeros error.

Zeros, factors & the factor theorem — Q5
Correct: **(D)**. *Traps:* **(A)** factor/zero sign error; **(B)** number-of-zeros error; **(C)** factor/zero sign error.

Polynomial graphs: end behavior, zeros, multiplicity — Q1
Correct: **(D)**. *Traps:* **(A)** end-behavior error; **(B)** end-behavior error; **(C)** end-behavior error.

Polynomial graphs: end behavior, zeros, multiplicity — Q2
Correct: **(C)**. *Traps:* **(A)** zeros-as-x-intercepts error; **(B)** zeros-as-x-intercepts error; **(D)** zeros-as-x-intercepts error.

Polynomial graphs: end behavior, zeros, multiplicity — Q3
Correct: **(A)**. *Traps:* **(B)** multiplicity error; **(C)** multiplicity error; **(D)** multiplicity error.

Polynomial graphs: end behavior, zeros, multiplicity — Q4
Correct: **(A)**. *Traps:* **(B)** end-behavior error; **(C)** end-behavior error; **(D)** end-behavior error.

Polynomial graphs: end behavior, zeros, multiplicity — Q5
Correct: **(B)**. *Traps:* **(A)** degree-vs-zero-count error; **(C)** degree-vs-zero-count error; **(D)** degree-vs-zero-count error.

Polynomial division & the remainder theorem — Q1
Correct: **(D)**. *Traps:* **(A)** remainder-theorem sign error; **(B)** remainder-theorem sign error; **(C)** remainder-theorem sign error.

Polynomial division & the remainder theorem — Q2
Correct: **(A)**. *Traps:* **(B)** remainder-vs-factor direction error; **(C)** remainder-vs-factor direction error; **(D)** remainder-vs-factor direction error.

Polynomial division & the remainder theorem — Q3
Correct: **(C)**. *Traps:* **(A)** division mechanics error; **(B)** division mechanics error; **(D)** division mechanics error.

Polynomial division & the remainder theorem — Q4
Correct: **(B)**. *Traps:* **(A)** nonzero-remainder interpretation error; **(C)** nonzero-remainder interpretation error; **(D)** nonzero-remainder interpretation error.

Polynomial division & the remainder theorem — Q5
Correct: **(D)**. *Traps:* **(A)** remainder-theorem sign error; **(B)** remainder-vs-factor direction error; **(C)** remainder-vs-factor direction error.

Synthetic division & the quotient — Q1
Correct: **(B)**. *Traps:* **(A)** used the divisor; **(C)** used the coefficient; **(D)** sign error.

Synthetic division & the quotient — Q2
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the divisor; **(D)** used the coefficient.

Synthetic division & the quotient — Q3
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the coefficient; **(D)** used the divisor.

Synthetic division & the quotient — Q4
Correct: **(C)**. *Traps:* **(A)** sign error; **(B)** used the divisor; **(D)** used the coefficient.

Synthetic division & the quotient — Q5
Correct: **(B)**. *Traps:* **(A)** sign error; **(C)** used the divisor; **(D)** used the coefficient.

Binomial expansion (perfect squares) — Q1
Correct: **(D)**. *Traps:* **(A)** did not square the constant; **(B)** dropped the middle term; **(C)** did not double.

Binomial expansion (perfect squares) — Q2
Correct: **(D)**. *Traps:* **(A)** dropped the middle term; **(B)** did not square the constant; **(C)** did not double.

Binomial expansion (perfect squares) — Q3
Correct: **(D)**. *Traps:* **(A)** did not double; **(B)** did not square the constant; **(C)** dropped the middle term.

Binomial expansion (perfect squares) — Q4
Correct: **(D)**. *Traps:* **(A)** did not square the constant; **(B)** did not double; **(C)** dropped the middle term.

Binomial expansion (perfect squares) — Q5
Correct: **(C)**. *Traps:* **(A)** did not double; **(B)** dropped the middle term; **(D)** did not square the constant.

Special products & factoring patterns — Q1
Correct: **(C)**. *Traps:* **(A)** kept a middle term; **(B)** did not square k; **(D)** wrong sign.

Special products & factoring patterns — Q2
Correct: **(B)**. *Traps:* **(A)** wrong sign; **(C)** did not square k; **(D)** kept a middle term.

Special products & factoring patterns — Q3
Correct: **(B)**. *Traps:* **(A)** wrong sign; **(C)** kept a middle term; **(D)** did not square k.

Special products & factoring patterns — Q4
Correct: **(D)**. *Traps:* **(A)** did not square k; **(B)** wrong sign; **(C)** kept a middle term.

Special products & factoring patterns — Q5
Correct: **(B)**. *Traps:* **(A)** did not square k; **(C)** kept a middle term; **(D)** wrong sign.

Sum & product of roots — Q1
Correct: **(A)**. *Traps:* **(B)** swapped sum & product; **(C)** product sign error; **(D)** sum sign error.

Sum & product of roots — Q2
Correct: **(D)**. *Traps:* **(A)** swapped sum & product; **(B)** product sign error; **(C)** sum sign error.

Sum & product of roots — Q3
Correct: **(D)**. *Traps:* **(A)** swapped sum & product; **(B)** product sign error; **(C)** sum sign error.

Sum & product of roots — Q4
Correct: **(A)**. *Traps:* **(B)** swapped sum & product; **(C)** sum sign error; **(D)** product sign error.

Sum & product of roots — Q5
Correct: **(B)**. *Traps:* **(A)** sum sign error; **(C)** product sign error; **(D)** swapped sum & product.

Exponent rules & properties — Q1
Correct: **(D)**. *Traps:* **(A)** Adding and multiplying exponents mixed up; **(B)** Coefficient or sum handled wrong; **(C)** Adding and multiplying exponents mixed up.

Exponent rules & properties — Q2
Correct: **(B)**. *Traps:* **(A)** Adding and multiplying exponents mixed up; **(C)** Adding and multiplying exponents mixed up; **(D)** Coefficient or sum handled wrong.

Exponent rules & properties — Q3
Correct: **(C)**. *Traps:* **(A)** Negative exponent read as a negative number; **(B)** Negative exponent read as a negative number; **(D)** Negative exponent read as a negative number.

Exponent rules & properties — Q4
Correct: **(B)**. *Traps:* **(A)** Zero or fractional exponent mishandled; **(C)** Zero or fractional exponent mishandled; **(D)** Zero or fractional exponent mishandled.

Exponent rules & properties — Q5
Correct: **(A)**. *Traps:* **(B)** Adding and multiplying exponents mixed up; **(C)** Coefficient or sum handled wrong; **(D)** Coefficient or sum handled wrong.

Scientific notation — Q1
Correct: **(B)**. *Traps:* **(A)** Decimal moved the wrong number of places; **(C)** Wrong sign on the exponent; **(D)** Decimal moved the wrong number of places.

Scientific notation — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong sign on the exponent; **(C)** Coefficient left out of range; **(D)** Decimal moved the wrong number of places.

Scientific notation — Q3
Correct: **(D)**. *Traps:* **(A)** Exponents combined the wrong way; **(B)** Exponents combined the wrong way; **(C)** Exponents combined the wrong way.

Scientific notation — Q4
Correct: **(A)**. *Traps:* **(B)** Coefficient left out of range; **(C)** Coefficient left out of range; **(D)** Coefficient left out of range.

Scientific notation — Q5
Correct: **(C)**. *Traps:* **(A)** Decimal moved the wrong number of places; **(B)** Exponents combined the wrong way; **(D)** Exponents combined the wrong way.

Exponential functions ($y = a*b^x$) — Q1
Correct: **(A)**. *Traps:* **(B)** Starting value and base swapped; **(C)** Coefficient folded into the base; **(D)** Exponential pattern treated as linear.

Exponential functions ($y = a*b^x$) — Q2
Correct: **(D)**. *Traps:* **(A)** Growth and decay direction reversed; **(B)** Growth and decay direction reversed; **(C)** Growth and decay direction reversed.

Exponential functions ($y = a*b^x$) — Q3
Correct: **(A)**. *Traps:* **(B)** Coefficient folded into the base; **(C)** Coefficient folded into the base; **(D)** Exponential pattern treated as linear.

Exponential functions ($y = a*b^x$) — Q4
Correct: **(B)**. *Traps:* **(A)** Growth and decay direction reversed; **(C)** Exponential pattern treated as linear; **(D)** Exponential pattern treated as linear.

Exponential functions ($y = a*b^x$) — Q5
Correct: **(D)**. *Traps:* **(A)** Starting value and base swapped; **(B)** Exponential pattern treated as linear; **(C)** Starting value and base swapped.

Exponential growth & decay in context — Q1
Correct: **(B)**. *Traps:* **(A)** Growth and decay base reversed; **(C)** Rate used as the base; **(D)** Rate used as the base.

Exponential growth & decay in context — Q2
Correct: **(A)**. *Traps:* **(B)** Simple interest used for compound change; **(C)** Growth and decay base reversed; **(D)** Rate used as the base.

Exponential growth & decay in context — Q3
Correct: **(B)**. *Traps:* **(A)** Simple interest used for compound change; **(C)** Rate used as the base; **(D)** Simple interest used for compound change.

Exponential growth & decay in context — Q4
Correct: **(D)**. *Traps:* **(A)** Simple interest used for compound change; **(B)** Simple interest used for compound change; **(C)** Rate used as the base.

Exponential growth & decay in context — Q5
Correct: **(A)**. *Traps:* **(B)** Rate used as the base; **(C)** Simple interest used for compound change; **(D)** Rate used as the base.

Solving exponential equations by matching bases — Q1
Correct: **(A)**. *Traps:* **(B)** Exponential solved as if linear; **(C)** Exponential solved as if linear; **(D)** Exponential solved as if linear.

Solving exponential equations by matching bases — Q2
Correct: **(C)**. *Traps:* **(A)** Exponential solved as if linear; **(B)** Number rewritten as the wrong power; **(D)** Number rewritten as the wrong power.

Solving exponential equations by matching bases — Q3
Correct: **(B)**. *Traps:* **(A)** Exponents equated without matching bases; **(C)** Exponents equated without matching bases; **(D)** Exponents equated without matching bases.

Solving exponential equations by matching bases — Q4
Correct: **(A)**. *Traps:* **(B)** Reciprocal or fraction exponent mishandled; **(C)** Reciprocal or fraction exponent mishandled; **(D)** Reciprocal or fraction exponent mishandled.

Solving exponential equations by matching bases — Q5
Correct: **(C)**. *Traps:* **(A)** Exponents equated without matching bases; **(B)** Exponents equated without matching bases; **(D)** Exponents equated without matching bases.

Graphs of exponential functions — Q1
Correct: **(A)**. *Traps:* **(B)** Base used as the intercept; **(C)** Coefficient times base used; **(D)** Assumed the graph passes through the origin.

Graphs of exponential functions — Q2
Correct: **(B)**. *Traps:* **(A)** Coefficient times base used; **(C)** Assumed the graph passes through the origin; **(D)** Base used as the intercept.

Graphs of exponential functions — Q3
Correct: **(D)**. *Traps:* **(A)** Coefficient times base used; **(B)** Assumed the graph passes through the origin; **(C)** Base used as the intercept.

Graphs of exponential functions — Q4
Correct: **(A)**. *Traps:* **(B)** Base used as the intercept; **(C)** Coefficient times base used; **(D)** Assumed the graph passes through the origin.

Graphs of exponential functions — Q5
Correct: **(C)**. *Traps:* **(A)** Assumed the graph passes through the origin; **(B)** Base used as the intercept; **(D)** Coefficient times base used.

Compound interest — Q1
Correct: **(C)**. *Traps:* **(A)** Percent not converted to a decimal; **(B)** Simple interest used instead of compound; **(D)** Dropped the 1 in the base.

Compound interest — Q2
Correct: **(C)**. *Traps:* **(A)** Percent not converted to a decimal; **(B)** Simple interest used instead of compound; **(D)** Dropped the 1 in the base.

Compound interest — Q3
Correct: **(A)**. *Traps:* **(B)** Simple interest used instead of compound; **(C)** Dropped the 1 in the base; **(D)** Percent not converted to a decimal.

Compound interest — Q4
Correct: **(D)**. *Traps:* **(A)** Simple interest used instead of compound; **(B)** Percent not converted to a decimal; **(C)** Dropped the 1 in the base.

Compound interest — Q5
Correct: **(D)**. *Traps:* **(A)** Dropped the 1 in the base; **(B)** Percent not converted to a decimal; **(C)** Simple interest used instead of compound.

Half-life & doubling time — Q1
Correct: **(C)**. *Traps:* **(A)** Multiplied instead of divided in the exponent; **(B)** Decay modeled linearly, not as a power; **(D)** Growth base used for decay.

Half-life & doubling time — Q2
Correct: **(B)**. *Traps:* **(A)** Decay modeled linearly, not as a power; **(C)** Multiplied instead of divided in the exponent; **(D)** Growth base used for decay.

Half-life & doubling time — Q3
Correct: **(D)**. *Traps:* **(A)** Multiplied instead of divided in the exponent; **(B)** Growth base used for decay; **(C)** Decay modeled linearly, not as a power.

Half-life & doubling time — Q4
Correct: **(A)**. *Traps:* **(B)** Multiplied instead of divided in the exponent; **(C)** Growth base used for decay; **(D)** Decay modeled linearly, not as a power.

Half-life & doubling time — Q5
Correct: **(C)**. *Traps:* **(A)** Growth base used for decay; **(B)** Decay modeled linearly, not as a power; **(D)** Multiplied instead of divided in the exponent.

Radicals & rational exponents (simplify & convert) — Q1
Correct: **(C)**. *Traps:* **(A)** Root and power swapped; **(B)** Root and power swapped; **(D)** Root and power swapped.

Radicals & rational exponents (simplify & convert) — Q2
Correct: **(C)**. *Traps:* **(A)** Root split over a sum; **(B)** Root split over a sum; **(D)** Radical simplified incorrectly.

Radicals & rational exponents (simplify & convert) — Q3
Correct: **(B)**. *Traps:* **(A)** Radical simplified incorrectly; **(C)** Radical simplified incorrectly; **(D)** Radical simplified incorrectly.

Radicals & rational exponents (simplify & convert) — Q4
Correct: **(B)**. *Traps:* **(A)** Root and power swapped; **(C)** Negative exponent's reciprocal dropped; **(D)** Negative exponent's reciprocal dropped.

Radicals & rational exponents (simplify & convert) — Q5
Correct: **(C)**. *Traps:* **(A)** Negative exponent's reciprocal dropped; **(B)** Root and power swapped; **(D)** Root and power swapped.

Radical & rational equations (solving with extraneous checks) — Q1
Correct: **(B)**. *Traps:* **(A)** Extraneous root not checked; **(C)** Extraneous root not checked; **(D)** Extraneous root not checked.

Radical & rational equations (solving with extraneous checks) — Q2
Correct: **(D)**. *Traps:* **(A)** Squaring or expansion done wrong; **(B)** Squaring or expansion done wrong; **(C)** Squaring or expansion done wrong.

Radical & rational equations (solving with extraneous checks) — Q3
Correct: **(C)**. *Traps:* **(A)** Denominator-zero value kept; **(B)** Denominator-zero value kept; **(D)** Denominator-zero value kept.

Radical & rational equations (solving with extraneous checks) — Q4
Correct: **(D)**. *Traps:* **(A)** Squaring or expansion done wrong; **(B)** Extraneous root not checked; **(C)** Extraneous root not checked.

Radical & rational equations (solving with extraneous checks) — Q5
Correct: **(A)**. *Traps:* **(B)** Denominator-zero value kept; **(C)** Squaring or expansion done wrong; **(D)** Extraneous root not checked.

Simplify radicals (perfect-square factors) — Q1
Correct: **(B)**. *Traps:* **(A)** Radical dropped or square left outside; **(C)** Simplification left unfinished; **(D)** Radical dropped or square left outside.

Simplify radicals (perfect-square factors) — Q2
Correct: **(C)**. *Traps:* **(A)** Radical dropped or square left outside; **(B)** Simplification left unfinished; **(D)** Radical dropped or square left outside.

Simplify radicals (perfect-square factors) — Q3
Correct: **(C)**. *Traps:* **(A)** Simplification left unfinished; **(B)** Radical dropped or square left outside; **(D)** Radical dropped or square left outside.

Simplify radicals (perfect-square factors) — Q4
Correct: **(D)**. *Traps:* **(A)** Simplification left unfinished; **(B)** Radical dropped or square left outside; **(C)** Radical dropped or square left outside.

Simplify radicals (perfect-square factors) — Q5
Correct: **(C)**. *Traps:* **(A)** Simplification left unfinished; **(B)** Radical dropped or square left outside; **(D)** Perfect square not rooted.

Add & subtract like radicals — Q1
Correct: **(C)**. *Traps:* **(A)** Like radicals treated as uncombinable; **(B)** Coefficients multiplied instead of added; **(D)** Numbers under the root added.

Add & subtract like radicals — Q2
Correct: **(C)**. *Traps:* **(A)** Coefficients multiplied instead of added; **(B)** Like radicals treated as uncombinable; **(D)** Numbers under the root added.

Add & subtract like radicals — Q3
Correct: **(A)**. *Traps:* **(B)** Like radicals treated as uncombinable; **(C)** Numbers under the root added; **(D)** Coefficients multiplied instead of added.

Add & subtract like radicals — Q4
Correct: **(C)**. *Traps:* **(A)** Numbers under the root added; **(B)** Like radicals treated as uncombinable; **(D)** Coefficients multiplied instead of added.

Add & subtract like radicals — Q5
Correct: **(D)**. *Traps:* **(A)** Numbers under the root added; **(B)** Like radicals treated as uncombinable; **(C)** Coefficients multiplied instead of added.

Multiply radicals & rationalize denominators — Q1
Correct: **(C)**. *Traps:* **(A)** Radical lost or numerator dropped; **(B)** Rational denominator dropped; **(D)** Radical lost or numerator dropped.

Multiply radicals & rationalize denominators — Q2
Correct: **(B)**. *Traps:* **(A)** Radical lost or numerator dropped; **(C)** Radical lost or numerator dropped; **(D)** Rational denominator dropped.

Multiply radicals & rationalize denominators — Q3
Correct: **(A)**. *Traps:* **(B)** Radical lost or numerator dropped; **(C)** Rational denominator dropped; **(D)** Radical lost or numerator dropped.

Multiply radicals & rationalize denominators — Q4
Correct: **(D)**. *Traps:* **(A)** Radical lost or numerator dropped; **(B)** Radical lost or numerator dropped; **(C)** Rational denominator dropped.

Multiply radicals & rationalize denominators — Q5
Correct: **(D)**. *Traps:* **(A)** Radical left in the denominator; **(B)** Rational denominator dropped; **(C)** Radical lost or numerator dropped.

nth roots & cube roots — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong operation for the root; **(B)** Wrong operation for the root; **(C)** Multiplied by the index.

nth roots & cube roots — Q2
Correct: **(D)**. *Traps:* **(A)** Multiplied by the index; **(B)** Wrong operation for the root; **(C)** Wrong operation for the root.

nth roots & cube roots — Q3
Correct: **(A)**. *Traps:* **(B)** Wrong operation for the root; **(C)** Wrong operation for the root; **(D)** Multiplied by the index.

nth roots & cube roots — Q4
Correct: **(B)**. *Traps:* **(A)** Multiplied by the index; **(C)** Divided by the index; **(D)** Wrong operation for the root.

nth roots & cube roots — Q5
Correct: **(C)**. *Traps:* **(A)** Multiplied by the index; **(B)** Divided by the index; **(D)** Wrong operation for the root.

Operations with rational exponents — Q1
Correct: **(B)**. *Traps:* **(A)** Power skipped; **(C)** Root skipped; **(D)** Root skipped.

Operations with rational exponents — Q2
Correct: **(C)**. *Traps:* **(A)** Root skipped; **(B)** Root skipped; **(D)** Power skipped.

Operations with rational exponents — Q3
Correct: **(A)**. *Traps:* **(B)** Root skipped; **(C)** Power skipped; **(D)** Root skipped.

Operations with rational exponents — Q4
Correct: **(C)**. *Traps:* **(A)** Root skipped; **(B)** Power skipped; **(D)** Root skipped.

Operations with rational exponents — Q5
Correct: **(D)**. *Traps:* **(A)** Root skipped; **(B)** Exponent used as a plain multiplier; **(C)** Power skipped.

Radical functions: domain & graphs — Q1
Correct: **(B)**. *Traps:* **(A)** Restriction ignored; **(C)** Sign error on the constant; **(D)** Inequality direction flipped.

Radical functions: domain & graphs — Q2
Correct: **(A)**. *Traps:* **(B)** Restriction ignored; **(C)** Sign error on the constant; **(D)** Inequality direction flipped.

Radical functions: domain & graphs — Q3
Correct: **(C)**. *Traps:* **(A)** Restriction ignored; **(B)** Inequality direction flipped; **(D)** Sign error on the constant.

Radical functions: domain & graphs — Q4
Correct: **(C)**. *Traps:* **(A)** Sign error on the constant; **(B)** Restriction ignored; **(D)** Inequality direction flipped.

Radical functions: domain & graphs — Q5
Correct: **(B)**. *Traps:* **(A)** Inequality direction flipped; **(C)** Restriction ignored; **(D)** Sign error on the constant.

Rational expressions: simplify & operate — Q1
Correct: **(B)**. *Traps:* **(A)** Canceling terms instead of factors; **(C)** Canceling terms instead of factors; **(D)** Canceling terms instead of factors.

Rational expressions: simplify & operate — Q2
Correct: **(D)**. *Traps:* **(A)** Combining straight across when adding; **(B)** Combining straight across when adding; **(C)** Combining straight across when adding.

Rational expressions: simplify & operate — Q3
Correct: **(C)**. *Traps:* **(A)** Dividing without flipping; **(B)** Dividing without flipping; **(D)** Dividing without flipping.

Rational expressions: simplify & operate — Q4
Correct: **(C)**. *Traps:* **(A)** Missing or wrong excluded values; **(B)** Missing or wrong excluded values; **(D)** Missing or wrong excluded values.

Rational expressions: simplify & operate — Q5
Correct: **(D)**. *Traps:* **(A)** Missing or wrong excluded values; **(B)** Missing or wrong excluded values; **(C)** Canceling terms instead of factors.

Multiply & divide rational expressions — Q1
Correct: **(D)**. *Traps:* **(A)** Canceled, then multiplied the leftovers; **(B)** Flipped the final answer; **(C)** Multiplied without flipping.

Multiply & divide rational expressions — Q2
Correct: **(A)**. *Traps:* **(B)** Flipped the final answer; **(C)** Canceled, then multiplied the leftovers; **(D)** Multiplied without flipping.

Multiply & divide rational expressions — Q3
Correct: **(C)**. *Traps:* **(A)** Multiplied without flipping; **(B)** Flipped the final answer; **(D)** Canceled, then multiplied the leftovers.

Multiply & divide rational expressions — Q4
Correct: **(D)**. *Traps:* **(A)** Multiplied without flipping; **(B)** Canceled, then multiplied the leftovers; **(C)** Flipped the final answer.

Multiply & divide rational expressions — Q5
Correct: **(A)**. *Traps:* **(B)** Multiplied without flipping; **(C)** Flipped the final answer; **(D)** Canceled, then multiplied the leftovers.

Add & subtract rational expressions — Q1
Correct: **(C)**. *Traps:* **(A)** Added the denominators; **(B)** Multiplied the numerators; **(D)** Multiplied the denominators.

Add & subtract rational expressions — Q2
Correct: **(A)**. *Traps:* **(B)** Added the denominators; **(C)** Multiplied the numerators; **(D)** Multiplied the denominators.

Add & subtract rational expressions — Q3
Correct: **(B)**. *Traps:* **(A)** Added the denominators; **(C)** Multiplied the numerators; **(D)** Multiplied the denominators.

Add & subtract rational expressions — Q4
Correct: **(B)**. *Traps:* **(A)** Multiplied the denominators; **(C)** Multiplied the numerators; **(D)** Added the denominators.

Add & subtract rational expressions — Q5
Correct: **(D)**. *Traps:* **(A)** Multiplied the denominators; **(B)** Multiplied the numerators; **(C)** Added the denominators.

Complex (compound) fractions — Q1
Correct: **(B)**. *Traps:* **(A)** Gave the reciprocal of the answer; **(C)** Multiplied the denominators; **(D)** Multiplied straight across.

Complex (compound) fractions — Q2
Correct: **(B)**. *Traps:* **(A)** Gave the reciprocal of the answer; **(C)** Multiplied straight across; **(D)** Multiplied the denominators.

Complex (compound) fractions — Q3
Correct: **(C)**. *Traps:* **(A)** Multiplied straight across; **(B)** Gave the reciprocal of the answer; **(D)** Multiplied the denominators.

Complex (compound) fractions — Q4
Correct: **(C)**. *Traps:* **(A)** Multiplied straight across; **(B)** Multiplied the denominators; **(D)** Gave the reciprocal of the answer.

Complex (compound) fractions — Q5
Correct: **(A)**. *Traps:* **(B)** Gave the reciprocal of the answer; **(C)** Multiplied the denominators; **(D)** Multiplied straight across.

Solve rational equations — Q1
Correct: **(B)**. *Traps:* **(A)** Multiplied instead of dividing; **(C)** Inverted the answer; **(D)** Dropped the coefficient.

Solve rational equations — Q2
Correct: **(C)**. *Traps:* **(A)** Multiplied instead of dividing; **(B)** Dropped the coefficient; **(D)** Inverted the answer.

Solve rational equations — Q3
Correct: **(D)**. *Traps:* **(A)** Inverted the answer; **(B)** Multiplied instead of dividing; **(C)** Dropped the coefficient.

Solve rational equations — Q4
Correct: **(C)**. *Traps:* **(A)** Inverted the answer; **(B)** Dropped the coefficient; **(D)** Multiplied instead of dividing.

Solve rational equations — Q5
Correct: **(D)**. *Traps:* **(A)** Dropped the coefficient; **(B)** Inverted the answer; **(C)** Multiplied instead of dividing.

Excluded values & domain — Q1
Correct: **(B)**. *Traps:* **(A)** Sign error on the excluded value; **(C)** Missed the exclusion; **(D)** Used the numerator.

Excluded values & domain — Q2
Correct: **(A)**. *Traps:* **(B)** Sign error on the excluded value; **(C)** Used the numerator; **(D)** Missed the exclusion.

Excluded values & domain — Q3
Correct: **(D)**. *Traps:* **(A)** Used the numerator; **(B)** Missed the exclusion; **(C)** Sign error on the excluded value.

Excluded values & domain — Q4
Correct: **(A)**. *Traps:* **(B)** Missed the exclusion; **(C)** Used the numerator; **(D)** Sign error on the excluded value.

Excluded values & domain — Q5
Correct: **(C)**. *Traps:* **(A)** Used the numerator; **(B)** Missed the exclusion; **(D)** Sign error on the excluded value.

Rational functions: asymptotes & holes — Q1
Correct: **(B)**. *Traps:* **(A)** Used the numerator; **(C)** Sign error; **(D)** Horizontal and vertical mixed up.

Rational functions: asymptotes & holes — Q2
Correct: **(A)**. *Traps:* **(B)** Sign error; **(C)** Used the numerator; **(D)** Horizontal and vertical mixed up.

Rational functions: asymptotes & holes — Q3
Correct: **(C)**. *Traps:* **(A)** Used the numerator; **(B)** Sign error; **(D)** Horizontal and vertical mixed up.

Rational functions: asymptotes & holes — Q4
Correct: **(C)**. *Traps:* **(A)** Sign error; **(B)** Used the numerator; **(D)** Horizontal and vertical mixed up.

Rational functions: asymptotes & holes — Q5
Correct: **(D)**. *Traps:* **(A)** Horizontal and vertical mixed up; **(B)** Sign error; **(C)** Used the numerator.

Direct & inverse variation — Q1
Correct: **(C)**. *Traps:* **(A)** Reused the given y; **(B)** Added instead of scaling; **(D)** Scaled without finding k.

Direct & inverse variation — Q2
Correct: **(A)**. *Traps:* **(B)** Added instead of scaling; **(C)** Scaled without finding k; **(D)** Reused the given y.

Direct & inverse variation — Q3
Correct: **(C)**. *Traps:* **(A)** Reused the given y; **(B)** Scaled without finding k; **(D)** Added instead of scaling.

Direct & inverse variation — Q4
Correct: **(D)**. *Traps:* **(A)** Scaled without finding k; **(B)** Added instead of scaling; **(C)** Reused the given y.

Direct & inverse variation — Q5
Correct: **(A)**. *Traps:* **(B)** Scaled without finding k; **(C)** Added instead of scaling; **(D)** Reused the given y.

Nonlinear systems: line & parabola — Q1
Correct: **(C)**. *Traps:* **(A)** Adding instead of substituting; **(B)** Adding instead of substituting; **(D)** Adding instead of substituting.

Nonlinear systems: line & parabola — Q2
Correct: **(C)**. *Traps:* **(A)** Giving x instead of the point; **(B)** Giving x instead of the point; **(D)** Giving x instead of the point.

Nonlinear systems: line & parabola — Q3
Correct: **(C)**. *Traps:* **(A)** Miscounting the intersections; **(B)** Miscounting the intersections; **(D)** Miscounting the intersections.

Nonlinear systems: line & parabola — Q4
Correct: **(D)**. *Traps:* **(A)** Dropping the second solution; **(B)** Dropping the second solution; **(C)** Dropping the second solution.

Nonlinear systems: line & parabola — Q5
Correct: **(B)**. *Traps:* **(A)** Dropping the second solution; **(C)** Miscounting the intersections; **(D)** Giving x instead of the point.

Solve line & parabola by substitution — Q1
Correct: **(B)**. *Traps:* **(A)** Sign error on the roots; **(C)** Stopping at one solution; **(D)** Reading off the coefficients.

Solve line & parabola by substitution — Q2
Correct: **(C)**. *Traps:* **(A)** Reading off the coefficients; **(B)** Stopping at one solution; **(D)** Sign error on the roots.

Solve line & parabola by substitution — Q3
Correct: **(D)**. *Traps:* **(A)** Reading off the coefficients; **(B)** Stopping at one solution; **(C)** Sign error on the roots.

Solve line & parabola by substitution — Q4
Correct: **(A)**. *Traps:* **(B)** Sign error on the roots; **(C)** Stopping at one solution; **(D)** Reading off the coefficients.

Solve line & parabola by substitution — Q5
Correct: **(A)**. *Traps:* **(B)** Sign error on the roots; **(C)** Stopping at one solution; **(D)** Reading off the coefficients.

Line & circle systems — Q1
Correct: **(A)**. *Traps:* **(B)** Using the line's value; **(C)** Using R-squared as x; **(D)** Keeping only one point.

Line & circle systems — Q2
Correct: **(D)**. *Traps:* **(A)** Keeping only one point; **(B)** Using R-squared as x; **(C)** Using the line's value.

Line & circle systems — Q3
Correct: **(A)**. *Traps:* **(B)** Using the line's value; **(C)** Keeping only one point; **(D)** Using R-squared as x.

Line & circle systems — Q4
Correct: **(D)**. *Traps:* **(A)** Using R-squared as x; **(B)** Using the line's value; **(C)** Keeping only one point.

Line & circle systems — Q5
Correct: **(B)**. *Traps:* **(A)** Using the line's value; **(C)** Using R-squared as x; **(D)** Keeping only one point.

Number of intersections (discriminant) — Q1
Correct: **(B)**. *Traps:* **(A)** Dropping the c; **(C)** Adding instead of subtracting; **(D)** Dropping the 4.

Number of intersections (discriminant) — Q2
Correct: **(C)**. *Traps:* **(A)** Dropping the c; **(B)** Dropping the 4; **(D)** Adding instead of subtracting.

Number of intersections (discriminant) — Q3
Correct: **(D)**. *Traps:* **(A)** Adding instead of subtracting; **(B)** Dropping the c; **(C)** Dropping the 4.

Number of intersections (discriminant) — Q4
Correct: **(B)**. *Traps:* **(A)** Adding instead of subtracting; **(C)** Dropping the 4; **(D)** Dropping the c.

Number of intersections (discriminant) — Q5
Correct: **(C)**. *Traps:* **(A)** Dropping the 4; **(B)** Adding instead of subtracting; **(D)** Dropping the c.

Report intersection points (x and y) — Q1
Correct: **(B)**. *Traps:* **(A)** Swapping the coordinates; **(C)** Reusing x as y; **(D)** Giving only the x-values.

Report intersection points (x and y) — Q2
Correct: **(B)**. *Traps:* **(A)** Giving only the x-values; **(C)** Swapping the coordinates; **(D)** Reusing x as y.

Report intersection points (x and y) — Q3
Correct: **(A)**. *Traps:* **(B)** Reusing x as y; **(C)** Giving only the x-values; **(D)** Swapping the coordinates.

Report intersection points (x and y) — Q4
Correct: **(D)**. *Traps:* **(A)** Swapping the coordinates; **(B)** Reusing x as y; **(C)** Giving only the x-values.

Report intersection points (x and y) — Q5
Correct: **(A)**. *Traps:* **(B)** Reusing x as y; **(C)** Swapping the coordinates; **(D)** Giving only the x-values.

Nonlinear systems in context — Q1
Correct: **(C)**. *Traps:* **(A)** Swapping sum and product; **(B)** Wrong sign on the sum; **(D)** Wrong sign on the product.

Nonlinear systems in context — Q2
Correct: **(B)**. *Traps:* **(A)** Wrong sign on the product; **(C)** Swapping sum and product; **(D)** Wrong sign on the sum.

Nonlinear systems in context — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong sign on the sum; **(B)** Wrong sign on the product; **(C)** Swapping sum and product.

Nonlinear systems in context — Q4
Correct: **(B)**. *Traps:* **(A)** Wrong sign on the product; **(C)** Wrong sign on the sum; **(D)** Swapping sum and product.

Nonlinear systems in context — Q5
Correct: **(D)**. *Traps:* **(A)** Wrong sign on the sum; **(B)** Wrong sign on the product; **(C)** Swapping sum and product.

Function transformations — Q1
Correct: **(B)**. *Traps:* **(A)** Inside and outside mixed up; **(C)** Inside and outside mixed up; **(D)** Inside and outside mixed up.

Function transformations — Q2
Correct: **(A)**. *Traps:* **(B)** Inside and outside mixed up; **(C)** Horizontal shift goes the wrong way; **(D)** Inside and outside mixed up.

Function transformations — Q3
Correct: **(B)**. *Traps:* **(A)** Reflection across the wrong axis; **(C)** Reflection across the wrong axis; **(D)** Reflection across the wrong axis.

Function transformations — Q4
Correct: **(B)**. *Traps:* **(A)** Inside and outside mixed up; **(C)** Stretch and compression reversed; **(D)** Stretch and compression reversed.

Function transformations — Q5
Correct: **(A)**. *Traps:* **(B)** Horizontal shift goes the wrong way; **(C)** Inside and outside mixed up; **(D)** Inside and outside mixed up.

Horizontal shifts — Q1
Correct: **(A)**. *Traps:* **(B)** Inside change treated as vertical; **(C)** Inside change treated as vertical; **(D)** Horizontal shift goes the wrong way.

Horizontal shifts — Q2
Correct: **(B)**. *Traps:* **(A)** Inside change treated as vertical; **(C)** Inside change treated as vertical; **(D)** Horizontal shift goes the wrong way.

Horizontal shifts — Q3
Correct: **(B)**. *Traps:* **(A)** Horizontal shift goes the wrong way; **(C)** Inside change treated as vertical; **(D)** Inside change treated as vertical.

Horizontal shifts — Q4
Correct: **(C)**. *Traps:* **(A)** Inside change treated as vertical; **(B)** Inside change treated as vertical; **(D)** Horizontal shift goes the wrong way.

Horizontal shifts — Q5
Correct: **(C)**. *Traps:* **(A)** Horizontal shift goes the wrong way; **(B)** Inside change treated as vertical; **(D)** Inside change applied to the output.

Reflection over the x-axis — Q1
Correct: **(C)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** Reflected across the wrong axis; **(D)** Reflected across the wrong axis.

Reflection over the x-axis — Q2
Correct: **(A)**. *Traps:* **(B)** Reflected across the wrong axis; **(C)** Wrong coordinate changed; **(D)** Reflected across the wrong axis.

Reflection over the x-axis — Q3
Correct: **(B)**. *Traps:* **(A)** Reflected across the wrong axis; **(C)** Reflected across the wrong axis; **(D)** Wrong coordinate changed.

Reflection over the x-axis — Q4
Correct: **(C)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** Reflected across the wrong axis; **(D)** Reflected across the wrong axis.

Reflection over the x-axis — Q5
Correct: **(D)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** No reflection applied; **(C)** Reflected across the wrong axis.

Reflection over the y-axis — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** Reflected across the wrong axis; **(C)** Reflected across the wrong axis.

Reflection over the y-axis — Q2
Correct: **(B)**. *Traps:* **(A)** Reflected across the wrong axis; **(C)** Wrong coordinate changed; **(D)** Reflected across the wrong axis.

Reflection over the y-axis — Q3
Correct: **(A)**. *Traps:* **(B)** Wrong coordinate changed; **(C)** Reflected across the wrong axis; **(D)** Reflected across the wrong axis.

Reflection over the y-axis — Q4
Correct: **(D)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** Reflected across the wrong axis; **(C)** Reflected across the wrong axis.

Reflection over the y-axis — Q5
Correct: **(C)**. *Traps:* **(A)** Wrong coordinate changed; **(B)** Output negated instead of input; **(D)** Reflected across the wrong axis.

Vertical stretch & compression — Q1
Correct: **(B)**. *Traps:* **(A)** Stretch and compression reversed; **(C)** Vertical scaling treated as horizontal; **(D)** Vertical scaling treated as horizontal.

Vertical stretch & compression — Q2
Correct: **(A)**. *Traps:* **(B)** Vertical scaling treated as horizontal; **(C)** Stretch and compression reversed; **(D)** Vertical scaling treated as horizontal.

Vertical stretch & compression — Q3
Correct: **(A)**. *Traps:* **(B)** Stretch and compression reversed; **(C)** Vertical scaling treated as horizontal; **(D)** Vertical scaling treated as horizontal.

Vertical stretch & compression — Q4
Correct: **(B)**. *Traps:* **(A)** Vertical scaling treated as horizontal; **(C)** Scaling turned into a shift; **(D)** Stretch and compression reversed.

Vertical stretch & compression — Q5
Correct: **(D)**. *Traps:* **(A)** Stretch and compression reversed; **(B)** Vertical scaling treated as horizontal; **(C)** Scaling turned into a shift.

Horizontal stretch & compression — Q1
Correct: **(B)**. *Traps:* **(A)** Horizontal scaling treated as vertical; **(C)** Stretch and compression reversed; **(D)** Horizontal scaling treated as vertical.

Horizontal stretch & compression — Q2
Correct: **(D)**. *Traps:* **(A)** Horizontal scaling treated as vertical; **(B)** Stretch and compression reversed; **(C)** Horizontal scaling treated as vertical.

Horizontal stretch & compression — Q3
Correct: **(A)**. *Traps:* **(B)** Stretch and compression reversed; **(C)** Horizontal scaling treated as vertical; **(D)** Horizontal scaling treated as vertical.

Horizontal stretch & compression — Q4
Correct: **(D)**. *Traps:* **(A)** Horizontal scaling treated as vertical; **(B)** Scaling confused with a shift; **(C)** Stretch and compression reversed.

Horizontal stretch & compression — Q5
Correct: **(D)**. *Traps:* **(A)** Stretch and compression reversed; **(B)** Horizontal scaling treated as vertical; **(C)** Scaling confused with a shift.

Combined transformations — Q1
Correct: **(A)**. *Traps:* **(B)** Horizontal and vertical mixed up; **(C)** Horizontal and vertical mixed up; **(D)** Horizontal shift goes the wrong way.

Combined transformations — Q2
Correct: **(A)**. *Traps:* **(B)** Horizontal and vertical mixed up; **(C)** Horizontal and vertical mixed up; **(D)** Horizontal shift goes the wrong way.

Combined transformations — Q3
Correct: **(B)**. *Traps:* **(A)** Horizontal and vertical mixed up; **(C)** Horizontal and vertical mixed up; **(D)** Horizontal shift goes the wrong way.

Combined transformations — Q4
Correct: **(C)**. *Traps:* **(A)** Horizontal shift goes the wrong way; **(B)** Horizontal and vertical mixed up; **(D)** Horizontal and vertical mixed up.

Combined transformations — Q5
Correct: **(B)**. *Traps:* **(A)** Horizontal shift goes the wrong way; **(C)** Horizontal and vertical mixed up; **(D)** Vertical shift goes the wrong way.

Evaluating transformed functions — Q1
Correct: **(B)**. *Traps:* **(A)** Wrong operation or sign; **(C)** Forgot to apply the change; **(D)** Wrong operation or sign.

Evaluating transformed functions — Q2
Correct: **(D)**. *Traps:* **(A)** Forgot to apply the change; **(B)** Wrong operation or sign; **(C)** Wrong operation or sign.

Evaluating transformed functions — Q3
Correct: **(C)**. *Traps:* **(A)** Wrong operation or sign; **(B)** Wrong operation or sign; **(D)** Forgot to apply the change.

Evaluating transformed functions — Q4
Correct: **(A)**. *Traps:* **(B)** Wrong operation or sign; **(C)** Forgot to apply the change; **(D)** Wrong operation or sign.

Evaluating transformed functions — Q5
Correct: **(A)**. *Traps:* **(B)** Used the input instead of the output; **(C)** Wrong operation or sign; **(D)** Forgot to apply the change.

Complex numbers: arithmetic with i — Q1
Correct: **(C)**. *Traps:* **(A)** i-squared sign error; **(B)** i-squared sign error; **(D)** i-squared sign error.

Complex numbers: arithmetic with i — Q2
Correct: **(C)**. *Traps:* **(A)** combine real/imaginary error; **(B)** combine real/imaginary error; **(D)** combine real/imaginary error.

Complex numbers: arithmetic with i — Q3
Correct: **(D)**. *Traps:* **(A)** multiplication/FOIL error; **(B)** i-squared sign error; **(C)** i-squared sign error.

Complex numbers: arithmetic with i — Q4
Correct: **(C)**. *Traps:* **(A)** square-root-of-negative error; **(B)** square-root-of-negative error; **(D)** square-root-of-negative error.

Complex numbers: arithmetic with i — Q5
Correct: **(D)**. *Traps:* **(A)** i-squared sign error; **(B)** multiplication/FOIL error; **(C)** i-squared sign error.

Powers of i & complex division — Q1
Correct: **(D)**. *Traps:* **(A)** power-cycle error; **(B)** power-cycle error; **(C)** power-cycle error.

Powers of i & complex division — Q2
Correct: **(A)**. *Traps:* **(B)** power-cycle error; **(C)** power-cycle error; **(D)** power-cycle error.

Powers of i & complex division — Q3
Correct: **(D)**. *Traps:* **(A)** conjugate error; **(B)** conjugate error; **(C)** conjugate error.

Powers of i & complex division — Q4
Correct: **(C)**. *Traps:* **(A)** conjugate error; **(B)** conjugate error; **(D)** conjugate error.

Powers of i & complex division — Q5
Correct: **(C)**. *Traps:* **(A)** division-method error; **(B)** division-method error; **(D)** division-method error.

Add & subtract complex numbers — Q1
Correct: **(A)**. *Traps:* **(B)** swapped the parts; **(C)** merged real & imaginary; **(D)** multiplied the i-parts.

Add & subtract complex numbers — Q2
Correct: **(B)**. *Traps:* **(A)** merged real & imaginary; **(C)** multiplied the i-parts; **(D)** swapped the parts.

Add & subtract complex numbers — Q3
Correct: **(A)**. *Traps:* **(B)** merged real & imaginary; **(C)** swapped the parts; **(D)** multiplied the i-parts.

Add & subtract complex numbers — Q4
Correct: **(D)**. *Traps:* **(A)** multiplied the i-parts; **(B)** merged real & imaginary; **(C)** swapped the parts.

Add & subtract complex numbers — Q5
Correct: **(C)**. *Traps:* **(A)** multiplied the i-parts; **(B)** swapped the parts; **(D)** merged real & imaginary.

Multiply complex numbers (FOIL with i) — Q1
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** straight-across product; **(D)** i-squared as +1.

Multiply complex numbers (FOIL with i) — Q2
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** i-squared as +1; **(D)** straight-across product.

Multiply complex numbers (FOIL with i) — Q3
Correct: **(A)**. *Traps:* **(B)** straight-across product; **(C)** one cross term; **(D)** i-squared as +1.

Multiply complex numbers (FOIL with i) — Q4
Correct: **(B)**. *Traps:* **(A)** one cross term; **(C)** i-squared as +1; **(D)** straight-across product.

Multiply complex numbers (FOIL with i) — Q5
Correct: **(C)**. *Traps:* **(A)** i-squared as +1; **(B)** straight-across product; **(D)** one cross term.

Complex conjugates & division — Q1
Correct: **(D)**. *Traps:* **(A)** dropped the b^2 term; **(B)** i-squared as +1; **(C)** left an imaginary unit.

Complex conjugates & division — Q2
Correct: **(D)**. *Traps:* **(A)** dropped the b^2 term; **(B)** left an imaginary unit; **(C)** i-squared as +1.

Complex conjugates & division — Q3
Correct: **(B)**. *Traps:* **(A)** i-squared as +1; **(C)** left an imaginary unit; **(D)** dropped the b^2 term.

Complex conjugates & division — Q4
Correct: **(A)**. *Traps:* **(B)** i-squared as +1; **(C)** dropped the b^2 term; **(D)** left an imaginary unit.

Complex conjugates & division — Q5
Correct: **(D)**. *Traps:* **(A)** i-squared as +1; **(B)** left an imaginary unit; **(C)** dropped the b^2 term.

Complex solutions of quadratics — Q1
Correct: **(A)**. *Traps:* **(B)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q2
Correct: **(A)**. *Traps:* **(B)** stopped at 'no solution'; **(C)** forgot the square root; **(D)** dropped the imaginary unit.

Complex solutions of quadratics — Q3
Correct: **(B)**. *Traps:* **(A)** forgot the square root; **(C)** dropped the imaginary unit; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q4
Correct: **(B)**. *Traps:* **(A)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

Complex solutions of quadratics — Q5
Correct: **(B)**. *Traps:* **(A)** dropped the imaginary unit; **(C)** forgot the square root; **(D)** stopped at 'no solution'.

DOMAIN 3

Problem-Solving & Data Analysis

Reading quantities and the stories they tell

"It is a capital mistake to theorize before one has data."
— Arthur Conan Doyle

About 15% of the section. Ratios, rates, proportions, percentages, units, and one- and two-variable data, probability, and inference.

Ratios & equivalent ratios

node: sat.ratios.R1

The skill. A ratio compares two amounts, like 2 to 3, written 2:3. Equivalent ratios come from multiplying (or dividing) both parts by the same number — never from adding. A part-to-part ratio like 2:3 has $2 + 3 = 5$ total parts, so 2:3 does not mean "2 out of 3." To simplify a ratio, divide both parts by their largest common factor, and always keep the order the question gives.

Worked example

The ratio of red to blue marbles is 3:5. There are 18 red marbles. How many blue marbles are there?

Read. We compare red to blue in the ratio 3:5, and we know there are 18 red marbles. We want the number of blue marbles.

Plan.

1. Find the number that multiplies the red part 3 up to 18.
2. Multiply the blue part 5 by that same number.

Solve.

$$\begin{aligned} \text{multiplier} &= \frac{18}{3} && \text{match the red count to its ratio number} \\ &= 6 && \text{the ratio was scaled up by 6} \\ \text{blue} &= 5 \times 6 && \text{scale the blue part by the same 6} \\ &= 30 && \text{multiply} \end{aligned}$$

Check. Check the ratio: 18 to 30 divides by 6 to give 3 to 5, the original ratio.
Answer: there are 30 blue marbles.

Approach Read · Plan · Solve · Check

Read. Decide what two things are being compared, and whether the ratio compares one part to another part or a part to the whole.

Plan.

1. Find the number that multiplies a known part up to (or down to) its ratio value.
2. Multiply every part of the ratio by that same number.
3. For a total, add the ratio numbers to get the number of parts, then find one part.

Solve.

1. Multiply to scale a ratio up or down — never add.
2. Divide both parts by their common factor to simplify.
3. Keep the order the question states.

Check. Did you scale by multiplying? Is the ratio part-to-part or part-to-whole? Is the order right and the ratio fully reduced?

Common traps — every wrong answer is one of these

additive scaling.
Scales a ratio by adding the same amount to both parts instead of multiplying both by the same factor. For 2:3, if the 2 grows to 8 (times 4), the 3 must grow to 12 (times 4), not 3 + 6 = 9.

part-part vs part-whole confusion.
Reads a part-to-part ratio like 2:3 as "2 out of 3." The parts are 2 and 3, so the whole is 2 + 3 = 5; that part is 2/5 of the total, not 2/3.

order/inversion error.
Writes or applies the ratio in the wrong order, or matches a quantity to the wrong part. If the question asks sugar to flour, put sugar first.

simplification error.
Leaves a ratio unreduced when simplest form is asked, or divides the two parts by different

numbers. Divide both parts by the same common factor.

Practice

1. *The ratio of cats to dogs is 2:3. There are 8 cats.*

How many dogs are there?

(A) 12
(B) 16
(C) 9
(D) 16/3 (about 5.3)

2. *The ratio of boys to girls in a class is 3:5, and there are 40 students.*

How many are boys?

(A) 25
(B) 24
(C) 15
(D) 8

3. *A recipe uses 6 cups of flour to 4 cups of sugar.*

What is the ratio of sugar to flour in simplest form?

(A) 6:4
(B) 4:6
(C) 3:2
(D) 2:3

4. *A mix has sand:cement:gravel = 2:3:5. A batch uses 30 kg of cement.*

How much gravel is used?

(A) 5 kg
(B) 75 kg
(C) 32 kg
(D) 50 kg

5. *A photo 4 in wide by 6 in tall is enlarged proportionally to be 15 in tall.*

How wide is the enlargement?

(A) 9 in

(B) 10 in

(C) 22.5 in

(D) 13 in

Unit rates

node: sat.ratios.R2

The skill. A unit rate tells you how much of one thing there is for exactly one of another — miles in one hour, dollars for one item. Find it by dividing the first amount by the second, keeping the order the words give: miles per hour means miles divided by hours, not hours divided by miles. To compare two deals, first put each on the same "per one" basis; do not compare raw totals or package sizes.

Worked example

A car travels 210 miles in 3.5 hours at a steady speed. What is its speed in miles per hour?

Read. We know the car covers 210 miles in 3.5 hours. We want the speed in miles per hour — the miles it covers in one hour.

Plan.

1. Divide the miles by the hours to get the miles for one hour.
2. Label the answer as miles per hour.

Solve.

$$\begin{aligned} \text{speed} &= \frac{210 \text{ miles}}{3.5 \text{ hours}} && \text{miles divided by hours, in that order} \\ &= 60 \text{ miles per hour} && \text{divide 210 by 3.5} \end{aligned}$$

Check. At 60 miles per hour, 3.5 hours gives 60 x 3.5 = 210 miles, which matches.
Answer: the speed is 60 miles per hour.

Approach Read · Plan · Solve · Check

Read. Name the two amounts and decide which one you want "one" of — that amount goes on the bottom.

Plan.

1. Divide the first amount by the second to get the amount per one unit.
2. Attach the correct unit label, like miles per hour or dollars per apple.
3. To compare options, reduce each one to the same per-one rate first.

Solve.

1. Divide in the order the question asks.

2. If a different amount is wanted, multiply the unit rate by that amount.

Check. Is the rate really per ONE unit, in the right order, with the right label? For a comparison, did you put both on the same basis before deciding?

Common traps — every wrong answer is one of these

rate inversion.
Divides in the wrong order and reports the flipped rate — hours per mile instead of miles per hour. Put the unit you want "one" of on the bottom.

failure to reduce to per-one.
Reports a total, or an amount for several units, instead of dividing all the way down to a single unit.

units/label error.
Gets the right number but the wrong or missing label, like "50" or "50 miles" instead of "50 miles per hour."

compare-without-normalizing.
Compares raw totals or package sizes instead of the per-unit rate when deciding which option is the better deal.

Practice

1. *A car travels 150 miles in 3 hours.*

What is its speed in miles per hour?

(A) 150 miles per hour
(B) 0.02 hours per mile
(C) 50 miles
(D) 50 miles per hour

2. *6 apples cost $4.50.*

What is the price per apple?

(A) 1.33 apples per dollar
(B) $4.50
(C) $0.90
(D) $0.75 per apple

3. *Store A sells 12 oz for $3.00; Store B sells 20 oz for $4.50.*

Which is the better deal (lower price per ounce)?

(A) Store B (lower price per ounce)

(B) Store A, because \$3.00 is less than \$4.50

(C) Store B, because it has more ounces

(D) Store A (comparing ounces per dollar inverted)

4. *A printer prints 240 pages in 8 minutes (a rate of 30 pages per minute).*

How many pages does it print in 5 minutes?

(A) 150 pages

(B) 30 pages

(C) 1200 pages

(D) About 0.17 pages

5. *A recipe needs 3 cups of flour for 12 cookies.*

How much flour is needed for 30 cookies?

(A) 120 cups

(B) 2.5 cups

(C) 0.25 cups

(D) 7.5 cups

Proportions & solving

node: sat.ratios.R3

The skill. A proportion says two ratios are equal, like $3/4 = x/20$. Solve it by cross-multiplying — multiply each top by the other bottom, set the products equal, and solve the resulting equation. You can also find the number that scales one ratio to the other. The key is lining up matching quantities: whatever is on top of the first fraction must be the same kind of thing on top of the second.

Worked example

Solve the proportion $5/8 = x/24$.

Read. The two ratios $5/8$ and $x/24$ are set equal. We want the value of x.

Plan.

1. Cross-multiply 5 by 24 and 8 by x.
2. Set the products equal and solve for x.

Solve.

$$\frac{5}{8} = \frac{x}{24} \qquad \text{the given proportion}$$
$$5 \times 24 = 8 \times x \qquad \text{cross-multiply, each top by the other bottom}$$
$$120 = 8x \qquad \text{multiply on the left}$$
$$x = 15 \qquad \text{divide both sides by 8}$$

Check. Put 15 back in: 15/24 reduces by 3 to 5/8, matching the left side.
Answer: x = 15.

Approach Read · Plan · Solve · Check

Read. Write the two equal ratios so matching quantities sit in matching spots — the same kind of thing on top in both.

Plan.

1. Cross-multiply: multiply each numerator by the other denominator.
2. Set the two products equal to form a simple equation.
3. Solve that equation for the unknown.

Solve.

1. Carry out the cross-multiplication.
2. Divide to isolate the unknown.

Check. Do the units line up across both ratios? Is the answer's size sensible next to the known ratio?

Common traps — every wrong answer is one of these

misaligned setup.
Puts quantities in mismatched spots — miles over hours on one side but hours over miles on the other. Keep the same kind of quantity on top in both ratios.

cross-multiply structure error.
Multiplies straight across (top times top) or multiplies a top by its own bottom, instead of each top by the OTHER bottom.

additive proportion error.
Solves by adding the gap between two terms instead of multiplying. Proportions scale by multiplying, not by adding a constant difference.

Practice

1. *Solve the proportion 3/4 = x/20.*

What is x?

(A) 15

(B) 19

(C) 26.67

(D) 60

2. *If 5 pens cost $2, how much do 15 pens cost? (Set up 5/2 = 15/x or 5/15 = 2/x.)*

What is the cost of 15 pens?

(A) $37.50

(B) $12

(C) $2.67

(D) $6

3. *Solve 8/x = 6/9.*

What is x?

(A) 5.33

(B) 11

(C) 12

(D) 0.75

4. *A map uses a scale where 2 inches represents 50 miles. Two cities are 7 inches apart on the map.*

What is the actual distance between the cities?

(A) 14.29 miles

(B) 55 miles

(C) 175 miles

(D) 100 miles

5. *A recipe for 4 servings uses 6 oz of cheese. You want to make 10 servings.*

How much cheese is needed?

(A) 12 oz

(B) 40 oz

(C) 6.67 oz

(D) 15 oz

Proportional relationships (y = kx)

node: sat.ratios.R4

The skill. Two amounts are proportional when one is always the same number times the other: y = kx. Their graph is a straight line through the origin, and y divided by x gives the same constant k at every point. That k is called the constant of proportionality, and it equals the unit rate and the slope. Find k by dividing y by x. If the line has a starting value that is not zero (y = mx + b with b not zero), the relationship is NOT proportional.

Worked example

y is proportional to x, and y = 21 when x = 6. Find the constant of proportionality, then find y when x = 10.

Read. Because y is proportional to x, we can write y = kx. We are told y = 21 when x = 6, and we want k and then y at x = 10.

Plan.

1. Divide y by x to find the constant k.
2. Multiply k by the new x of 10 to find y.

Solve.

$$
\begin{aligned}
k &= \frac{y}{x} && \text{the constant is y divided by x} \\
&= \frac{21}{6} && \text{use the point where x = 6, y = 21} \\
&= 3.5 && \text{divide} \\
y &= 3.5 \times 10 && \text{apply y = kx at x = 10} \\
&= 35 && \text{multiply}
\end{aligned}
$$

Check. Check with the first point: 3.5 x 6 = 21, which matches the given value.
Answer: k = 3.5, and y = 35 when x = 10.

Approach Read · Plan · Solve · Check

Read. Check the two things that make a relationship proportional: it passes through the origin, and y divided by x is constant.

Plan.

1. Pick any point and divide y by x to get the constant k.
2. Write the rule as y = kx.
3. Multiply by k to find a y, or divide by k to find an x.

Solve.

1. Divide y by x for the constant k.
2. Multiply or divide by k to answer the question.

Check. Is k found as y over x, not x over y? Does the line really go through the origin, so it is truly proportional?

Common traps — every wrong answer is one of these

constant-of-proportionality error.
Finds k as x over y (flipped) or otherwise reads the constant wrong. The constant of proportionality is y divided by x.

proportional vs non-proportional confusion.
Treats a relationship with a nonzero starting value (or a curved one) as proportional without checking that it passes through the origin with a constant y over x.

application error.
Uses k the wrong way — divides when it should multiply, or the reverse, or applies it in the wrong direction.

Practice

1. *A proportional relationship has the values $x = 2$, $y = 6$ and $x = 5$, $y = 15$.*

What is the constant of proportionality?

(A) It is not proportional, so k is undefined
(B) 1/3
(C) 8
(D) 3

2. *Consider $y = 4x$, $y = 4x + 3$, and $y = x^2$.*

Which represents a proportional relationship?

(A) y = 4x + 3
(B) y = 4x only
(C) Both y = 4x and y = 4x + 3
(D) $y = x^2$

3. *y is proportional to x, and when $x = 8$, $y = 20$.*

Which equation models the relationship?

(A) y = 20x
(B) y = 2.5x
(C) y = 0.4x

(D) y = x + 12

4. *Cost y is proportional to weight x, with constant of proportionality k = $1.50 per pound.*

How much does 12 pounds cost?

(A) $0.125
(B) $13.50
(C) $18
(D) $8

5. *A proportional relationship passes through the origin and the point (4, 10).*

What is the constant of proportionality (unit rate)?

(A) 2.5
(B) 0.4
(C) 6
(D) It cannot be found without more points

Percentages

node: sat.ratios.R5

The skill. A percent is a number out of 100, so 25 percent means 25 per 100 = 0.25 = 1/4. To find a percent OF a number, change the percent to a decimal and multiply. To find what percent one number is of another, divide the part by the whole and read the result as a percent. To find the whole when you know a part and its percent, divide the part by the decimal form of the percent. Watch the decimal point, and be clear about which number is the whole (the base).

Worked example

What is 30 percent of 150?

Read. We want 30 percent of 150. Here 150 is the whole, and 30 percent is the rate applied to it.

Plan.

1. Change 30 percent to the decimal 0.30.
2. Multiply 150 by 0.30.

Solve.

$$\begin{aligned} 30\% \text{ of } 150 &= 0.30 \times 150 && \text{write the percent as a decimal, then multiply} \\ &= 45 && \text{multiply} \end{aligned}$$

Check. 10 percent of 150 is 15, so 30 percent is 3 x 15 = 45, which matches.
Answer: 30 percent of 150 is 45.

Approach Read · Plan · Solve · Check

Read. Sort out the three pieces — the part, the whole (base), and the percent — and decide which one is unknown.

Plan.

1. Percent OF a number: multiply the whole by the percent written as a decimal.
2. What percent one number is of another: divide the part by the whole.
3. Find the whole: divide the part by the percent written as a decimal.

Solve.

1. Change the percent to a decimal by dividing by 100.
2. Multiply or divide as the plan says.

Check. Is the percent converted with the decimal point in the right place? Did you use the correct number as the whole?

Common traps — every wrong answer is one of these

percent-decimal conversion error.
Turns a percent into a decimal in the wrong place — 25 percent as 2.5 or 0.025 — or uses the percent number as if it were already a decimal. 25 percent is 0.25.

base/part inversion.
Uses the wrong number as the whole — divides the whole by the part, or treats the part as if it were the whole.

reverse-percent operation error.
When finding the whole from a part and a percent, multiplies instead of dividing. Finding the whole means dividing the part by the decimal percent.

Practice

1. *Find 25% of 80.*

What is the result?

(A) 200
(B) 3.2
(C) 2000
(D) 20

2. *What percent of 40 is 10?*

What is the percent?

(A) 25%

(B) 30%
(C) 4%
(D) 400%

3. *Write 0.6 as a percent.*

What is the percent?

(A) 600%
(B) 60%
(C) 6%
(D) 0.6%

4. *30 is 20% of what number?*

What is the number?

(A) 1.5
(B) 150
(C) 6
(D) 600

5. *A shirt is on sale for $45, which is 75% of its original price.*

What was the original price?

(A) $60
(B) $36
(C) $56.25
(D) $33.75

Percent change

node: sat.ratios.R6

The skill. Percent change is the change divided by the ORIGINAL value, then read as a percent — always over the original amount, never the new one. To apply a change, multiply by (1 + r) to increase or (1 - r) to decrease, where r is the percent written as a decimal. When several percent changes happen in a row, MULTIPLY their factors: a 20 percent rise then a 30 percent rise is 1.2 x 1.3 = 1.56, a 56 percent increase, not 50 percent. Successive changes never simply add.

Worked example

A price rises from $80 to $100. What is the percent increase?

Read. The price starts at 80 dollars (the original) and rises to 100 dollars. We want the

percent increase.

Plan.

1. Find the change: new value minus original.
2. Divide the change by the ORIGINAL value.
3. Read the result as a percent.

Solve.

$$\begin{aligned} \text{change} &= 100 - 80 && \text{new value minus original} \\ &= 20 && \text{subtract} \\ \text{percent increase} &= \frac{20}{80} && \text{divide the change by the original 80} \\ &= 0.25 && \text{divide} \\ &= 25\% && \text{write the decimal as a percent} \end{aligned}$$

Check. 25 percent of 80 is 20, and 80 + 20 = 100, the new price.
Answer: the price increased by 25 percent.

Approach Read · Plan · Solve · Check

Read. Find the original (starting) value, and note whether each change is a rise or a fall.

Plan.

1. For a percent change, divide the change by the original value.
2. To get a new value, multiply by (1 + r) for a rise or (1 - r) for a fall.
3. For several changes in a row, multiply the factors together.

Solve.

1. Compute change divided by original, or multiply the chain of factors.
2. Read the final decimal back as a percent if asked.

Check. Is the ORIGINAL value the base? Did you multiply successive changes instead of adding them? Is the direction right?

Common traps — every wrong answer is one of these

wrong base.
Uses the new value (or some other amount) as the bottom of the percent change instead of the original. Percent change is always over the original value.

successive-change additive error.
Adds percent changes that happen in a row instead of multiplying the factors — treating 20 percent then 30 percent as 50 percent, or plus 20 percent then minus 20 percent as back to the start.

direction/value error.
Applies a change the wrong way, adds the raw percent number instead of the percent amount, or reports the size of the change instead of the new value.

Practice

1. *A price rises from \$40 to \$50.*

What is the percent increase?

(A) 20%
(B) 125%
(C) 25%
(D) 10%

2. *A \$200 item increases by 15%.*

What is the new price?

(A) \$230
(B) \$215
(C) \$30
(D) \$170

3. *A population grows 10% one year, then 20% the next.*

What is the total percent growth?

(A) 132%
(B) 30%
(C) 32%
(D) 200%

4. *A \$100 item drops 20%, then the sale price rises 20%.*

What is the final price?

(A) \$104
(B) \$96
(C) \$80
(D) \$100

5. *A stock falls from \$50 to \$40. You want it to return to \$50.*

By what percent must it now rise?

(A) 25%

(B) 0% – it returns on its own

(C) 10%

(D) 20%

Apply ratios, rates & percent in context (multi-step)

node: sat.ratios.R7

The skill. Some problems chain rates, proportions, and percents together. Work one step at a time: find each in-between amount, carry the units along, and make sure you answer the exact quantity the question asks for. Use the correct base for every percent step, and do not stop early and report an in-between value as if it were the final answer.

Worked example

A car uses 5 gallons to go 150 miles. Gas costs $4 per gallon. What is the fuel cost of a 240-mile trip at the same efficiency?

Read. The car goes 150 miles on 5 gallons, so 30 miles per gallon. Gas is 4 dollars a gallon. We want the fuel cost for 240 miles, in dollars.

Plan.

1. Find the mileage: miles per gallon.
2. Divide the trip distance by the mileage to get the gallons needed.
3. Multiply the gallons by the price per gallon for the cost.

Solve.

$$\begin{aligned} \text{mileage} &= \frac{150}{5} && \text{miles divided by gallons} \\ &= 30 \text{ miles per gallon} && \text{divide} \\ \text{gallons} &= \frac{240}{30} && \text{trip miles divided by miles per gallon} \\ &= 8 \text{ gallons} && \text{divide} \\ \text{cost} &= 8 \times 4 && \text{gallons times price per gallon} \\ &= 32 && \text{multiply} \end{aligned}$$

Check. 8 gallons at 4 dollars is 32 dollars, and 8 gallons x 30 miles per gallon = 240 miles, the trip length.

Answer: the fuel cost is 32 dollars.

Approach Read · Plan · Solve · Check

Read. List what you are given and what is asked, with units, and map out the steps between them.

Plan.

1. Find the first in-between amount, such as a rate or a count.
2. Use it to find the next amount, tracking the base for any percent step.
3. Continue until you reach the exact quantity asked.

Solve.

1. Carry each step through with its units.
2. Combine percents that happen in a row by multiplying their factors.

Check. Did you answer the quantity actually asked, not an in-between one? Right units? Correct base at each percent step?

Common traps — every wrong answer is one of these

multi-step setup error.
Chooses the wrong operation or order, or skips a needed step — like multiplying distance by price without first going through the rate to find the amount used.

answer-interpretation/units.
Reports the wrong quantity, or drops or mislabels the units, so the answer does not match what was asked.

intermediate-value-as-final.
Stops at an in-between result and gives it as the final answer, or carries the wrong base from one step to the next.

Practice

1. *A car travels 180 miles using 6 gallons of fuel. Fuel costs $4 per gallon.*

What is the fuel cost for a 300-mile trip at the same efficiency?

(A) $40
(B) $1200
(C) $24
(D) 10 gallons

2. *An $80 jacket is marked 25% off, then 8% sales tax is applied to the sale price.*

What is the final cost?

(A) \$62.40

(B) \$64.80

(C) \$60

(D) 64.8 (no units)

3. *3 machines make 600 parts in 4 hours, all at the same constant rate.*

How many parts do 5 machines make in 6 hours?

(A) 1000

(B) 1500

(C) 900

(D) 3000

4. *A town's population is 8000. It grows 25% one year, then falls 20% the next.*

What is the final population?

(A) 10000

(B) 8000

(C) 8400

(D) 6400

5. *A recipe serves 4 and needs 2 cups of flour. You scale it for 10 servings, and flour costs \$0.50 per cup.*

What is the flour cost for the scaled recipe?

(A) \$2.50

(B) \$1.25

(C) 5 cups

(D) \$5.00

Unit conversion & dimensional analysis

node: sat.ratios.R8

The skill. To change units, multiply by conversion factors written as fractions that equal 1, set up so the unit you want to get rid of cancels — that unit goes on the bottom. String several factors together for a multi-step change, cancelling as you go. For a compound rate like miles per hour to feet per second, you must convert BOTH the top unit and the bottom unit. A quick check: moving to a smaller unit makes the number bigger, and moving to a larger unit makes it smaller.

Worked example

A faucet drips 3 milliliters per second. How many liters drip in one minute? (60 seconds in a minute, 1000 milliliters in a liter.)

Read. The drip rate is 3 milliliters each second. We want liters in one minute. A minute is 60 seconds, and a liter is 1000 milliliters.

Plan.

1. Multiply by 60 seconds per minute so seconds cancel, giving milliliters per minute.
2. Divide by 1000 milliliters per liter so milliliters cancel, giving liters per minute.

Solve.

$$\begin{aligned} \text{rate} &= \frac{3\ \text{mL}}{1\ \text{s}} \times \frac{60\ \text{s}}{1\ \text{min}} && \text{seconds cancel} \\ &= \frac{180\ \text{mL}}{1\ \text{min}} && \text{multiply 3 by 60} \\ &= \frac{180\ \text{mL}}{1\ \text{min}} \times \frac{1\ \text{L}}{1000\ \text{mL}} && \text{milliliters cancel} \\ &= 0.18\ \text{L per minute} && \text{divide 180 by 1000} \end{aligned}$$

Check. 180 milliliters a minute is a bit under a fifth of a liter, and 0.18 liter matches that.
Answer: 0.18 liter drips in one minute.

Approach Read · Plan · Solve · Check

Read. Note the units you start with and the units you want, and whether it is a single amount or a compound rate with two units to change.

Plan.

1. Write each conversion as a fraction equal to 1, with the unwanted unit on the bottom so it cancels.
2. Multiply the chain of factors together.
3. For a rate, convert the top unit and the bottom unit both.

Solve.

1. Cancel the matching units across the chain.
2. Multiply and divide the numbers.

Check. Did the unwanted units cancel? Did you convert EVERY unit in a rate? Did the number move the expected direction? Are the conversion values right?

Common traps — every wrong answer is one of these

factor-orientation error.
Uses a conversion factor upside down — divides when it should multiply, or the reverse — so the units do not cancel and the number moves the wrong way.

incomplete-chain error.
Converts only one unit of a compound rate, or stops partway through a multi-step chain, leaving a unit unconverted.

unit-tracking/label error.
Loses track of the units along the chain, reports the wrong unit, or forgets to convert the final unit.

conversion-factor value error.
Uses a wrong equivalence, like 1 hour = 100 minutes or 1 foot = 10 inches, or misremembers the factor.

Practice

1. *There are 60 minutes in an hour.*

How many minutes are in 3 hours?

(A) 0.05
(B) 180
(C) 300
(D) 20

2. *A car travels at 60 miles per hour.*

How far does it travel in 30 minutes?

(A) 120 miles
(B) 30 miles
(C) 1800 miles
(D) 2 miles

3. *A speed is given as 60 miles per hour.*

What is this speed in miles per minute?

(A) 1 mile per minute
(B) 60 miles per minute
(C) 3600 miles per minute
(D) 1/60 mile per minute

4. *There are 12 inches in a foot.*

How many inches are in 4 feet?

(A) 16 inches

(B) 1/3 inch

(C) 40 inches

(D) 48 inches

5. *A faucet drips 2 milliliters per second. There are 1000 milliliters in a liter.*

How many liters drip in one hour?

(A) 7.2 liters

(B) 2 liters

(C) 120 liters

(D) 7200 liters

One-variable data: measures of center & spread

node: sat.data.D1

The skill. The mean is the sum of all the values divided by how many values there are. The median is the middle value once the data is put in order from smallest to largest; when there is an even number of values, the median is the average of the two middle ones. The mode is the value that appears most often. The range is the largest value minus the smallest value, and it measures how spread out the data is, not where its center is. Because the mean uses every value, one very large or very small value (an outlier) pulls it noticeably. The median only cares about the middle position, so an outlier barely moves it. If you know the mean and the number of values, you can find the total by multiplying the mean by the count — that is how you recover a missing value.

Worked example

Find the median of the data set 9, 2, 14, 5, 2, 8.

Read. We are given six values: 9, 2, 14, 5, 2, 8. The median is the middle value once the data is in order. Because there are six values (an even count), the median will be the average of the two middle ones.

Plan.

1. Put the six values in order from smallest to largest.
2. Find the two values in the middle.
3. Average those two values.

Solve.

$$\begin{aligned} \text{ordered} &= 2,\ 2,\ 5,\ 8,\ 9,\ 14 && \text{sort from smallest to largest} \\ \text{middle two} &= 5 \text{ and } 8 && \text{the 3rd and 4th of six values} \\ \text{median} &= \frac{5+8}{2} && \text{average the two middle values} \\ &= 6.5 && \text{add, then divide by 2} \end{aligned}$$

Check. Three values (2, 2, 5) fall below 6.5 and three (8, 9, 14) fall above it, so 6.5 splits the data evenly — exactly what a median should do.

Answer: the median is 6.5.

Approach Read · Plan · Solve · Check

Read. Decide first whether the question asks for a center (mean, median, or mode) or for spread (the range). Also notice whether there is an outlier and whether the count of values is even or odd.

Plan.

1. For the mean, add every value, then divide by the count.
2. For the median, put the values in order, then take the middle value (or average the two middle values if the count is even).
3. For the mode, find the value that appears most often.
4. For the range, subtract the smallest value from the largest.
5. For a missing value, multiply the target mean by the count to get the total the values must add to.

Solve.

1. Carry out the addition, division, or subtraction on the ordered values.
2. Keep the center and the spread separate — do not mix them up.

Check. Did you answer the measure that was actually asked? Did you order the data before taking the median, and average the two middle values when the count was even? Remember an outlier moves the mean much more than the median, and the range (largest minus smallest) is a spread, not a center.

Common traps — every wrong answer is one of these

mean/median confusion.
Reports the mean when the median or mode was asked (or the reverse), averages only some of the values, or assumes a missing value simply equals the mean. Fix: reread which measure is asked, and for a missing value work from the total, not the mean itself.

median-without-ordering error. 1. Leaves the data in the order it was given instead of sorting it.

2. Points to whatever value happens to sit in the middle of that unsorted list.
3. With an even count, picks one middle value instead of averaging the two.
4. Fix: always sort the values first, then average the two middle values when the count is even.

outlier-resistance error.
Believes an outlier shifts the median as much as it shifts the mean, or that the mean is the resistant one. Fix: the mean uses every value, so an outlier drags it, while the median stays near the middle position.

range/spread vs center confusion.
Confuses the range (a spread) with a center, computes the range as largest plus smallest, or reports a spread when a center was asked. Fix: the range is largest minus smallest, and it is not a center.

Practice

1. *The data set is 4, 8, 8, 10, 20.*

What is the mean?

(A) 16
(B) 20
(C) 10
(D) 8

2. *Consider the data set 7, 2, 9, 4.*

What is the median?

(A) 7
(B) 5.5
(C) 5
(D) 9

3. *A data set of scores is 80, 82, 85, 88, 90. A new score of 20 is then added to the set.*

Which statement is true?

(A) Neither the mean nor the median changes
(B) The mean and median decrease by the same amount
(C) The mean decreases more than the median
(D) The median decreases more than the mean

4. *The data set is 12, 5, 19, 8, 5.*

What is the range?

(A) 9.8

(B) 14

(C) 5

(D) 24

5. *Four numbers have a mean of 10. Three of the numbers are 6, 8, and 12.*

What is the fourth number?

(A) 26

(B) 14

(C) 8.67

(D) 10

Distributions & standard deviation (comparing spread)

node: sat.data.D2

The skill. Standard deviation measures how far the values typically sit from the mean. A larger standard deviation means the values are more spread out; a smaller one means they cluster close to the mean, and it is zero only when every value is the same. On the SAT you compare standard deviations by reasoning about spread, not by using a formula. The range and the standard deviation both describe spread, but they are not the same — two data sets can share the same range yet have different standard deviations. The shape of the data also matters: when the data is roughly symmetric the mean and median are about equal; a long tail of high values (a right skew) pulls the mean above the median, and a long tail of low values (a left skew) pulls the mean below the median. Adding the same number to every value shifts the mean but leaves the spread — both range and standard deviation — unchanged.

Worked example

Set A is 5, 5, 5, 5 and Set B is 1, 3, 7, 9. Both sets have the same mean of 5. Which set has the larger standard deviation?

Read. Both sets have four values and the same mean, 5. Standard deviation measures how far the values typically fall from that mean, so we compare how far each set's values sit from 5.

Plan.

1. Confirm each set's mean is 5.
2. Look at how far each value sits from the mean in each set.
3. The set whose values sit farther from the mean has the larger standard deviation.

Solve.

Set A distances from $5 = 0,\ 0,\ 0,\ 0$	every value equals the mean
Set B distances from $5 = 4,\ 2,\ 2,\ 4$	each value differs from 5
spread of B $>$ spread of A	B's values sit farther from the mean

Check. Set A has no spread at all (all values identical), so its standard deviation is 0. Set B's values are clearly spread out, so it must have the larger standard deviation.

Answer: Set B has the larger standard deviation.

Approach Read · Plan · Solve · Check

Read. Decide whether the question is about spread (standard deviation or range) or about shape (skew), and whether you are comparing two data sets or changing one.

Plan.

1. To compare standard deviations, judge how tightly the values cluster around the mean, not just the extremes.
2. To place the mean against the median, follow the direction of the long tail.
3. If the same number is added to every value, remember the spread does not change.

Solve. Reason about the spread or shape directly; only work out a mean or median if you need it to anchor the comparison.

Check. More spread means a larger standard deviation, not a smaller one. The same range does not force the same standard deviation. A right skew puts the mean above the median. Adding the same number to every value leaves the standard deviation unchanged.

Common traps — every wrong answer is one of these

SD-direction error.
Reverses what standard deviation means — thinks tightly clustered data has a large one, or that more spread gives a smaller one, or that a higher mean forces a higher standard deviation. Fix: more spread from the mean means a larger standard deviation.

range-as-SD error.
Judges standard deviation only from the range or the two extremes, assuming an equal range forces an equal standard deviation or that the larger maximum always wins. Fix: standard deviation depends on how all the values sit around the mean, not just the two ends.

skew/mean-median error.
Gets the skew backward — thinks a right skew pulls the mean below the median, or that the mean always equals the median no matter the shape. Fix: the long tail pulls the mean toward it, so a right skew puts the mean above the median.

spread-invariance error.
Thinks adding the same number to every value changes the standard deviation, confusing a shift of the center with a change in spread. Fix: shifting every value by the same amount moves the center but leaves the spread unchanged.

Practice

1. *Set A is 10, 10, 10, 10. Set B is 2, 8, 12, 18.*

Which set has the greater standard deviation?

(A) Set B
(B) Set A
(C) Cannot be determined
(D) They are equal

2. *Set A is 1, 1, 1, 9 and Set B is 1, 4, 6, 9. Both sets have a range of 8.*

Which statement is true?

(A) Standard deviation cannot differ when the range is equal
(B) The two sets can have different standard deviations
(C) The set with the larger maximum has the larger standard deviation
(D) The sets must have the same standard deviation because they have the same range

3. *A data set is skewed to the right (it has a long tail of high values).*

How do the mean and median compare?

(A) The mean is less than the median
(B) The mean equals the median
(C) The mean is greater than the median
(D) The median is greater than the maximum

4. *Every value in a data set is increased by 5.*

What happens to the standard deviation?

(A) It stays the same
(B) It increases by 5
(C) It cannot be determined
(D) It increases, but not by 5

5. *Two classes have the same mean test score. Class A's scores cluster tightly near the mean; Class B's scores are widely spread out.*

Which class has the larger standard deviation?

(A) Class A

(B) Class B

(C) They are equal, because the means are equal

(D) Cannot be determined without the exact values

Two-variable data: scatterplots & line of best fit

node: sat.data.D3

The skill. A scatterplot plots paired data as points, one dot for each (x, y) pair, and a line of best fit is a straight line drawn to summarize the overall trend. For a line written as y = mx + b, the slope m is the predicted change in y for each one-unit increase in x — a rate — and b is the predicted value of y when x is 0. To predict a y-value, put the x-value into the whole equation. A residual is the actual value minus the predicted value, so a point sitting above the line has a positive residual. The sign of the slope tells you the direction of the association (rising or falling), and how closely the points hug the line tells you how strong it is. Choose the model from the pattern: a roughly constant change each step is linear, while repeated multiplying (doubling or halving, a curving shape) is exponential. Finally, an association between two variables does not by itself prove that one causes the other.

Worked example

A line of best fit is y = 4x + 7, where y is a bakery's daily sales in dollars and x is the number of loaves of a new bread put out. How much in sales does the model predict when 6 loaves are put out?

Read. The model is y = 4x + 7. Here x is the number of loaves, which is 6, and y is the predicted sales. To predict, we substitute x = 6 into the whole equation.

Plan.

1. Substitute x = 6 into y = 4x + 7.
2. Multiply first, then add the intercept.

Solve.

$y = 4x + 7$	start from the model
$= 4(6) + 7$	substitute x = 6
$= 24 + 7$	multiply first
$= 31$	add the intercept

Check. The slope 4 adds 4 for each loaf and the intercept 7 is the sales when x is 0, so 6

loaves add 6 times 4, or 24, above the starting 7 — a total of 31. Using the whole equation, not just the 24, gives the right prediction.

Answer: the model predicts 31 dollars in sales.

Approach Read · Plan · Solve · Check

Read. Figure out what is being asked: interpret the slope or the intercept, predict a value, compare an actual point to the prediction (a residual), judge the association, or choose the type of model.

Plan.

1. Read the slope as the change in y for each one-unit increase in x, and the intercept as the value when x is 0.
2. To predict, substitute the x-value into the whole equation.
3. For a residual, subtract the predicted value from the actual value.
4. Pick a linear model for constant change and an exponential model for repeated multiplying.

Solve. Substitute and work it out, keeping the slope (a rate) separate from any single predicted value.

Check. Did you read the slope as a rate rather than a total or the intercept? Did the prediction use the whole equation? Is the residual the actual minus the predicted, with the right sign? Does the model type match the pattern? Did you avoid claiming cause from a mere association?

Common traps — every wrong answer is one of these

best-fit slope interpretation error.
Misreads the slope of the line of best fit — treats it as a total, as a single predicted y-value, as the intercept (the value when x is 0), or as the x-variable, instead of the predicted change in y for each one-unit increase in x. Fix: the slope is a per-unit rate of change in y.

intercept/prediction misuse.
Predicts with only part of the equation — uses m times x but drops the intercept, or uses only the intercept, reads the wrong axis, or reports an actual data value as if it were the prediction. Fix: substitute x into the whole equation $y = mx + b$.

model-type choice error.
Fits a straight line to data that is clearly curving or multiplying, or forces an exponential model onto data that changes by a constant amount. Fix: constant additive change is linear; repeated multiplication is exponential.

association/causation misread.
Reads a falling association as a rising one (or the reverse), treats a correlation as proof of cause, or claims the model gives an exact rather than an estimated value. Fix: the slope's sign gives

the direction, and an association alone never proves causation.

Practice

1. *A scatterplot of plant height (y, in cm) versus time (x, in days) has line of best fit y = 1.5x + 4.*

What does 1.5 best represent?

(A) The predicted height when the time is 0 days
(B) The number of days
(C) The total height of the plant
(D) The predicted increase in height, in cm, for each additional day

2. *The line of best fit is y = 1.5x + 4, where y is height in cm and x is time in days.*

What height does the model predict at day 10?

(A) 4 cm
(B) 19 cm
(C) 15 cm
(D) 1.5 cm

3. *A quantity is measured over equal time periods as 4, 8, 16, 32 (it doubles each period).*

Which model best fits the data?

(A) An exponential model
(B) A linear model
(C) A line of best fit with constant slope
(D) No model can fit the data

4. *A scatterplot shows that as weekly hours of TV (x) increase, test score (y) decreases, with line of best fit y = -2x + 90.*

Which is the best interpretation?

(A) Each additional hour of TV is associated with a 2-point increase in score
(B) Watching TV causes scores to drop by exactly 2 points
(C) Test scores have no relationship with hours of TV
(D) Each additional hour of TV is associated with a predicted 2-point decrease in score

5. *The line of best fit for a car's value (y, in thousands of dollars) versus age (x, in years) is y = -1.2x + 18. A particular 3-year-old car actually sold for $16,000.*

What is the residual for this car?

(A) \$14,400

(B) \$2,000

(C) \$1,600 (the actual value is above the predicted value)

(D) -\$1,600

Probability & two-way tables

node: sat.data.D4

The skill. A probability is the number of favorable outcomes divided by the total number of outcomes, so it is always between 0 and 1. In a two-way table, a joint probability counts a single cell over the grand total (the chance of being in both categories at once). A marginal probability uses a whole row or column total over the grand total (the chance of being in one category overall). A conditional probability narrows the group first: the chance of A given B is the count that are both A and B divided by the total in group B. Wording like 'selected from Grade 10' or 'given that the student plays a sport' tells you the denominator is that group's total, not the grand total. The complement rule says the chance of not A is 1 minus the chance of A.

Worked example

In a class of 100 students, 50 are in Grade 10; of those, 35 play a sport. If a Grade 10 student is chosen at random, what is the probability the student plays a sport?

Read. The student is chosen from Grade 10, so the group is limited to the 50 Grade 10 students. Of those, 35 play a sport. We want the chance of playing a sport within that group — a conditional probability.

Plan.

1. Set the denominator to the size of the given group, the 50 Grade 10 students.
2. Set the numerator to how many of that group play a sport, 35.
3. Divide.

Solve.

$$
\begin{aligned}
P(\text{sport} \mid \text{Grade 10}) &= \frac{\text{Grade 10 who play a sport}}{\text{all Grade 10}} && \text{conditional: restrict to the given group} \\
&= \frac{35}{50} && \text{read the counts from the table} \\
&= 0.7 && \text{divide}
\end{aligned}
$$

Check. Because the student is known to be in Grade 10, we divide by 50, that group's total, not by 100. 35 out of 50 is 0.7, a sensible probability between 0 and 1.

Answer: the probability is 0.7.

Approach Read · Plan · Solve · Check

Read. Sort the question into one of three types: joint (a single cell over the grand total), marginal (a row or column total over the grand total), or conditional (limited to a given group).

Plan.

1. Find the correct count for the numerator.
2. Choose the denominator: the grand total for a joint or marginal probability, or the given group's total for a conditional.
3. If the question says 'not', subtract the probability from 1.

Solve. Read the right cell and total from the table, then form the ratio of favorable to total.

Check. Does the denominator match the group the selection comes from? Does a conditional use the subgroup total instead of the grand total? Is the ratio favorable over total, not flipped? Did you take the complement when the question said 'not'?

Common traps — every wrong answer is one of these

conditional-denominator error.
For a conditional probability, divides by the grand total or by the wrong subgroup total instead of the total of the group the selection is limited to. Fix: the denominator is the given group's total.

joint/conditional confusion.
Uses a single cell over the grand total (a joint probability) when a conditional was asked, or works out a conditional when a joint was asked. Fix: 'given' signals a conditional with a restricted denominator, while 'and' signals a joint over the grand total.

ratio-inversion/complement error.
Flips the ratio (total over favorable, or favorable over unfavorable), or forgets the complement and reports the chance of A when asked for the chance of not A. Fix: probability is favorable over total, and 'not A' is 1 minus the chance of A.

table-reading error.
Reads the wrong cell, row, or column, or uses a row or column total where a single cell count is needed. Fix: locate the exact cell the question describes before dividing.

Practice

1. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

If one student is selected at random, what is the probability the student plays a sport?

(A) 0.45

(B) 0.55

(C) 100/55

(D) 55/45

2. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

If a Grade 10 student is selected at random, what is the probability the student plays a sport?

(A) 0.5

(B) 0.7

(C) 0.55

(D) 0.35

3. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

Among students who play a sport, what is the probability a randomly selected one is in Grade 9?

(A) 55/20

(B) 20/50 (0.40)

(C) 20/100 (0.20)

(D) 20/55 (about 0.36)

4. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

What is the probability that a randomly selected student is in Grade 9 AND plays a sport?

(A) 0.20

(B) 0.55

(C) 0.50

(D) 0.70

5. *A two-way table of 100 students: Grade 9 has 20 who play a sport and 30 who do not (50 total); Grade 10 has 35 who play a sport and 15 who do not (50 total); overall 55 play a sport and 45 do not.*

What is the probability that a randomly selected student does NOT play a sport?

(A) 0.55

(B) 0.30

(C) 0.90

(D) 0.45

Inference, margin of error & evaluating statistical claims

node: sat.data.D5

The skill. A sample statistic is an estimate of a value for a whole population. The margin of error gives a range of plausible values around that estimate — you read it as the estimate plus or minus the margin. A larger random sample makes the margin of error smaller. A result can be generalized only to the population that the sample was randomly drawn from, not to a wider group. To claim that one thing causes another, you need a randomized controlled experiment, meaning subjects are randomly assigned to a treatment group or a control group. An observational study, which only watches what happens, can show that two things go together but cannot show that one causes the other. In short, random selection lets you generalize to the population, and random assignment lets you claim cause.

Worked example

A random sample of voters gives 45% support for a proposal, with a margin of error of 4 percentage points. What is the plausible range for the true level of support, and what would make that range narrower?

Read. The estimate is 45%, and the margin of error is 4 percentage points. The margin of error sets a range around the estimate, read as 45% plus or minus 4%. We also want to know how to narrow that range.

Plan.

1. Subtract the margin from the estimate for the low end.
2. Add the margin to the estimate for the high end.
3. Recall that a larger random sample shrinks the margin of error.

Solve.

$$\begin{aligned} \text{low end} &= 45\% - 4\% = 41\% && \text{estimate minus the margin} \\ \text{high end} &= 45\% + 4\% = 49\% && \text{estimate plus the margin} \\ \text{plausible range} &= 41\% \text{ to } 49\% && \text{the interval around the estimate} \end{aligned}$$

Check. The interval is centered on 45% and reaches 4 points on each side, which matches 'plus or minus 4 percentage points.' To make it narrower, survey a larger random sample, since more data tightens the estimate.

Answer: the true support is plausibly between 41% and 49%, and a larger random sample would narrow that range.

Approach Read · Plan · Solve · Check

Read. Decide what the question is really claiming: a range around an estimate (margin of error), a generalization to a population, or a cause-and-effect conclusion.

Plan.

1. For a margin of error, build the interval as the estimate plus or minus the margin, and recall that a larger sample shrinks it.
2. For a generalization, check which population was randomly sampled.
3. For a causal claim, check whether subjects were randomly assigned (an experiment) or only observed.

Solve. State the plausible interval, name the population the claim can reach, and say whether cause or only association is supported.

Check. Is the margin of error a plausible range rather than an absolute limit? Does the generalization stay within the sampled population? Is a causal claim made only when there was random assignment?

Common traps — every wrong answer is one of these

margin-of-error meaning error.
Treats the margin of error as an absolute limit the true value cannot cross, takes the estimate as exact, or thinks a larger sample gives a larger margin. Fix: it is a plausible range, and a larger sample makes it smaller.

generalization-scope error.
Stretches the result to a population wider than the one randomly sampled, or generalizes from a biased, non-random sample. Fix: conclusions reach only the population that was randomly sampled.

causation-from-observation error.
Draws a cause-and-effect conclusion from an observational study, treating an observed association as proof of cause. Fix: without random assignment you can claim association but not causation.

selection-vs-assignment confusion.
Mixes up random selection with random assignment — thinks random selection allows a causal claim, or that random assignment allows generalizing to a broader population. Fix: selection supports generalization; assignment supports causation.

Practice

1. *A poll estimates that 60% of voters support a measure, with a margin of error of 3 percentage points.*

Which is the best interpretation?

(A) Exactly 3% of the responses were recorded incorrectly

(B) The true level of support is plausibly between 57% and 63%

(C) Support is definitely 60%

(D) Support absolutely cannot be outside 57% to 63%

2. *A researcher wants the margin of error of an estimate to be smaller.*

Which change would achieve this?

(A) Decrease the sample size

(B) Increase the sample size

(C) Survey fewer people but more carefully

(D) Sample size has no effect on the margin of error

3. *Researchers randomly sampled students at one high school and found that 40% bike to school.*

To which group can this result be appropriately generalized?

(A) All high school students in the country

(B) Students at that high school

(C) All residents of the city

(D) All adults who commute to work

4. *An observational study finds that people who drink more coffee tend to report higher productivity.*

What can be appropriately concluded?

(A) Higher productivity causes people to drink more coffee

(B) There is an association, but the study cannot establish that coffee causes higher productivity

(C) Coffee and productivity are unrelated

(D) Drinking more coffee causes higher productivity

5. *In an experiment, volunteers from one college were randomly ASSIGNED to use either a new study method or the usual one. The new-method group scored higher.*

What is appropriate to conclude?

(A) The new method likely caused the higher scores, but the result may not generalize beyond similar volunteers

(B) The result generalizes to all adults but no causal claim can be made

(C) The new method caused higher scores for all college students everywhere

(D) No causal claim is possible; this is only an association

Weighted averages & combined means

node: sat.data.D6

The skill. A weighted average gives each value an importance, or weight, set by how often it occurs or how large its group is. You multiply each value by its weight, add those products, and divide by the total of the weights. Simply averaging two group means gives the overall mean only when the groups are the same size; when the groups differ in size you must weight by the counts. The key idea is that a total equals a mean times a count: rebuild each group's total, add the totals, and divide by the total number of members. To find a missing value or weight that produces a target average, work backward from the total the values must reach, not from the averages themselves.

Worked example

Class A has 10 students with an average score of 80. Class B has 30 students with an average score of 90. What is the average score of all 40 students?

Read. The two classes are different sizes: 10 students at an average of 80 and 30 students at an average of 90. Because the sizes differ, we weight by the number of students rather than just averaging 80 and 90.

Plan.

1. Rebuild each class's total points as its average times its number of students.
2. Add the two totals.
3. Divide by the total number of students, 40.

Solve.

$$\begin{aligned} \text{Class A total} &= 80 \times 10 = 800 && \text{mean times count} \\ \text{Class B total} &= 90 \times 30 = 2700 && \text{mean times count} \\ \text{combined mean} &= \frac{800 + 2700}{40} && \text{all points over all students} \\ &= \frac{3500}{40} && \text{add the totals} \\ &= 87.5 && \text{divide} \end{aligned}$$

Check. The plain average of 80 and 90 is 85, but Class B has more students, so the combined mean should sit closer to 90. 87.5 does, which confirms the weighting.

Answer: the combined average is 87.5.

Approach Read · Plan · Solve · Check

Read. Check whether the groups or categories are the same size. If they are not, averaging the means directly is wrong. Identify each value and the weight that goes with it (a count or a percent).

Plan.

1. Multiply each value by its weight and add the products.
2. Divide by the total of the weights.
3. As a check, rebuild each group's total as mean times count, add the totals, and divide by the total count.
4. For a missing piece, set the required total equal to the target mean times the total count, then solve.

Solve. Carry out the multiplications and the division, or rebuild and combine the totals.

Check. Did you weight by the group sizes instead of just averaging the means? Did you pair each weight with the right value? Did you divide by the total weight? For a reverse problem, did you work from the totals?

Common traps — every wrong answer is one of these

average-of-averages error.
Averages the group means directly even though the groups are different sizes, instead of weighting by the counts. Fix: weight each mean by its group size before combining.

weight/value swap.
Pairs a value with the wrong weight, or treats the weight as the value and the value as the weight. Fix: keep each value matched to its own weight.

total-vs-mean error.
Mishandles total equals mean times count — reports a total where a mean belongs, forgets to divide by the total count, or adds means or counts wrongly. Fix: after adding the totals, divide by the total count to get back to a mean.

reverse weighted-average error.
When solving for a missing value that gives a target average, reads the new average as the missing value or uses the change in the average, instead of working from the required total. Fix: use total equals target mean times count, then subtract the known total.

Practice

1. *Class A has 10 students averaging 80. Class B has 30 students averaging 90.*

What is the combined average of all 40 students?

(A) 87.5
(B) 3500
(C) 85
(D) 170

2. *A course grade is 30% homework and 70% exam. A student scores 90 on homework and 80 on*

the exam.

What is the weighted course grade?

(A) 87
(B) 83
(C) 170
(D) 85

3. *In a weighted average, the value 4 has weight 3 and the value 10 has weight 1.*

What is the weighted average?

(A) 8.5
(B) 7
(C) 5.5
(D) 22

4. *After 4 tests a student's average is 85. After a 5th test, the average rises to 86.*

What did the student score on the 5th test?

(A) 1
(B) 86
(C) 90
(D) 87

5. *Group X has 5 people with mean 12. Group Y has 15 people with mean 20.*

What is the mean of all 20 people?

(A) 32
(B) 16
(C) 360
(D) 18

Line of best fit: prediction & residuals

node: sat.data.D7

The skill. To predict from a line of best fit written as y = mx + b, put the input in for x and work out y. A residual is the actual value minus the predicted value: a point above the line has a positive residual, meaning the model predicted too low there, and a point below the line has a negative residual, meaning the model predicted too high. Predicting for an input inside the range of the data you collected is called interpolation and is reasonable. Predicting far outside that range is

called extrapolation and is less trustworthy, because the pattern may not continue. The slope of the fitted line is the modeled rate of change, measured in units of y for each unit of x.

Worked example

A line of best fit is $y = 3x + 5$. At $x = 4$ the actual data value is 20. Find the model's prediction at $x = 4$ and the residual for this point.

Read. The model is $y = 3x + 5$. At $x = 4$ we first predict y, then compare it to the actual value of 20. The residual is the actual value minus the predicted value.

Plan.

1. Substitute $x = 4$ into $y = 3x + 5$ to get the prediction.
2. Subtract the predicted value from the actual value, 20.
3. Read the sign of the residual.

Solve.

$$\begin{aligned} \text{predicted} &= 3(4) + 5 && \text{substitute } x = 4 \\ &= 17 && \text{multiply, then add} \\ \text{residual} &= \text{actual} - \text{predicted} && \text{definition of a residual} \\ &= 20 - 17 && \text{substitute the actual and predicted values} \\ &= 3 && \text{subtract} \end{aligned}$$

Check. The actual value 20 is above the predicted 17, so the point lies above the line and the residual should be positive. It is +3, meaning the model predicted 3 too low here.

Answer: the prediction is 17 and the residual is +3.

Approach Read · Plan · Solve · Check

Read. Decide which task it is: a prediction (put in x), a residual (compare the actual to the predicted), a reliability judgment (inside the data range or far outside it), or reading the slope.

Plan.

1. To predict, substitute the x-value into $y = mx + b$ and evaluate.
2. For a residual, subtract the predicted value from the actual value, and read the sign as predicting too low or too high.
3. To judge reliability, check whether the input sits inside the range of the collected data.
4. To read the slope, state the rate of change with its units of y per x.

Solve. Evaluate or subtract as the task needs, keeping the sign of a residual meaningful.

Check. Did you substitute into the right variable? Is the residual the actual minus the predicted, so that a point above the line is positive? Is the input inside the data range? Are the slope's units y per x?

Common traps — every wrong answer is one of these

prediction computation error.
Misuses the equation — drops the intercept, evaluates in the wrong order, or solves for x instead of finding y. Fix: substitute the x-value into the whole equation and evaluate y.

residual sign/direction error.
Reverses the residual — calls a point above the line negative, subtracts actual from predicted, or swaps predicting too low and too high. Fix: residual is actual minus predicted, so a point above the line is positive.

extrapolation-reliability error.
Treats a prediction far outside the data range as just as trustworthy as one inside it, not seeing that it is an extrapolation. Fix: predictions far beyond the data are less reliable because the pattern may not hold.

slope-as-rate misread.
Misreads the slope's meaning or units — using the intercept as the rate, or flipping the units. Fix: the slope is the change in y per one unit of x, in units of y per x.

Practice

1. *A line of best fit is $y = 3x + 5$.*

What does the model predict for y when x = 4?

(A) 12
(B) 27
(C) x = -1/3
(D) 17

2. *A data point lies above the line of best fit.*

What is true of its residual?

(A) It is positive; the model underpredicts this point
(B) It is negative; the model overpredicts this point
(C) It is zero
(D) It cannot be determined

3. *For a particular x, the actual y is 20 and the model predicts 17.*

What is the residual?

(A) 37
(B) -3

(C) 3

(D) about 1.18

4. *A model is built from data with x between 1 and 10, then used to predict y at $x = 100$.*

How should this prediction be regarded?

(A) As just as reliable as predictions within the data range

(B) As guaranteed accurate because it uses the best-fit line

(C) As an extrapolation that may be unreliable

(D) As impossible to compute

5. *A line of best fit $y = 2.5x + 10$ models a plant's height in centimeters versus its age in weeks.*

What does the slope mean, and what height does the model give at week 6?

(A) Height grows about 10 cm per week; 25 cm at week 6

(B) Height grows about 2.5 cm per week; 25 cm at week 6

(C) Height grows about 2.5 cm per week; 15 cm at week 6

(D) Height grows about 2.5 weeks per cm; 25 cm at week 6

Compound & conditional probability

node: sat.data.D8

The skill. For independent events — events that do not affect each other — the chance that both happen is found by multiplying: the chance of A and B is the chance of A times the chance of B. The chance that either one happens is found by adding and then subtracting the overlap: the chance of A or B is the chance of A plus the chance of B minus the chance of A and B; when the events cannot happen together (mutually exclusive) that overlap is zero. The complement rule says the chance of not A is 1 minus the chance of A. A conditional probability, the chance of A given B, is the chance of A and B divided by the chance of B. In short, 'and' multiplies for independent events, 'or' adds while removing any overlap, and a complement subtracts from 1.

Worked example

For two events, P(A) = 0.6, P(B) = 0.3, and P(A and B) = 0.1. What is P(A or B)?

Read. We know the chance of A, the chance of B, and the chance of both together (the overlap). The 'or' rule adds the two chances and then removes the overlap so the shared part is not counted twice.

Plan.

1. Add P(A) and P(B).
2. Subtract the overlap P(A and B).

Solve.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) && \text{the addition rule for 'or'} \\ &= 0.6 + 0.3 - 0.1 && \text{substitute the given probabilities} \\ &= 0.8 && \text{add, then subtract the overlap} \end{aligned}$$

Check. Adding 0.6 and 0.3 gives 0.9, but that counts the 0.1 overlap twice; removing it once leaves 0.8, a valid probability between 0 and 1.

Answer: P(A or B) is 0.8.

Approach Read · Plan · Solve · Check

Read. Decide which kind of question it is: 'and' (both happen), 'or' (either happens), a complement ('not'), or a conditional ('given'). Then note whether the events are independent or mutually exclusive.

Plan.

1. For 'and' with independent events, multiply the probabilities.
2. For 'or', add the probabilities and subtract the overlap.
3. For a complement, subtract from 1.
4. For a conditional, divide the joint probability by the probability of the given event.

Solve. Apply the matching rule with the probabilities you are given.

Check. Did you multiply for 'and' and add for 'or', not the reverse? Did you subtract the overlap in an 'or'? Is the complement 1 minus the probability? Is the conditional's denominator the given event?

Common traps — every wrong answer is one of these

and/or operation swap.
Uses the wrong combining rule — adds for 'and', or multiplies for 'or'. Fix: 'and' multiplies (for independent events) and 'or' adds.

independence/overlap error.
Treats dependent events as independent, or works out an 'or' without subtracting the overlap and double-counts the shared outcomes. Fix: subtract the overlap in an 'or', and check that events are really independent before multiplying.

complement error.
Computes not A wrongly — as the chance of A, as its negative, or as 1 divided by it, instead of 1 minus it. Fix: the chance of not A is 1 minus the chance of A.

conditional-denominator error.
Uses the wrong denominator for a conditional — the grand total or the joint probability instead of the probability of the given event. Fix: divide the joint probability by the probability of the

given event.

Practice

1. *A fair coin and a fair six-sided die are each tossed once.*

What is the probability of getting heads AND a 3?

(A) 7/12
(B) 2/3
(C) 1/12
(D) 1/8

2. *A fair six-sided die is rolled once.*

What is the probability of rolling a 2 OR a 5?

(A) 2/5
(B) 1/6
(C) 1/36
(D) 1/3

3. *The probability that it rains tomorrow is 0.3.*

What is the probability that it does NOT rain?

(A) 0.3
(B) -0.3
(C) 0.7
(D) about 3.33

4. *In a group, P(likes tea and coffee) = 0.2 and P(likes coffee) = 0.5.*

What is P(likes tea GIVEN likes coffee)?

(A) 0.4
(B) 0.7
(C) 0.1
(D) 0.2

5. *For two events, P(A) = 0.6, P(B) = 0.3, and P(A and B) = 0.1.*

What is P(A or B)?

(A) 0.1

(B) 0.9

(C) 0.18

(D) 0.8

Answer Key — Problem-Solving & Data Analysis

Ratios & equivalent ratios — Q1
Correct: **(A)**. *Traps:* **(B)** order/inversion error; **(C)** additive scaling; **(D)** order/inversion error.

Ratios & equivalent ratios — Q2
Correct: **(C)**. *Traps:* **(A)** order/inversion error; **(B)** part-part vs part-whole confusion; **(D)** part-part vs part-whole confusion.

Ratios & equivalent ratios — Q3
Correct: **(D)**. *Traps:* **(A)** order/inversion error; **(B)** simplification error; **(C)** order/inversion error.

Ratios & equivalent ratios — Q4
Correct: **(D)**. *Traps:* **(A)** additive scaling; **(B)** order/inversion error; **(C)** additive scaling.

Ratios & equivalent ratios — Q5
Correct: **(B)**. *Traps:* **(A)** additive scaling; **(C)** order/inversion error; **(D)** additive scaling.

Unit rates — Q1
Correct: **(D)**. *Traps:* **(A)** failure to reduce to per-one; **(B)** rate inversion; **(C)** units/label error.

Unit rates — Q2
Correct: **(D)**. *Traps:* **(A)** rate inversion; **(B)** failure to reduce to per-one; **(C)** Dividing the wrong way.

Unit rates — Q3
Correct: **(A)**. *Traps:* **(B)** compare-without-normalizing; **(C)** compare-without-normalizing; **(D)** rate inversion.

Unit rates — Q4
Correct: **(A)**. *Traps:* **(B)** failure to reduce to per-one; **(C)** failure to reduce to per-one; **(D)** rate inversion.

Unit rates — Q5
Correct: **(D)**. *Traps:* **(A)** rate inversion; **(B)** failure to reduce to per-one; **(C)** failure to reduce to per-one.

Proportions & solving — Q1
Correct: **(A)**. *Traps:* **(B)** additive proportion error; **(C)** cross-multiply structure error; **(D)** Dividing the wrong way.

Proportions & solving — Q2
Correct: **(D)**. *Traps:* **(A)** misaligned setup; **(B)** additive proportion error; **(C)** misaligned setup.

Proportions & solving — Q3
Correct: **(C)**. *Traps:* **(A)** cross-multiply structure error; **(B)** additive proportion error; **(D)** Dividing the wrong way.

Proportions & solving — Q4
Correct: **(C)**. *Traps:* **(A)** misaligned setup; **(B)** additive proportion error; **(D)** cross-multiply structure error.

Proportions & solving — Q5
Correct: **(D)**. *Traps:* **(A)** additive proportion error; **(B)** cross-multiply structure error; **(C)** misaligned setup.

Proportional relationships (y = kx) — Q1
Correct: **(D)**. *Traps:* **(A)** proportional vs non-proportional confusion; **(B)** constant-of-proportionality error; **(C)** application error.

Proportional relationships (y = kx) — Q2
Correct: **(B)**. *Traps:* **(A)** proportional vs non-proportional confusion; **(C)** proportional vs non-proportional confusion; **(D)** proportional vs non-proportional confusion.

Proportional relationships (y = kx) — Q3
Correct: **(B)**. *Traps:* **(A)** application error; **(C)** constant-of-proportionality error; **(D)** proportional vs non-proportional confusion.

Proportional relationships (y = kx) — Q4
Correct: **(C)**. *Traps:* **(A)** constant-of-proportionality error; **(B)** application error; **(D)** application error.

Proportional relationships (y = kx) — Q5
Correct: **(A)**. *Traps:* **(B)** constant-of-proportionality error; **(C)** application error; **(D)** proportional vs non-proportional confusion.

Percentages — Q1
Correct: **(D)**. *Traps:* **(A)** percent-decimal conversion error; **(B)** base/part inversion; **(C)** percent-decimal conversion error.

Percentages — Q2
Correct: **(A)**. *Traps:* **(B)** reverse-percent operation error; **(C)** percent-decimal conversion error; **(D)** base/part inversion.

Percentages — Q3
Correct: **(B)**. *Traps:* **(A)** percent-decimal conversion error; **(C)** percent-decimal conversion error; **(D)** percent-decimal conversion error.

Percentages — Q4
Correct: **(B)**. *Traps:* **(A)** reverse-percent operation error; **(C)** reverse-percent operation error; **(D)** reverse-percent operation error.

Percentages — Q5
Correct: **(A)**. *Traps:* **(B)** base/part inversion; **(C)** reverse-percent operation error; **(D)** reverse-percent operation error.

Percent change — Q1
Correct: **(C)**. *Traps:* **(A)** wrong base; **(B)** wrong base; **(D)** wrong base.

Percent change — Q2
Correct: **(A)**. *Traps:* **(B)** direction/value error; **(C)** direction/value error; **(D)** direction/value error.

Percent change — Q3
Correct: **(C)**. *Traps:* **(A)** wrong base; **(B)** successive-change additive error; **(D)** successive-change additive error.

Percent change — Q4
Correct: **(B)**. *Traps:* **(A)** direction/value error; **(C)** direction/value error; **(D)** successive-change additive error.

Percent change — Q5
Correct: **(A)**. *Traps:* **(B)** successive-change additive error; **(C)** wrong base; **(D)** wrong base.

Apply ratios, rates & percent in context (multi-step) — Q1
Correct: **(A)**. *Traps:* **(B)** multi-step setup error; **(C)** intermediate-value-as-final; **(D)** intermediate-value-as-final.

Apply ratios, rates & percent in context (multi-step) — Q2
Correct: **(B)**. *Traps:* **(A)** multi-step setup error; **(C)** intermediate-value-as-final; **(D)** answer-interpretation/units.

Apply ratios, rates & percent in context (multi-step) — Q3
Correct: **(B)**. *Traps:* **(A)** intermediate-value-as-final; **(C)** intermediate-value-as-final; **(D)** multi-step setup error.

Apply ratios, rates & percent in context (multi-step) — Q4
Correct: **(B)**. *Traps:* **(A)** intermediate-value-as-final; **(C)** answer-interpretation/units; **(D)** multi-step setup error.

Apply ratios, rates & percent in context (multi-step) — Q5
Correct: **(A)**. *Traps:* **(B)** multi-step setup error; **(C)** answer-interpretation/units; **(D)** intermediate-value-as-final.

Unit conversion & dimensional analysis — Q1
Correct: **(B)**. *Traps:* **(A)** factor-orientation error; **(C)** conversion-factor value error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q2
Correct: **(B)**. *Traps:* **(A)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q3
Correct: **(A)**. *Traps:* **(B)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** factor-orientation error.

Unit conversion & dimensional analysis — Q4
Correct: **(D)**. *Traps:* **(A)** conversion-factor value error; **(B)** factor-orientation error; **(C)** conversion-factor value error.

Unit conversion & dimensional analysis — Q5
Correct: **(A)**. *Traps:* **(B)** incomplete-chain error; **(C)** incomplete-chain error; **(D)** unit-tracking/label error.

One-variable data: measures of center & spread — Q1
Correct: **(C)**. *Traps:* **(A)** range/spread vs center confusion; **(B)** range/spread vs center confusion; **(D)** mean/median confusion.

One-variable data: measures of center & spread — Q2
Correct: **(B)**. *Traps:* **(A)** median-without-ordering error; **(C)** median-without-ordering error; **(D)** median-without-ordering error.

One-variable data: measures of center & spread — Q3
Correct: **(C)**. *Traps:* **(A)** outlier-resistance error; **(B)** outlier-resistance error; **(D)** outlier-resistance error.

One-variable data: measures of center & spread — Q4
Correct: **(B)**. *Traps:* **(A)** mean/median confusion; **(C)** range/spread vs center confusion; **(D)** range/spread vs center confusion.

One-variable data: measures of center & spread — Q5
Correct: **(B)**. *Traps:* **(A)** range/spread vs center confusion; **(C)** mean/median confusion; **(D)** mean/median confusion.

Distributions & standard deviation (comparing spread) — Q1
Correct: **(A)**. *Traps:* **(B)** SD-direction error; **(C)** SD-direction error; **(D)** SD-direction error.

Distributions & standard deviation (comparing spread) — Q2
Correct: **(B)**. *Traps:* **(A)** range-as-SD error; **(C)** range-as-SD error; **(D)** range-as-SD error.

Distributions & standard deviation (comparing spread) — Q3
Correct: **(C)**. *Traps:* **(A)** skew/mean-median error; **(B)** skew/mean-median error; **(D)** skew/mean-median error.

Distributions & standard deviation (comparing spread) — Q4
Correct: **(A)**. *Traps:* **(B)** spread-invariance error; **(C)** spread-invariance error; **(D)** spread-invariance error.

Distributions & standard deviation (comparing spread) — Q5
Correct: **(B)**. *Traps:* **(A)** SD-direction error; **(C)** spread-invariance error; **(D)** SD-direction error.

Two-variable data: scatterplots & line of best fit — Q1
Correct: **(D)**. *Traps:* **(A)** best-fit slope interpretation error; **(B)** best-fit slope interpretation error; **(C)** best-fit slope interpretation error.

Two-variable data: scatterplots & line of best fit — Q2
Correct: **(B)**. *Traps:* **(A)** intercept/prediction misuse; **(C)** intercept/prediction misuse; **(D)** intercept/prediction misuse.

Two-variable data: scatterplots & line of best fit — Q3
Correct: **(A)**. *Traps:* **(B)** model-type choice error; **(C)** model-type choice error; **(D)** model-type choice error.

Two-variable data: scatterplots & line of best fit — Q4
Correct: **(D)**. *Traps:* **(A)** association/causation misread; **(B)** association/causation misread; **(C)** association/causation misread.

Two-variable data: scatterplots & line of best fit — Q5
Correct: **(C)**. *Traps:* **(A)** intercept/prediction misuse; **(B)** best-fit slope interpretation error; **(D)** intercept/prediction misuse.

Probability & two-way tables — Q1
Correct: **(B)**. *Traps:* **(A)** table-reading error; **(C)** ratio-inversion/complement error; **(D)** ratio-inversion/complement error.

Probability & two-way tables — Q2
Correct: **(B)**. *Traps:* **(A)** table-reading error; **(C)** joint/conditional confusion; **(D)** conditional-denominator error.

Probability & two-way tables — Q3
Correct: **(D)**. *Traps:* **(A)** ratio-inversion/complement error; **(B)** conditional-denominator error; **(C)** joint/conditional confusion.

Probability & two-way tables — Q4
Correct: **(A)**. *Traps:* **(B)** table-reading error; **(C)** table-reading error; **(D)** joint/conditional confusion.

Probability & two-way tables — Q5
Correct: **(D)**. *Traps:* **(A)** ratio-inversion/complement error; **(B)** table-reading error; **(C)** conditional-denominator error.

Inference, margin of error & evaluating statistical claims — Q1
Correct: **(B)**. *Traps:* **(A)** margin-of-error meaning error; **(C)** margin-of-error meaning error; **(D)** margin-of-error meaning error.

Inference, margin of error & evaluating statistical claims — Q2
Correct: **(B)**. *Traps:* **(A)** margin-of-error meaning error; **(C)** margin-of-error meaning error; **(D)** margin-of-error meaning error.

Inference, margin of error & evaluating statistical claims — Q3
Correct: **(B)**. *Traps:* **(A)** generalization-scope error; **(C)** generalization-scope error; **(D)** generalization-scope error.

Inference, margin of error & evaluating statistical claims — Q4
Correct: **(B)**. *Traps:* **(A)** causation-from-observation error; **(C)** causation-from-observation error; **(D)** causation-from-observation error.

Inference, margin of error & evaluating statistical claims — Q5
Correct: **(A)**. *Traps:* **(B)** selection-vs-assignment confusion; **(C)** selection-vs-assignment confusion; **(D)** selection-vs-assignment confusion.

Weighted averages & combined means — Q1
Correct: **(A)**. *Traps:* **(B)** total-vs-mean error; **(C)** average-of-averages error; **(D)** total-vs-mean error.

Weighted averages & combined means — Q2
Correct: **(B)**. *Traps:* **(A)** weight/value swap; **(C)** total-vs-mean error; **(D)** average-of-averages error.

Weighted averages & combined means — Q3
Correct: **(C)**. *Traps:* **(A)** weight/value swap; **(B)** average-of-averages error; **(D)** total-vs-mean error.

Weighted averages & combined means — Q4
Correct: **(C)**. *Traps:* **(A)** reverse weighted-average error; **(B)** reverse weighted-average error; **(D)** reverse weighted-average error.

Weighted averages & combined means — Q5
Correct: **(D)**. *Traps:* **(A)** total-vs-mean error; **(B)** average-of-averages error; **(C)** total-vs-mean error.

Line of best fit: prediction & residuals — Q1
Correct: **(D)**. *Traps:* **(A)** prediction computation error; **(B)** prediction computation error; **(C)** prediction computation error.

Line of best fit: prediction & residuals — Q2
Correct: **(A)**. *Traps:* **(B)** residual sign/direction error; **(C)** residual sign/direction error; **(D)** residual sign/direction error.

Line of best fit: prediction & residuals — Q3
Correct: **(C)**. *Traps:* **(A)** residual sign/direction error; **(B)** residual sign/direction error; **(D)** residual sign/direction error.

Line of best fit: prediction & residuals — Q4
Correct: **(C)**. *Traps:* **(A)** extrapolation-reliability error; **(B)** extrapolation-reliability error; **(D)** extrapolation-reliability error.

Line of best fit: prediction & residuals — Q5
Correct: **(B)**. *Traps:* **(A)** slope-as-rate misread; **(C)** prediction computation error; **(D)** slope-as-rate misread.

Compound & conditional probability — Q1
Correct: **(C)**. *Traps:* **(A)** and/or operation swap; **(B)** and/or operation swap; **(D)** and/or operation swap.

Compound & conditional probability — Q2
Correct: **(D)**. *Traps:* **(A)** independence/overlap error; **(B)** independence/overlap error; **(C)** and/or operation swap.

Compound & conditional probability — Q3
Correct: **(C)**. *Traps:* **(A)** complement error; **(B)** complement error; **(D)** complement error.

Compound & conditional probability — Q4
Correct: **(A)**. *Traps:* **(B)** conditional-denominator error; **(C)** conditional-denominator error; **(D)** conditional-denominator error.

Compound & conditional probability — Q5
Correct: **(D)**. *Traps:* **(A)** independence/overlap error; **(B)** independence/overlap error; **(C)** and/or operation swap.

DOMAIN 4

Geometry & Trigonometry

Shape, measure, and the relations that fix them

"There is no royal road to geometry."
— Euclid

About 15% of the section. Lines and angles, triangles and similarity, circles, area and volume, right-triangle trigonometry, and radians.

Angles & lines

node: sat.geometry.G1

The skill. Angles that fit together in special ways have fixed relationships. Two angles are complementary when they add up to 90 degrees, and supplementary when they add up to 180 degrees. The angles along one side of a straight line add up to 180 degrees, and the angles all the way around a point add up to 360 degrees. When two lines cross, the angles opposite each other (the vertical angles) are equal. When one line, a transversal, crosses two parallel lines, corresponding angles and alternate angles are equal, while the two angles on the same side of the transversal between the parallel lines add up to 180 degrees. To solve, name the relationship, write it as an equation, and solve for the unknown.

Worked example

Two parallel lines are cut by a transversal. One same-side interior angle measures (3x + 20) degrees and the other measures (2x + 10) degrees. Find x and the two angle measures.

Read. The two angles are same-side interior angles between parallel lines, so they are supplementary and add up to 180 degrees. Their measures are given in terms of x.

Plan.

1. Recall that same-side interior angles add to 180.
2. Write the sum of the two expressions equal to 180.

3. Solve for x.
4. Substitute x back to get each angle.

Solve.

$$\begin{aligned} (3x + 20) + (2x + 10) &= 180 && \text{same-side interior angles are supplementary} \\ 5x + 30 &= 180 && \text{combine like terms} \\ 5x &= 150 && \text{subtract 30 from both sides} \\ x &= 30 && \text{divide both sides by 5} \end{aligned}$$

Check. With x = 30 the angles are 3(30) + 20 = 110 and 2(30) + 10 = 70, and 110 + 70 = 180, as required for same-side interior angles.

Answer: x = 30; the two angles are 110 degrees and 70 degrees.

Approach Read · Plan · Solve · Check

Read. Look at how the angles are related: are they complementary, supplementary, a linear pair, vertical angles, or a pair formed by parallel lines and a transversal?

Plan.

1. Choose the rule that fits: sum to 90, sum to 180, equal, or supplementary.
2. Write that relationship as an equation.

Solve.

1. Solve the equation for the unknown angle or variable.

Check. Check that you used 90 where you meant 90 and 180 where you meant 180. For parallel lines, make sure you chose equal or supplementary correctly for that pair.

Common traps — every wrong answer is one of these

Complementary and supplementary mixed up.
Uses 90 where 180 belongs, or 180 where 90 belongs — for example, treating supplementary angles as if they were complementary. Complementary angles add to 90 degrees; supplementary angles add to 180 degrees.

Wrong parallel-line angle pair.
Treats a pair that should be equal (corresponding or alternate angles) as adding to 180, or treats same-side interior angles as equal. Corresponding and alternate angles are equal; same-side interior angles are supplementary.

Wrong relationship or setup.
Uses the wrong angle rule — such as treating vertical angles as supplementary or a linear pair as equal — or writes the equation incorrectly.

Practice

1. *Angle A and angle B are complementary, and A = 35 degrees.*

What is B?

(A) 35 degrees
(B) 55 degrees
(C) 325 degrees
(D) 145 degrees

2. *Two lines intersect, forming an angle of 70 degrees.*

What is the measure of its vertical angle?

(A) 70 degrees
(B) 290 degrees
(C) 110 degrees
(D) 20 degrees

3. *Two parallel lines are cut by a transversal, and one angle measures 65 degrees.*

What is the measure of its alternate interior angle?

(A) 65 degrees
(B) 130 degrees
(C) 115 degrees
(D) 25 degrees

4. *Two parallel lines are cut by a transversal. Two same-side interior angles are formed, and one measures 110 degrees.*

What is the measure of the other?

(A) 250 degrees
(B) 70 degrees
(C) 110 degrees
(D) 20 degrees

5. *Two angles form a linear pair and measure (2x + 10) degrees and (3x - 5) degrees.*

What is x?

(A) 35
(B) 15
(C) 17
(D) 175

Triangles (angle sum, isosceles, exterior angle, inequality)

node: sat.geometry.G2

The skill. The three inside angles of any triangle add up to 180 degrees. A triangle with two equal sides (isosceles) also has two equal base angles, and this works in reverse. A triangle with all three sides equal (equilateral) has all three angles equal to 60 degrees. If you extend one side, the outside angle you form, called an exterior angle, equals the sum of the two inside angles that are not next to it. Finally, three lengths can actually form a triangle only when the two shorter lengths added together are longer than the longest one; this is the triangle inequality.

Worked example

In a triangle, an exterior angle measures 130 degrees. One of its two remote interior angles measures 50 degrees. Find the other remote interior angle.

Read. An exterior angle equals the sum of the two remote interior angles — the two inside angles not next to it. One of them is 50 degrees; call the other one y.

Plan.

1. Write exterior angle equal to the sum of the two remote interior angles.
2. Substitute the known values.
3. Solve for y.

Solve.

$130 = 50 + y$	exterior angle equals the sum of the two remote interior angles
$y = 130 - 50$	subtract 50 from both sides
$y = 80$	simplify

Check. The interior angle next to the exterior angle is 180 - 130 = 50, so the three interior angles are 50, 50, and 80, which sum to 180.

Answer: the other remote interior angle is 80 degrees.

Approach Read · Plan · Solve · Check

Read. Note what you are given — angles, sides, or an exterior angle — and which triangle fact fits.

Plan.

1. For a missing angle, use that the three angles add to 180.
2. For equal sides, set the matching base angles equal.
3. For an exterior angle, set it equal to the sum of the two remote interior angles.
4. For a set of three sides, test the triangle inequality.

Solve.

1. Set up the equation or inequality.
2. Solve for the unknown, or check whether the sides work.

Check. Make sure you subtracted from 180, not 360. For an isosceles triangle, check that the correct two angles are equal. For an exterior angle, confirm you added the two remote angles.

Common traps — every wrong answer is one of these

Angle-sum mistake.
Does not use the 180-degree total correctly: reports just the sum of the two given angles, or subtracts from 360 instead of 180. The three angles of a triangle always add to 180 degrees.

Isosceles base-angle mistake.
Does not turn equal sides into equal base angles, or picks the wrong pair of angles to be equal. In an isosceles triangle the two base angles, across from the two equal sides, are equal.

Exterior-angle mistake.
Sets an exterior angle equal to just one remote interior angle, or treats it as supplementary to their sum, instead of equal to the sum of the two remote interior angles.

Triangle-inequality mistake.
Assumes any three lengths can form a triangle. In fact the two shorter lengths must add up to more than the longest one.

Practice

1. *A triangle has two angles measuring 40 degrees and 75 degrees.*

What is the third angle?

(A) 115 degrees

(B) 65 degrees

(C) 40 degrees

(D) 245 degrees

2. *An isosceles triangle has a vertex angle of 40 degrees.*

What is the measure of each base angle?

(A) 70 degrees

(B) 40 degrees

(C) 140 degrees

(D) 100 degrees

3. *An exterior angle of a triangle measures 120 degrees, and the two remote interior angles are equal.*

What is the measure of each remote interior angle?

(A) 240 degrees

(B) 60 degrees

(C) 30 degrees

(D) 120 degrees

4. *Consider four sets of side lengths.*

Which set can form a triangle?

(A) 2, 3, 6

(B) 1, 1, 3

(C) 4, 4, 10

(D) 5, 6, 9

5. *The angles of a triangle measure x, 2x, and 3x degrees.*

What is the largest angle?

(A) 60 degrees

(B) 90 degrees

(C) 180 degrees

(D) 30 degrees

Right triangles & the Pythagorean theorem

node: sat.geometry.G3

The skill. In a right triangle the two short sides are the legs, and the longest side, across from the right angle, is the hypotenuse. The Pythagorean theorem says leg squared plus leg squared equals hypotenuse squared, written $a^2 + b^2 = c^2$. To find the hypotenuse, add the two squared legs and take the square root; to find a missing leg, subtract instead, using $\text{leg}^2 = \text{hypotenuse}^2$ - other leg^2. A few whole-number right triangles come up often: 3-4-5, 5-12-13, and 8-15-17. Two special triangles have fixed side ratios: a 45-45-90 triangle has two equal legs and a hypotenuse equal to a leg times $\sqrt{2}$, and a 30-60-90 triangle has sides in the ratio short leg, short leg times $\sqrt{3}$, and twice the short leg.

Worked example

A right triangle has a hypotenuse of 10 and one leg of 6. Find the length of the other leg.

Read. The hypotenuse, the longest side across from the right angle, is 10, and one leg is 6. Because the hypotenuse is known, we subtract to find the missing leg.

Plan.

1. Write the Pythagorean theorem $a^2 + b^2 = c^2$.
2. Rearrange to isolate the missing leg.
3. Substitute the hypotenuse and known leg.
4. Square, subtract, and take the square root.

Solve.

$a^2 + b^2 = c^2$	Pythagorean theorem
$b^2 = c^2 - a^2$	isolate the missing leg
$b^2 = 10^2 - 6^2$	substitute the hypotenuse and known leg
$b^2 = 100 - 36$	square each value
$b^2 = 64$	subtract
$b = 8$	take the square root

Check. The leg 8 is shorter than the hypotenuse 10, as a leg must be, and 6, 8, 10 is a scaled 3-4-5 triple.

Answer: the other leg is 8.

Approach Read · Plan · Solve · Check

Read. Find the right angle and the hypotenuse (the longest side, across from it), and note whether a leg or the hypotenuse is missing. Check whether it is a special 45-45-90 or 30-60-90 triangle.

Plan.

1. To find the hypotenuse, add the two squared legs.
2. To find a leg, subtract: leg^2 = hypotenuse^2 - other leg^2.
3. For a special triangle, use the $\sqrt{2}$ or $\sqrt{3}$ side ratio instead.

Solve.

1. Square the known sides.
2. Add or subtract as needed.
3. Take the square root, or scale the special-triangle ratio.

Check. Check that the hypotenuse is the longest side and that you actually took the final square root. Confirm you used $\sqrt{2}$ for a 45-45-90 and $\sqrt{3}$ for the long leg of a 30-60-90.

Common traps — every wrong answer is one of these

Hypotenuse and leg mixed up.
Adds the hypotenuse in when finding a leg instead of subtracting. To find a leg, use leg^2 = hypotenuse^2 - other leg^2, not $a^2 + b^2 = c^2$.

Skipped a square or the square root.
Adds or subtracts the side lengths directly without squaring them first, or squares correctly but forgets to take the final square root.

Wrong special-triangle ratio.
Uses $\sqrt{3}$ where $\sqrt{2}$ belongs, or the reverse, or attaches the ratio to the wrong side. A 45-45-90 triangle uses $\sqrt{2}$; a 30-60-90 triangle uses $\sqrt{3}$ for the long leg.

Practice

1. *A right triangle has legs of length 3 and 4.*

What is the length of the hypotenuse?

(A) 12
(B) 5
(C) 25
(D) 7

2. *A right triangle has a hypotenuse of 13 and one leg of 5.*

What is the length of the other leg?

(A) 8
(B) about 13.9 (sqrt of 194)
(C) 18
(D) 12

3. *A right triangle has legs of length 6 and 8.*

What is the length of the hypotenuse?

(A) 10
(B) 100
(C) 14
(D) sqrt of 28

4. *A 45-45-90 right triangle has legs of length 5.*

What is the length of the hypotenuse?

(A) $5*\sqrt{2}$

(B) 5

(C) 10

(D) $5*\sqrt{3}$

5. *In a 30-60-90 triangle, the shorter leg has length 4.*

What is the length of the hypotenuse?

(A) $4*\sqrt{3}$

(B) 12

(C) 8

(D) $4*\sqrt{2}$

Similarity & congruence

node: sat.geometry.G4

The skill. Two figures are similar when they have the same shape but not necessarily the same size: matching angles are equal, and matching sides are all in the same ratio, called the scale factor k. Congruent figures are the special case where k is 1, so they are also the same size. To find a missing length, set up a proportion that keeps matching sides in matching positions. Lengths grow by the factor k, but areas grow by k squared and volumes grow by k cubed.

Worked example

Triangle ABC is similar to triangle DEF. Side AB = 5 corresponds to DE = 15, and BC = 7. Find EF.

Read. The triangles are similar, so matching sides are in the same ratio. AB matches DE, and BC matches EF. From the known pair AB = 5 and DE = 15, the scale factor is 15/5 = 3.

Plan.

1. Set up a proportion pairing EF with BC and DE with AB.
2. Substitute the known lengths.
3. Simplify the known ratio to the scale factor.
4. Multiply to solve for EF.

Solve.

$$\frac{EF}{BC} = \frac{DE}{AB} \qquad \text{corresponding sides are proportional}$$
$$\frac{EF}{7} = \frac{15}{5} \qquad \text{substitute the known lengths}$$
$$\frac{EF}{7} = 3 \qquad \text{the scale factor is 3}$$
$$EF = 21 \qquad \text{multiply both sides by 7}$$

Check. Every side of DEF is 3 times the matching side of ABC, and 7 times 3 is 21, which is larger, as expected for the bigger triangle.

Answer: EF = 21.

Approach Read · Plan · Solve · Check

Read. Confirm the two figures are similar, then decide which sides match and what the scale factor is.

Plan.

1. For a missing length, write a proportion with matching sides in matching positions.
2. For an area, plan to use k squared, not k.

Solve.

1. Solve the proportion by multiplying by the scale factor.
2. For an area, multiply by k squared.

Check. Check that matching sides line up and were not flipped. Confirm you multiplied by the scale factor rather than adding a difference, and used k squared for area.

Common traps — every wrong answer is one of these

Mismatched corresponding sides.
Pairs sides that do not actually correspond, or flips the ratio upside down, so the proportion is set up wrong.

Added instead of multiplied by the scale factor.
Adds the difference between matching sides instead of multiplying by the scale factor. Similar figures scale by multiplying, not by adding a fixed amount.

Wrong power for area.
Scales an area by the plain factor k (or by k cubed) instead of k squared. Areas grow by the square of the linear scale factor.

Practice

1. *Two triangles are similar. A side of length 4 in the first corresponds to a side of length 6 in the second.*

What length in the second triangle corresponds to a side of length 6 in the first?

(A) 8
(B) 4
(C) 4.5
(D) 9

2. *Triangle ABC is similar to triangle DEF, with $AB = 8$ corresponding to $DE = 12$ and $BC = 10$.*

What is the length of EF?

(A) 14
(B) 30
(C) about 6.7
(D) 15

3. *Two rectangles are similar. The smaller is 3 by 5, and the larger has a short side of 9.*

What is the long side of the larger rectangle?

(A) 45
(B) about 1.7
(C) 11
(D) 15

4. *Two similar triangles have a linear scale factor of 3, and the smaller has an area of 8.*

What is the area of the larger triangle?

(A) 24
(B) 8
(C) 72
(D) 216

5. *Triangle PQR is similar to triangle STU, with $PQ = 6$ corresponding to $ST = 9$. The area of triangle PQR is 20.*

What is the area of triangle STU?

(A) 30
(B) about 13.3

(C) 45

(D) 67.5

Area & perimeter

node: sat.geometry.G5

The skill. Perimeter is the distance all the way around a flat figure, measured in ordinary length units; area is the amount of space inside it, measured in square units. A rectangle has area equal to base times height and perimeter equal to 2 times (length plus width). A triangle has area one half times base times height. A parallelogram has area base times height, and a trapezoid has area one half times (first base plus second base) times height. For a figure made of several shapes, break it into rectangles and triangles and then add or subtract the pieces. In every case the height must be the perpendicular height, measured straight across, not the length of a slanted side.

Worked example

A trapezoid has parallel sides of length 8 and 12 and a perpendicular height of 5. Find its area.

Read. A trapezoid has two parallel sides, called bases, of length 8 and 12, and a perpendicular height of 5 between them.

Plan.

1. Recall the trapezoid area formula, one half times the sum of the bases times the height.
2. Substitute the two bases and the height.
3. Add the bases, then multiply and take half.

Solve.

$A = \frac{1}{2}(b_1 + b_2)h$	trapezoid area formula
$A = \frac{1}{2}(8 + 12)(5)$	substitute the bases and height
$A = \frac{1}{2}(20)(5)$	add the bases
$A = 50$	multiply

Check. The area 50 square units lies between an 8 by 5 rectangle (40) and a 12 by 5 rectangle (60), as it should.

Answer: the area is 50 square units.

Approach Read · Plan · Solve · Check

Read. Decide whether the question asks for area or perimeter, and find the base and the perpendicular height (not a slanted side).

Plan.

1. Pick the formula for the shape you have.
2. For a composite figure, split it into simple shapes and decide whether to add or subtract each piece.

Solve.

1. Put the numbers into the formula, keeping the one half for a triangle.
2. Combine the pieces of a composite figure.

Check. Check whether the answer should be in square units (area) or length units (perimeter). Confirm you used the perpendicular height and kept the one half, and that composite pieces were added or subtracted correctly.

Common traps — every wrong answer is one of these

Area and perimeter mixed up.
Finds perimeter when area is asked, or the reverse, or otherwise mixes up length units with square units.

Forgot the one half for a triangle.
Uses base times height for a triangle's area and leaves out the factor of one half.

Used a slanted side as the height.
Plugs in the length of a slanted side instead of the perpendicular height of a triangle or parallelogram. Area needs the straight-across height.

Composite-figure mistake.
Adds a piece that should have been removed, leaves out a region, or counts part of the figure twice when combining shapes.

Practice

1. *A rectangle measures 8 cm by 5 cm.*

What is its area?

(A) 40 square cm
(B) 13 cm
(C) 64 square cm
(D) 26 cm

2. *A triangle has a base of 12, a perpendicular height of 5, and a slanted side of 13.*

What is its area?

(A) 30

(B) 78

(C) 156

(D) 60

3. *A parallelogram has a base of 9, a perpendicular height of 4, and a slanted side of 5.*

What is its area?

(A) 28

(B) 36

(C) 45

(D) 18

4. *An L-shaped figure is a 6 by 8 rectangle with a 2 by 3 rectangular notch removed from one corner.*

What is the area of the L-shape?

(A) 48

(B) 54

(C) 42

(D) 6

5. *A figure is a 10 by 6 rectangle with a triangle (base 6, perpendicular height 4) attached along one side.*

What is the total area?

(A) 60

(B) 72

(C) 32

(D) 84

Circles

node: sat.geometry.G6

The skill. For a circle with radius r, the circumference (the distance around) is C = 2*pi*r, which is the same as pi times the diameter, and the area (the space inside) is A = pi*r^2. A slice of the circle set by a center angle of theta degrees is the fraction theta/360 of the whole circle, so its arc length is (theta/360) times the circumference and its sector area is (theta/360) times the area. A circle centered at (h, k) with radius r has the equation $(x - h)^2 + (y - k)^2 = r^2$; the center's coordinates come out with the opposite signs from the numbers in the parentheses, and the number on the right is the radius squared, not the radius itself.

Worked example

A circle has radius 10, and a sector has a central angle of 72 degrees. Find the area of the sector.

Read. A sector is a fraction of the whole circle set by its central angle. Here the radius is 10 and the central angle is 72 degrees, so the sector is 72/360 of the full area.

Plan.

1. Write the sector area as the fraction theta/360 times pi*r^2.
2. Substitute the angle and radius.
3. Simplify the fraction and square the radius.
4. Multiply.

Solve.

$$A = \frac{\theta}{360}\,\pi r^2 \qquad \text{sector area is a fraction of the whole circle}$$

$$A = \frac{72}{360}\,\pi(10)^2 \qquad \text{substitute the angle and radius}$$

$$A = \frac{1}{5}\,(100\pi) \qquad \text{simplify the fraction and square the radius}$$

$$A = 20\pi \qquad \text{multiply}$$

Check. 72 degrees is one fifth of 360, and the full area is 100*pi, so one fifth of it is 20*pi. *Answer:* the sector area is 20*pi.

Approach Read · Plan · Solve · Check

Read. Note whether you are given the radius or the diameter, and whether the question asks for circumference, area, an arc or sector, or the equation of the circle.

Plan.

1. Use 2*pi*r for the distance around and pi*r^2 for the space inside.
2. For an arc or sector, multiply by the fraction theta/360.
3. For the equation, read the center as (h, k) with flipped signs and take the radius as the square root of the right side.

Solve.

1. Put in the radius, not the diameter.
2. Apply the fraction for an arc or sector.
3. Take the square root for the radius when reading an equation.

Check. Check radius versus diameter, and circumference versus area. Confirm you included theta/360 for a part, flipped the center signs, and used the square root for the radius.

Common traps — every wrong answer is one of these

Radius and diameter mixed up.
Uses the diameter where the radius is needed, or the reverse — for instance, putting the diameter into pi*r^2 or 2*pi*r. The radius is half the diameter.

Circumference and area formulas swapped.
Uses 2*pi*r when the area is wanted, or pi*r^2 when the circumference is wanted.

Left out the arc or sector fraction.
Forgets the theta/360 fraction and gives the whole circle instead of the part, or applies the fraction incorrectly.

Circle-equation mistake.
Reads the center with the wrong signs, or takes the number on the right as the radius instead of the radius squared. The center flips the signs in the parentheses, and the radius is the square root of the right side.

Practice

1. *A circle has a radius of 5.*

What is its circumference?

(A) 10*pi
(B) 25*pi
(C) 5*pi
(D) 20*pi

2. *A circle has a radius of 4.*

What is its area?

(A) 16*pi
(B) 8*pi
(C) 64*pi
(D) 4*pi

3. *A circle has a radius of 6, and a sector has a central angle of 60 degrees.*

What is the area of the sector?

(A) 6*pi
(B) 12*pi
(C) 36*pi
(D) 2*pi

4. *A circle has the equation $(x - 3)^2 + (y + 2)^2 = 16$.*

What are its center and radius?

(A) Center (3, -2), radius 4

(B) Center (-3, 2), radius 4

(C) Center (-3, 2), radius 16

(D) Center (3, -2), radius 16

5. *A circle has center (1, -4) and passes through the point (1, 0).*

What is its equation?

(A) $(x - 1)^2 + (y + 4)^2 = 16$

(B) $(x - 1)^2 + (y + 4)^2 = 64$

(C) $(x + 1)^2 + (y - 4)^2 = 16$

(D) $(x - 1)^2 + (y + 4)^2 = 4$

Volume & surface area

node: sat.geometry.G7

The skill. Volume measures the space inside a solid, in cubic units, while surface area measures the total area of all its outer faces, in square units. A rectangular box has volume length times width times height. A cylinder has volume pi*r^2 times height. A cone has volume one third of pi*r^2 times height. A sphere has volume (4/3)*pi*r^3 and surface area 4*pi*r^2. Cones and pyramids always carry a factor of one third, and a sphere carries four thirds and uses the radius cubed. Always use the radius, which is half the diameter, not the diameter itself.

Worked example

A cone has a radius of 6 and a height of 5. Find its volume.

Read. A cone's volume is one third of pi*r^2 times its height. Here the radius is 6 and the height is 5.

Plan.

1. Write the cone volume formula, one third times pi*r^2 times height.
2. Substitute the radius and height.
3. Square the radius and multiply.
4. Take one third.

Solve.

$$V = \frac{1}{3}\pi r^2 h \qquad \text{cone volume formula}$$
$$V = \frac{1}{3}\pi(6)^2(5) \qquad \text{substitute radius and height}$$
$$V = \frac{1}{3}\pi(36)(5) \qquad \text{square the radius}$$
$$V = \frac{1}{3}(180\pi) \qquad \text{multiply}$$
$$V = 60\pi \qquad \text{take one third}$$

Check. A cylinder with the same radius and height has volume pi*36*5 = 180*pi, and a cone is one third of that, giving 60*pi.

Answer: the volume is 60*pi.

Approach Read · Plan · Solve · Check

Read. Identify the solid, whether volume or surface area is wanted, and the radius (not the diameter).

Plan.

1. Choose the formula that matches the solid.
2. Remember the one third for a cone or pyramid and the four thirds with r cubed for a sphere.

Solve.

1. Put in the radius and raise it to the correct power.
2. Include any fraction in front of the formula.

Check. Check whether the answer is in cubic units (volume) or square units (surface area). Confirm you kept the one third or four thirds, used the right power, and used the radius.

Common traps — every wrong answer is one of these

Volume and surface area mixed up.
Finds surface area when volume is asked, or the reverse, or uses a flat-area formula for a 3D quantity.

Left out the fraction in the formula.
Omits the one third for a cone or pyramid, or the four thirds for a sphere — for example, using the cylinder formula for a cone.

Wrong radius or wrong power.
Uses the diameter in place of the radius, or the wrong power such as r squared where r cubed belongs.

Practice

1. *A rectangular box measures 4 by 3 by 5.*

What is its volume?

(A) 94
(B) 60
(C) 47
(D) 12

2. *A cylinder has a radius of 3 and a height of 10.*

What is its volume?

(A) 60*pi
(B) 90*pi
(C) 30*pi
(D) 360*pi

3. *A cone has a radius of 3 and a height of 4.*

What is its volume?

(A) 48*pi
(B) 36*pi
(C) 4*pi
(D) 12*pi

4. *A sphere has a radius of 3.*

What is its volume?

(A) 12*pi
(B) 108*pi
(C) 27*pi
(D) 36*pi

5. *A cube has an edge length of 5.*

What is its surface area?

(A) 25
(B) 30
(C) 150
(D) 125

Right-triangle trigonometry

node: sat.geometry.G8

The skill. In a right triangle, pick one of the two acute angles. Relative to that angle, the three sides are the opposite side, the adjacent side, and the hypotenuse. The three basic ratios are sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, and tangent equals opposite over adjacent, remembered as SOH-CAH-TOA. To find a missing side, multiply a known side by the right ratio. The two acute angles always add to 90 degrees, so the sine of one equals the cosine of the other: sin(theta) = cos(90 - theta). The SAT uses only right-triangle trig, never the unit circle.

Worked example

A right triangle has an acute angle of 30 degrees and a hypotenuse of 14. Find the length of the side opposite the 30-degree angle. Use sin 30 = 0.5.

Read. Relative to the 30-degree angle, we know the hypotenuse (14) and want the opposite side. Sine links the opposite side and the hypotenuse.

Plan.

1. Choose sine, which is opposite over hypotenuse.
2. Write sin 30 equal to the opposite side over 14.
3. Multiply both sides by 14 to solve.

Solve.

$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{SOH: sine is opposite over hypotenuse}$$

$$0.5 = \frac{\text{opposite}}{14} \qquad \text{substitute the sine value and the hypotenuse}$$

$$\text{opposite} = 0.5 \times 14 \qquad \text{multiply both sides by 14}$$

$$\text{opposite} = 7 \qquad \text{simplify}$$

Check. The side opposite the smallest angle is the shortest side, and 7 is less than the hypotenuse 14, as expected.

Answer: the opposite side is 7.

Approach Read · Plan · Solve · Check

Read. Mark the acute angle you are working from, and label each side as opposite, adjacent, or hypotenuse relative to that angle.

Plan.

1. Pick the ratio that connects the known side to the unknown one, using SOH-CAH-TOA.
2. For two acute angles, use sin(theta) = cos(90 - theta).

Solve.

1. Write the ratio right side up.

2. Multiply to solve for the missing side.

Check. Check that the ratio fits what is asked and is not flipped, that opposite and adjacent are labeled from the correct angle, and that a cofunction step used the complement.

Common traps — every wrong answer is one of these

Wrong or upside-down ratio.
Picks the wrong trig ratio (sine for cosine, and so on) or flips it over — hypotenuse over opposite instead of opposite over hypotenuse — including dividing by a ratio when you should multiply.

Opposite and adjacent mislabeled.
Labels the wrong leg as opposite or adjacent, or treats a leg as the hypotenuse, relative to the given angle.

Cofunction mistake.
Treats sin(theta) as equal to cos(theta) instead of cos(90 - theta), or otherwise mishandles the fact that the two acute angles add to 90 degrees.

Practice

1. *In a right triangle, the acute angle theta has an opposite side of 3, an adjacent side of 4, and a hypotenuse of 5.*

What is sin(theta)?

(A) 4/5
(B) 5/3
(C) 3/4
(D) 3/5

2. *Same right triangle: opposite 3, adjacent 4, hypotenuse 5, angle theta.*

What is cos(theta)?

(A) 3/5
(B) 4/5
(C) 4/3
(D) 5/4

3. *A right triangle has a 30-degree angle and a hypotenuse of 10. (sin 30 = 0.5, cos 30 is about 0.87.)*

What is the length of the side opposite the 30-degree angle?

(A) 0.5
(B) 20
(C) about 8.7
(D) 5

4. *In a right triangle, sin(theta) = 0.6, and phi is the other acute angle.*

What is cos(phi)?

(A) 0.8
(B) about 1.67
(C) 0.6
(D) 0.4

5. *A 12-foot ladder leans against a wall, making a 60-degree angle with the ground. (sin 60 is about 0.87, cos 60 = 0.5.)*

How high up the wall does the ladder reach?

(A) 24 ft
(B) about 10.4 ft
(C) about 0.87 ft
(D) 6 ft

Radian measure & trigonometric relationships

node: sat.geometry.G9

The skill. Radians are a second way to measure angles, based on distance around a circle. A full circle is 2*pi radians, which equals 360 degrees, so 180 degrees equals pi radians. To change degrees to radians, multiply by pi/180; to change radians to degrees, multiply by 180/pi. The arc length formula s = r*theta works only when theta is measured in radians. Sine and cosine are cofunctions, meaning the sine of an angle equals the cosine of its complement, the angle that completes 90 degrees, so sin(theta) = cos(90 - theta). And for every angle, $\sin^2(\text{theta}) + \cos^2(\text{theta}) = 1$.

Worked example

An angle theta has sin(theta) = 4/5. Find $\cos^2(\text{theta})$.

Read. The Pythagorean identity $\sin^2(\text{theta}) + \cos^2(\text{theta}) = 1$ connects sine and cosine. We know the sine, so we can solve for $\cos^2$.

Plan.

1. Write the Pythagorean identity.
2. Isolate $\cos^2(\text{theta})$.
3. Substitute the sine value.

4. Square the fraction and subtract.

Solve.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 && \text{Pythagorean identity} \\ \cos^2\theta &= 1 - \sin^2\theta && \text{isolate cosine squared} \\ \cos^2\theta &= 1 - \left(\frac{4}{5}\right)^2 && \text{substitute the sine value} \\ \cos^2\theta &= 1 - \frac{16}{25} && \text{square the fraction} \\ \cos^2\theta &= \frac{9}{25} && \text{subtract} \end{aligned}$$

Check. $\cos^2 = 9/25$ gives $\cos = 3/5$, and $(3/5)^2 + (4/5)^2 = 9/25 + 16/25 = 1$, confirming the identity.

Answer: $\cos^2(\text{theta}) = 9/25$.

Approach Read · Plan · Solve · Check

Read. Decide which task it is: a unit conversion, a cofunction or complement relationship, the Pythagorean identity, or an arc length.

Plan.

1. To convert, multiply by pi/180 for degrees to radians, or 180/pi for radians to degrees.
2. For a cofunction, pair the angle with its 90-degree complement.
3. For the identity, use $\cos^2 = 1$ - $\sin^2$ after squaring.
4. For arc length, use s = r*theta with theta in radians.

Solve.

1. Apply the chosen rule.
2. Keep the units straight, using radians for s = r*theta.

Check. Check that you used the right conversion factor and kept pi, used the complement (90 - theta) rather than the supplement, squared before subtracting in the identity, and had theta in radians for the arc length.

Common traps — every wrong answer is one of these

Degree-radian conversion mistake.
Converts with the wrong factor or the wrong direction: multiplying by 180/pi when going from degrees to radians, dropping the pi, or reversing the conversion. Degrees to radians uses pi/180.

Complement or cofunction mistake.
Misuses the cofunction rule: treating sin(theta) as cos(theta), using the supplement 180 - theta instead of the complement 90 - theta, or subtracting the value from 1 instead of using the angle's complement.

Pythagorean-identity mistake.
Mishandles $\sin^2 + \cos^2 = 1$: forgetting to square (using 1 - sin instead of 1 - $\sin^2$), giving $\sin^2$ in place of $\cos^2$, or setting a single squared term equal to 1.

Arc length in degrees instead of radians.
Uses s = r*theta with theta in degrees instead of radians, or mixes up the arc length with the central angle itself.

Practice

1. *Convert 90 degrees to radians.*

What is the result?

(A) 2*pi
(B) 90*pi
(C) pi/2
(D) 180/pi

2. *Convert pi/3 radians to degrees.*

What is the result?

(A) 60*pi degrees
(B) 60 degrees
(C) 30 degrees
(D) 120 degrees

3. *It is given that sin(40 degrees) = 0.64.*

What is cos(50 degrees)?

(A) 0.36
(B) 0.64
(C) It cannot be determined
(D) -0.64

4. *An angle theta satisfies sin(theta) = 3/5.*

What is $\cos^2$(theta)?

(A) 1
(B) 2/5
(C) 9/25

(D) 16/25

5. *A circle has radius 6. A central angle of pi/3 radians subtends an arc.*

What is the length of that arc?

(A) 360

(B) 12*pi

(C) pi/3

(D) 2*pi

Coordinate geometry: distance & midpoint

node: sat.geometry.G10

The skill. The distance between two points (x1, y1) and (x2, y2) is $\sqrt{(x2 - x1^2}$ + (y2 - y1)^2). This is just the Pythagorean theorem using the horizontal gap and the vertical gap as the two legs, so you subtract the coordinates, square each gap, add them, and take the square root. The midpoint of the segment between the two points is ((x1 + x2)/2, (y1 + y2)/2): you average the x-values and average the y-values, so you add and divide by 2. Distance uses differences and finishes with a square root, while the midpoint uses sums and finishes by halving. Mixing up those two operations is the most common mistake.

Worked example

Find the distance between the points (-1, 2) and (3, 5).

Read. Distance is the square root of the sum of the squared horizontal and vertical gaps. Subtract carefully because one x-coordinate is negative.

Plan.

1. Write the distance formula.
2. Substitute the two points.
3. Subtract to get each gap, watching the signs.
4. Square each gap, add, and take the square root.

Solve.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	distance formula
$d = \sqrt{(3 - (-1))^2 + (5 - 2)^2}$	substitute the two points
$d = \sqrt{(4)^2 + (3)^2}$	subtract, watching the signs
$d = \sqrt{16 + 9}$	square each difference
$d = \sqrt{25}$	add
$d = 5$	take the square root

Check. The horizontal and vertical gaps are 4 and 3, a 3-4-5 right triangle, so the straight-line distance is 5.

Answer: the distance is 5.

Approach Read · Plan · Solve · Check

Read. Decide whether this is a distance (one length, uses differences and a square root) or a midpoint (a point, averages the coordinates).

Plan.

1. For distance, subtract the coordinates, square each difference, add them, and take the square root.
2. For a midpoint, add the matching coordinates and divide each sum by 2.

Solve.

1. Carry out the operation, pairing x with x and y with y.
2. Watch the signs when coordinates are negative.

Check. For a distance, check that you both squared and square-rooted. For a midpoint, check that you added and then halved, not subtracted. Confirm the signs and the coordinate pairing.

Common traps — every wrong answer is one of these

Distance-formula mistake.
Skips the squaring or the final square root when finding distance — for example, reporting the sum of the squares without rooting it, or just adding the horizontal and vertical gaps.

Distance and midpoint operations swapped.
Uses the wrong operation for the task: subtracting coordinates to find a midpoint, or averaging or adding coordinates to find a distance.

Midpoint mistake.
Adds the coordinates but forgets to divide by 2, halves only one coordinate, or otherwise mishandles the average.

Sign or pairing mistake.
Makes a sign slip with negative coordinates, such as treating a negative difference as positive, or pairs an x with a y by mistake.

Practice

1. *Consider the points (1, 2) and (4, 6).*

What is the distance between them?

(A) $\sqrt{7}$

(B) 7
(C) 25
(D) 5

2. *Consider the points (2, 3) and (8, 7).*

What is the midpoint of the segment joining them?

(A) (5, 5)
(B) (6, 4)
(C) (5, 10)
(D) (10, 10)

3. *Two points have coordinates (x1, y1) and (x2, y2).*

Which expression gives their midpoint?

(A) ((x1 - x2)/2, (y1 - y2)/2)
(B) (x2 - x1, y2 - y1)
(C) $\sqrt{(x2 - x1^2 + (y2 - y1)^2)}$
(D) ((x1 + x2)/2, (y1 + y2)/2)

4. *Consider the points (-2, 1) and (1, 5).*

What is the distance between them?

(A) $\sqrt{37}$
(B) $\sqrt{17}$
(C) $\sqrt{20}$
(D) 5

5. *Let M be the midpoint of the segment with endpoints (0, 0) and (6, 8).*

What is the distance from (0, 0) to M?

(A) 10
(B) $\sqrt{7}$
(C) 25
(D) 5

Circle angles: central, inscribed & arcs

node: sat.geometry.G11

The skill. Angles in a circle are tied to the arcs they cut off. A central angle, with its vertex at the center, has exactly the same degree measure as the arc it intercepts. An inscribed angle, with its

vertex on the circle, is half the measure of the arc it intercepts, so it is also half of a central angle that cuts off the same arc. One special case: an angle inscribed in a semicircle, whose two sides reach the ends of a diameter, cuts off a 180-degree arc and therefore measures 90 degrees. Keep in mind that the arc's measure in degrees is a different thing from the arc's length in units.

Worked example

An inscribed angle intercepts an arc that measures 140 degrees. Find the measure of the inscribed angle.

Read. An inscribed angle, with its vertex on the circle, is half the measure of the arc it intercepts. The intercepted arc is 140 degrees.

Plan.

1. Recall that an inscribed angle is half its intercepted arc.
2. Substitute the arc measure.
3. Take half.

Solve.

$$\begin{aligned} \text{inscribed angle} &= \frac{1}{2}\,(\text{arc}) && \text{an inscribed angle is half its intercepted arc} \\ &= \frac{1}{2}\,(140^\circ) && \text{substitute the arc measure} \\ &= 70^\circ && \text{take half} \end{aligned}$$

Check. A central angle on the same arc would be the full 140 degrees; the inscribed angle is half of that, 70 degrees.

Answer: the inscribed angle is 70 degrees.

Approach Read · Plan · Solve · Check

Read. Decide whether the angle is central (vertex at the center) or inscribed (vertex on the circle), and find which arc it cuts off.

Plan.

1. A central angle equals its intercepted arc.
2. An inscribed angle is half its intercepted arc, and half of a central angle on the same arc.
3. An angle inscribed in a semicircle is 90 degrees.

Solve.

1. Apply the matching relationship between the angle and its arc.

Check. Check that you halved for an inscribed angle but not for a central one, used the same intercepted arc, and recognized a semicircle as giving 90 degrees.

Common traps — every wrong answer is one of these

Inscribed-versus-central factor mistake.
Treats an inscribed angle as equal to the central angle (or the full arc) instead of half of it, or applies the factor of two in the wrong direction.

Arc measure versus central angle confused.
Does not set a central angle equal to its intercepted arc, halving or doubling it instead, or confuses an arc's measure in degrees with its length in units.

Semicircle mistake.
Does not recognize that an angle inscribed in a semicircle is 90 degrees, and instead uses the 180-degree arc directly or some other value.

Practice

1. *A central angle of a circle measures 80 degrees.*

What is the measure of its intercepted arc?

(A) 100 degrees
(B) 160 degrees
(C) 80 degrees
(D) 40 degrees

2. *An inscribed angle subtends the same arc as a central angle of 100 degrees.*

What is the measure of the inscribed angle?

(A) 25 degrees
(B) 100 degrees
(C) 50 degrees
(D) 200 degrees

3. *An inscribed angle intercepts an arc of 60 degrees.*

What is the measure of the inscribed angle?

(A) 15 degrees
(B) 120 degrees
(C) 60 degrees
(D) 30 degrees

4. *An angle is inscribed in a semicircle, with its rays passing through the two endpoints of a diameter.*

What is the measure of the angle?

(A) 90 degrees

(B) 180 degrees

(C) It cannot be determined

(D) 45 degrees

5. *A central angle of a circle measures 120 degrees.*

What are the measure of its intercepted arc and the measure of an inscribed angle on that same arc?

(A) Arc 60 degrees; inscribed angle 120 degrees

(B) Arc 120 degrees; inscribed angle 60 degrees

(C) Arc 120 degrees; inscribed angle 120 degrees

(D) Arc 240 degrees; inscribed angle 60 degrees

Scale factor: length, area & volume

node: sat.geometry.G12

The skill. When two figures are similar, the linear scale factor k is the ratio of any pair of matching lengths. Their areas are then in the ratio k squared, and their volumes are in the ratio k cubed. For example, if every length is doubled (k = 2), the area becomes 4 times as large and the volume becomes 8 times as large. Working backward, if you know the ratio of areas you recover the length ratio by taking a square root, and if you know the ratio of volumes you take a cube root. In short, lengths use k, areas use k squared, and volumes use k cubed.

Worked example

Two similar solids have corresponding edges in the ratio 2 : 5. Find the ratio of their volumes.

Read. The linear scale ratio of matching edges is $2 : 5$. Volumes scale by the cube of the linear ratio.

Plan.

1. Recall that volumes are in the ratio k cubed.
2. Cube each part of the linear ratio.

Solve.

$$\begin{aligned} \text{volume ratio} &= 2^3 : 5^3 && \text{volume scales by the cube of the linear ratio} \\ &= 8 : 125 && \text{cube each term} \end{aligned}$$

Check. Lengths use the first power $(2 : 5)$, so volumes use the third power, giving $8 : 125$, a much wider spread.

Answer: the volume ratio is $8 : 125$.

Approach Read · Plan · Solve · Check

Read. Decide whether you are going from a length ratio to an area or volume ratio, or backward from an area or volume ratio to a length ratio.

Plan.

1. Length to area: square the factor.
2. Length to volume: cube the factor.
3. Area to length: take the square root.
4. Volume to length: take the cube root.

Solve.

1. Apply the correct power or root of the scale factor.

Check. Check that you used k squared for area and k cubed for volume, matched the exponent to the dimension, and took the right root on a reverse problem.

Common traps — every wrong answer is one of these

Used the plain factor for area or volume.
Applies the linear scale factor k straight to area or volume — for instance, assuming doubling the sides doubles the area — instead of using k squared or k cubed.

Square and cube swapped.
Uses the square where the cube is needed, or the cube where the square is needed: k squared for volume or k cubed for area.

Reverse-scale mistake.
Given an area or volume ratio, forgets to take the square root or cube root to get back the length ratio, or stops at the length ratio when an area or volume answer is wanted.

Practice

1. *Two similar rectangles have corresponding sides in the ratio 1 : 3.*

What is the ratio of their areas?

(A) 1 : 9

(B) 1 : 6

(C) 1 : 27

(D) 1 : 3

2. *Two similar solids have corresponding edges in the ratio 1 : 2.*

What is the ratio of their volumes?

(A) 1 : 6
(B) 1 : 2
(C) 1 : 4
(D) 1 : 8

3. *Every dimension of a cube is doubled.*

Its volume becomes how many times as large?

(A) 4 times
(B) 6 times
(C) 8 times
(D) 2 times

4. *Two similar figures have areas in the ratio 25 : 1.*

What is the ratio of their corresponding sides?

(A) 5 : 1
(B) 25 : 1
(C) 625 : 1
(D) 12.5 : 1

5. *Two similar solids have volumes in the ratio 27 : 1.*

What is the ratio of their surface areas?

(A) 27 : 1
(B) 9 : 1
(C) 3 : 1
(D) 729 : 1

Circle equations: center & radius by completing the square

node: sat.geometry.G13

The skill. A circle written in general form as $x^2 + y^2 + Dx + Ey + F = 0$ can be rewritten in standard form $(x - h)^2 + (y - k)^2 = r^2$ by completing the square, once for the x-terms and once for the y-terms. Group the x-terms together and the y-terms together, then for each group take half of its linear coefficient, square that, and add it to both sides so the equation stays balanced. After rewriting each group as a perfect square, the center (h, k) comes out with the opposite signs from the numbers in the parentheses, and the radius is the square root of the number on the right, not that number itself.

Worked example

Write $x^2 + y^2 - 6x + 4y - 12 = 0$ in standard form, and give its center and radius.

Read. The equation is in general form. Complete the square in x and in y to reach standard form, then read the center and radius.

Plan.

1. Group the x-terms and y-terms and move the constant to the right.
2. For each group, add the square of half its coefficient to both sides.
3. Rewrite each group as a perfect square.
4. Read the center with flipped signs and the radius as the square root.

Solve.

$$x^2 - 6x + y^2 + 4y = 12 \qquad \text{group terms and move the constant to the right}$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4 \qquad \text{add (half of each coefficient) squared to both sides}$$
$$(x - 3)^2 + (y + 2)^2 = 25 \qquad \text{write each group as a perfect square}$$

Check. Half of -6 is -3 and $(-3)^2 = 9$; half of 4 is 2 and $2^2 = 4$; the right side becomes 25, so the radius is $\sqrt{25} = 5$ and the center, flipping the signs, is (3, -2).

Answer: standard form $(x - 3)^2 + (y + 2)^2 = 25$; center (3, -2), radius 5.

Approach Read · Plan · Solve · Check

Read. Decide whether the equation is already in standard form (just read the center and radius) or in general form (complete the square first).

Plan.

1. Group the x-terms together and the y-terms together.
2. For each group, add the square of half its linear coefficient, adding the same amount to the other side.
3. Rewrite each group as a perfect square and read off the center and radius.

Solve.

1. Complete both squares, keeping the equation balanced.
2. Read the center with flipped signs and take the square root for the radius.

Check. Check that you halved each coefficient before squaring, added the new constants to both sides, flipped the signs for the center, and took the square root of the right side for the radius.

Common traps — every wrong answer is one of these

Halve-then-square mistake.
Adds the wrong constant when completing a square — for example, the square of the whole

coefficient instead of the square of half of it. Take half of the linear coefficient first, then square.

Balance mistake.
Adds the completing constants to only one side, or mishandles the constant term, so the right-hand side comes out wrong. Whatever you add on the left you must also add on the right.

Center-sign or radius mistake.
Reads the center with the wrong signs, or takes the right-hand side as the radius instead of its square root.

Practice

1. *You are completing the square on the x-terms $x^2 + 6x$ of a circle's equation.*

What constant should be added?

(A) 36
(B) 6
(C) 9
(D) 3

2. *A circle has equation $(x - 2)^2 + (y + 3)^2 = 16$.*

What is its center?

(A) (-2, 3)
(B) (2, 3)
(C) (2, -3)
(D) (-2, -3)

3. *A circle has equation $(x - 1)^2 + (y - 4)^2 = 25$.*

What is its radius?

(A) 12.5
(B) 25
(C) 625
(D) 5

4. *Convert $x^2 + y^2 - 4x + 6y + 9 = 0$ to standard form.*

Which is the result?

(A) $(x - 2)^2 + (y + 3)^2 = 4$
(B) $(x - 2)^2 + (y + 3)^2 = 13$

(C) $(x - 2)^2 + (y + 3)^2 = 22$

(D) $(x - 2)^2 + (y + 3)^2 = -9$

5. *Consider the circle $x^2 + y^2 + 8x - 2y + 8 = 0$.*

What are its center and radius?

(A) Center (-4, 1), radius 3

(B) Center (-8, 2), radius 3

(C) Center (-4, 1), radius 9

(D) Center (4, -1), radius 3

Answer Key — Geometry & Trigonometry

Angles & lines — Q1
Correct: **(B)**. *Traps:* **(A)** Wrong relationship or setup; **(C)** Wrong relationship or setup; **(D)** Complementary and supplementary mixed up.

Angles & lines — Q2
Correct: **(A)**. *Traps:* **(B)** Wrong relationship or setup; **(C)** Wrong relationship or setup; **(D)** Complementary and supplementary mixed up.

Angles & lines — Q3
Correct: **(A)**. *Traps:* **(B)** Wrong relationship or setup; **(C)** Wrong parallel-line angle pair; **(D)** Complementary and supplementary mixed up.

Angles & lines — Q4
Correct: **(B)**. *Traps:* **(A)** Wrong relationship or setup; **(C)** Wrong parallel-line angle pair; **(D)** Complementary and supplementary mixed up.

Angles & lines — Q5
Correct: **(A)**. *Traps:* **(B)** Wrong relationship or setup; **(C)** Complementary and supplementary mixed up; **(D)** Wrong relationship or setup.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q1
Correct: **(B)**. *Traps:* **(A)** Angle-sum mistake; **(C)** Isosceles base-angle mistake; **(D)** Angle-sum mistake.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q2
Correct: **(A)**. *Traps:* **(B)** Isosceles base-angle mistake; **(C)** Angle-sum mistake; **(D)** Isosceles base-angle mistake.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q3
Correct: **(B)**. *Traps:* **(A)** Angle-sum mistake; **(C)** Exterior-angle mistake; **(D)** Exterior-angle mistake.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q4
Correct: **(D)**. *Traps:* **(A)** Triangle-inequality mistake; **(B)** Triangle-inequality mistake; **(C)** Triangle-inequality mistake.

Triangles (angle sum, isosceles, exterior angle, inequality) — Q5
Correct: **(B)**. *Traps:* **(A)** Angle-sum mistake; **(C)** Angle-sum mistake; **(D)** Angle-sum mistake.

Right triangles & the Pythagorean theorem — Q1
Correct: **(B)**. *Traps:* **(A)** Skipped a square or the square root; **(C)** Skipped a square or the square root; **(D)** Skipped a square or the square root.

Right triangles & the Pythagorean theorem — Q2
Correct: **(D)**. *Traps:* **(A)** Skipped a square or the square root; **(B)** Hypotenuse and leg mixed up; **(C)** Skipped a square or the square root.

Right triangles & the Pythagorean theorem — Q3
Correct: **(A)**. *Traps:* **(B)** Skipped a square or the square root; **(C)** Skipped a square or the square root; **(D)** Hypotenuse and leg mixed up.

Right triangles & the Pythagorean theorem — Q4
Correct: **(A)**. *Traps:* **(B)** Wrong special-triangle ratio; **(C)** Skipped a square or the square root; **(D)** Wrong special-triangle ratio.

Right triangles & the Pythagorean theorem — Q5
Correct: **(C)**. *Traps:* **(A)** Wrong special-triangle ratio; **(B)** Wrong special-triangle ratio; **(D)** Wrong special-triangle ratio.

Similarity & congruence — Q1
Correct: **(D)**. *Traps:* **(A)** Added instead of multiplied by the scale factor; **(B)** Mismatched corresponding sides; **(C)** Mismatched corresponding sides.

Similarity & congruence — Q2
Correct: **(D)**. *Traps:* **(A)** Added instead of multiplied by the scale factor; **(B)** Mismatched corresponding sides; **(C)** Mismatched corresponding sides.

Similarity & congruence — Q3
Correct: **(D)**. *Traps:* **(A)** Mismatched corresponding sides; **(B)** Mismatched corresponding sides; **(C)** Added instead of multiplied by the scale factor.

Similarity & congruence — Q4
Correct: **(C)**. *Traps:* **(A)** Wrong power for area; **(B)** Wrong power for area; **(D)** Wrong power for area.

Similarity & congruence — Q5
Correct: **(C)**. *Traps:* **(A)** Wrong power for area; **(B)** Mismatched corresponding sides; **(D)** Wrong power for area.

Area & perimeter — Q1
Correct: **(A)**. *Traps:* **(B)** Area and perimeter mixed up; **(C)** Used a slanted side as the height; **(D)** Area and perimeter mixed up.

Area & perimeter — Q2
Correct: **(A)**. *Traps:* **(B)** Used a slanted side as the height; **(C)** Used a slanted side as the height; **(D)** Forgot the one half for a triangle.

Area & perimeter — Q3
Correct: **(B)**. *Traps:* **(A)** Area and perimeter mixed up; **(C)** Used a slanted side as the height; **(D)** Forgot the one half for a triangle.

Area & perimeter — Q4
Correct: **(C)**. *Traps:* **(A)** Composite-figure mistake; **(B)** Composite-figure mistake; **(D)** Composite-figure mistake.

Area & perimeter — Q5
Correct: **(B)**. *Traps:* **(A)** Composite-figure mistake; **(C)** Area and perimeter mixed up; **(D)** Forgot the one half for a triangle.

Circles — Q1
Correct: **(A)**. *Traps:* **(B)** Circumference and area formulas swapped; **(C)** Radius and diameter mixed up; **(D)** Radius and diameter mixed up.

Circles — Q2
Correct: **(A)**. *Traps:* **(B)** Circumference and area formulas swapped; **(C)** Radius and diameter mixed up; **(D)** Circumference and area formulas swapped.

Circles — Q3
Correct: **(A)**. *Traps:* **(B)** Left out the arc or sector fraction; **(C)** Left out the arc or sector fraction; **(D)** Circumference and area formulas swapped.

Circles — Q4
Correct: **(A)**. *Traps:* **(B)** Circle-equation mistake; **(C)** Circle-equation mistake; **(D)** Circle-equation mistake.

Circles — Q5
Correct: **(A)**. *Traps:* **(B)** Radius and diameter mixed up; **(C)** Circle-equation mistake; **(D)** Circle-equation mistake.

Volume & surface area — Q1
Correct: **(B)**. *Traps:* **(A)** Volume and surface area mixed up; **(C)** Volume and surface area mixed up; **(D)** Volume and surface area mixed up.

Volume & surface area — Q2
Correct: **(B)**. *Traps:* **(A)** Volume and surface area mixed up; **(C)** Wrong radius or wrong power; **(D)** Wrong radius or wrong power.

Volume & surface area — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong radius or wrong power; **(B)** Left out the fraction in the formula; **(C)** Wrong radius or wrong power.

Volume & surface area — Q4
Correct: **(D)**. *Traps:* **(A)** Wrong radius or wrong power; **(B)** Left out the fraction in the formula; **(C)** Left out the fraction in the formula.

Volume & surface area — Q5
Correct: **(C)**. *Traps:* **(A)** Volume and surface area mixed up; **(B)** Volume and surface area mixed up; **(D)** Volume and surface area mixed up.

Right-triangle trigonometry — Q1
Correct: **(D)**. *Traps:* **(A)** Wrong or upside-down ratio; **(B)** Wrong or upside-down ratio; **(C)** Wrong or upside-down ratio.

Right-triangle trigonometry — Q2
Correct: **(B)**. *Traps:* **(A)** Wrong or upside-down ratio; **(C)** Opposite and adjacent mislabeled; **(D)** Wrong or upside-down ratio.

Right-triangle trigonometry — Q3
Correct: **(D)**. *Traps:* **(A)** Wrong or upside-down ratio; **(B)** Wrong or upside-down ratio; **(C)** Opposite and adjacent mislabeled.

Right-triangle trigonometry — Q4
Correct: **(C)**. *Traps:* **(A)** Cofunction mistake; **(B)** Wrong or upside-down ratio; **(D)** Cofunction mistake.

Right-triangle trigonometry — Q5
Correct: **(B)**. *Traps:* **(A)** Wrong or upside-down ratio; **(C)** Wrong or upside-down ratio; **(D)** Opposite and adjacent mislabeled.

Radian measure & trigonometric relationships — Q1
Correct: **(C)**. *Traps:* **(A)** Degree-radian conversion mistake; **(B)** Degree-radian conversion mistake; **(D)** Degree-radian conversion mistake.

Radian measure & trigonometric relationships — Q2
Correct: **(B)**. *Traps:* **(A)** Degree-radian conversion mistake; **(C)** Degree-radian conversion mistake; **(D)** Degree-radian conversion mistake.

Radian measure & trigonometric relationships — Q3
Correct: **(B)**. *Traps:* **(A)** Complement or cofunction mistake; **(C)** Complement or cofunction mistake; **(D)** Complement or cofunction mistake.

Radian measure & trigonometric relationships — Q4
Correct: **(D)**. *Traps:* **(A)** Pythagorean-identity mistake; **(B)** Pythagorean-identity mistake; **(C)** Pythagorean-identity mistake.

Radian measure & trigonometric relationships — Q5
Correct: **(D)**. *Traps:* **(A)** Arc length in degrees instead of radians; **(B)** Arc length in degrees instead of radians; **(C)** Arc length in degrees instead of radians.

Coordinate geometry: distance & midpoint — Q1
Correct: **(D)**. *Traps:* **(A)** Distance-formula mistake; **(B)** Distance-formula mistake; **(C)** Distance-formula mistake.

Coordinate geometry: distance & midpoint — Q2
Correct: **(A)**. *Traps:* **(B)** Distance and midpoint operations swapped; **(C)** Midpoint mistake; **(D)** Midpoint mistake.

Coordinate geometry: distance & midpoint — Q3
Correct: **(D)**. *Traps:* **(A)** Distance and midpoint operations swapped; **(B)** Distance and midpoint operations swapped; **(C)** Distance and midpoint operations swapped.

Coordinate geometry: distance & midpoint — Q4
Correct: **(D)**. *Traps:* **(A)** Sign or pairing mistake; **(B)** Sign or pairing mistake; **(C)** Sign or pairing mistake.

Coordinate geometry: distance & midpoint — Q5
Correct: **(D)**. *Traps:* **(A)** Midpoint mistake; **(B)** Distance-formula mistake; **(C)** Distance-formula mistake.

Circle angles: central, inscribed & arcs — Q1
Correct: **(C)**. *Traps:* **(A)** Arc measure versus central angle confused; **(B)** Arc measure versus central angle confused; **(D)** Arc measure versus central angle confused.

Circle angles: central, inscribed & arcs — Q2
Correct: **(C)**. *Traps:* **(A)** Inscribed-versus-central factor mistake; **(B)** Inscribed-versus-central factor mistake; **(D)** Inscribed-versus-central factor mistake.

Circle angles: central, inscribed & arcs — Q3
Correct: **(D)**. *Traps:* **(A)** Inscribed-versus-central factor mistake; **(B)** Inscribed-versus-central factor mistake; **(C)** Inscribed-versus-central factor mistake.

Circle angles: central, inscribed & arcs — Q4
Correct: **(A)**. *Traps:* **(B)** Semicircle mistake; **(C)** Semicircle mistake; **(D)** Semicircle mistake.

Circle angles: central, inscribed & arcs — Q5
Correct: **(B)**. *Traps:* **(A)** Inscribed-versus-central factor mistake; **(C)** Inscribed-versus-central factor mistake; **(D)** Arc measure versus central angle confused.

Scale factor: length, area & volume — Q1
Correct: **(A)**. *Traps:* **(B)** Used the plain factor for area or volume; **(C)** Square and cube swapped; **(D)** Used the plain factor for area or volume.

Scale factor: length, area & volume — Q2
Correct: **(D)**. *Traps:* **(A)** Square and cube swapped; **(B)** Used the plain factor for area or volume; **(C)** Square and cube swapped.

Scale factor: length, area & volume — Q3
Correct: **(C)**. *Traps:* **(A)** Square and cube swapped; **(B)** Used the plain factor for area or volume; **(D)** Used the plain factor for area or volume.

Scale factor: length, area & volume — Q4
Correct: **(A)**. *Traps:* **(B)** Reverse-scale mistake; **(C)** Reverse-scale mistake; **(D)** Reverse-scale mistake.

Scale factor: length, area & volume — Q5
Correct: **(B)**. *Traps:* **(A)** Reverse-scale mistake; **(C)** Reverse-scale mistake; **(D)** Square and cube swapped.

Circle equations: center & radius by completing the square — Q1
Correct: **(C)**. *Traps:* **(A)** Halve-then-square mistake; **(B)** Halve-then-square mistake; **(D)** Halve-then-square mistake.

Circle equations: center & radius by completing the square — Q2
Correct: **(C)**. *Traps:* **(A)** Center-sign or radius mistake; **(B)** Center-sign or radius mistake; **(D)** Center-sign or radius mistake.

Circle equations: center & radius by completing the square — Q3
Correct: **(D)**. *Traps:* **(A)** Center-sign or radius mistake; **(B)** Center-sign or radius mistake; **(C)** Center-sign or radius mistake.

Circle equations: center & radius by completing the square — Q4
Correct: **(A)**. *Traps:* **(B)** Balance mistake; **(C)** Balance mistake; **(D)** Balance mistake.

Circle equations: center & radius by completing the square — Q5
Correct: **(A)**. *Traps:* **(B)** Halve-then-square mistake; **(C)** Center-sign or radius mistake; **(D)** Center-sign or radius mistake.

DOMAIN 5

Student-Produced Response

The points you lose without missing the math

"Take care of the sense, and the sounds will take care of themselves."
— Lewis Carroll

Cross-cutting test mechanics. About one question in four is a grid-in (no choices): the answer-entry rules that quietly cost points.

SPR answer entry: format & symbols

node: sat.spr.S1

The skill. On grid-in (student-produced response) questions, your answer must be typed in an accepted form, or it is marked wrong even when the math is right. The field holds up to 5 characters, or 6 if there is a negative sign. Use no symbols — no dollar sign, comma, or percent sign. Enter a mixed number as an improper fraction or a decimal (three and a half becomes 7/2 or 3.5); typing "3 1/2" is read as the fraction 31/2. If a fraction is too long to fit, enter its decimal instead. If a decimal is too long, fill the grid by keeping digits through the fourth place, either cut off or rounded (2/3 becomes .6666 or .6667, with or without the leading zero, never .66). Enter a percent as its decimal (45 percent becomes .45).

Worked example

A problem's answer works out to two and three-quarters. How should it be entered on the grid?

Read. The computed value is two and three-quarters, a mixed number. The grid does not accept a space between the whole number and the fraction, so the mixed number must be converted.

Plan.

1. Turn the mixed number into an improper fraction, or into a decimal.
2. Enter that form with no space and no extra symbols.

Solve.

$$2\frac{3}{4} = \frac{2\times 4+3}{4} \qquad \text{convert the mixed number to an improper fraction}$$
$$= \frac{11}{4} \qquad 8 \text{ plus } 3 \text{ is } 11, \text{ over } 4$$
$$= 2.75 \qquad \text{or divide } 11 \text{ by } 4 \text{ for the decimal form}$$

Check. Both 11/4 and 2.75 fit the field and use only accepted characters; "2 3/4" would be misread as 23/4.

Answer: enter 11/4 or 2.75.

Approach Read · Plan · Solve · Check

Read. Once you have the value, decide its form — whole number, fraction, or decimal — and whether it fits the field.

Plan.

1. Turn any mixed number into an improper fraction or a decimal.
2. Remove every symbol: dollar sign, comma, percent sign.
3. If a fraction will not fit, enter its decimal instead.
4. If a decimal will not fit, keep digits through the fourth place, cut off or rounded.
5. Write a percent as its decimal.

Solve.

1. Type only accepted characters.
2. Count the characters to stay within the limit.

Check. No symbols? Mixed number converted? A long decimal filled to four places? Percent written as a decimal? Within 5 characters, or 6 with a sign?

Common traps — every wrong answer is one of these

mixed-number entry error.
Enters a mixed number literally, like "3 1/2" or "31/2," instead of the improper fraction 7/2 or the decimal 3.5. The grid reads "3 1/2" as the fraction 31/2.

symbol inclusion error.
Includes a symbol the grid does not accept, such as a dollar sign, comma, or percent sign — like $2.53 or 68,132.

decimal precision error.
1. Has a value like 2/3 that does not fit the grid as a decimal.
2. Enters only .66 or .67, too few digits, so the grid is under-filled.
3. Fix: keep digits through the fourth place, .6666 or .6667.

percent-not-converted error.
Enters a percent answer as the percent itself rather than its decimal — 45 percent or 45 instead

of .45 — or shifts the decimal point wrongly (4.5).

Practice

1. *You solve a problem and get the value 3 and one half.*

Which is an accepted way to enter it?

(A) 31/2
(B) 3-1/2
(C) 3 1/2
(D) 7/2

2. *A problem's answer is the number 68,132.*

Which is an accepted way to enter it?

(A) 68132
(B) 68.132
(C) 68,132
(D) 68 132

3. *Your answer is 2/3 and you choose to enter it as a decimal.*

Which is an accepted entry?

(A) .66
(B) .67
(C) .6667
(D) 0.7

4. *A problem's answer is 45%.*

Which is an accepted way to enter it?

(A) 45%
(B) 45
(C) .45
(D) 4.5

5. *Your answer is 7/4.*

Which is an accepted entry?

(A) 1 3/4

(B) 7/4

(C) 1.7

(D) 13/4

Entering negatives & decimals

node: sat.spr.S2

The skill. On grid-in questions you may enter a leading minus sign and a decimal point, and both matter. For a negative decimal like -5.2, keep the minus sign, use a decimal point (not a space) between the whole and fractional parts, and include the full fractional part — do not round it off. Dropping the sign, using a space, or cutting the decimal short each change the value.

Worked example

A problem's answer is -3.75. How should it be entered on the grid?

Read. The value is negative three point seven five. Both the minus sign and the decimal point must appear in the entry.

Plan.

1. Write the minus sign first.
2. Follow with the digits, using a decimal point, not a space.

Solve.

value $= -3.75$	the exact answer, sign and all
grid entry = -3.75	minus sign, then 3, decimal point, 7, 5

Check. Dropping the sign gives 3.75 (wrong value), "-3 75" is unreadable, and "-3" loses the .75; only -3.75 is right.

Answer: enter -3.75.

Approach Read · Plan · Solve · Check

Read. Check whether the value is negative, a decimal, or both.

Plan.

1. Keep the leading minus sign for a negative value.
2. Use a decimal point, not a space, before the fractional part.
3. Include the whole fractional part.

Solve. Enter it in the form -a.b.

Check. Minus sign kept? Decimal point instead of a space? Full value with no rounding?

Common traps — every wrong answer is one of these

dropped the sign.
Leaves off the negative sign, turning a negative value into a positive one.

space not decimal.
Types a space where the decimal point belongs, so the grid cannot read the decimal.

dropped the decimal part.
Rounds off or drops the fractional part, entering a whole number instead of the exact decimal.

Practice

1. *Your grid-in answer is the value negative 5 point 2 (that is, -5.2). Which is an accepted SPR entry?*

   Which is correct?

   (A) -5 2
   (B) -5
   (C) -5.2
   (D) 5.2

2. *Your grid-in answer is the value negative 7 point 2 (that is, -7.2). Which is an accepted SPR entry?*

   Which is correct?

   (A) -7 2
   (B) -7.2
   (C) -7
   (D) 7.2

3. *Your grid-in answer is the value negative 5 point 8 (that is, -5.8). Which is an accepted SPR entry?*

   Which is correct?

   (A) 5.8
   (B) -5 8
   (C) -5.8
   (D) -5

4. *Your grid-in answer is the value negative 7 point 8 (that is, -7.8). Which is an accepted SPR entry?*

   Which is correct?

(A) -7.8

(B) -7

(C) 7.8

(D) -7 8

5. *Your grid-in answer is -3.75. Which is an accepted SPR entry?*

Which is correct?

(A) -3 75

(B) 3.75

(C) -3.75

(D) -3

Fractions vs decimals: rounding & truncation

node: sat.spr.S3

The skill. When an answer is a repeating decimal, like a number over 3, the safest entry is the exact fraction — a fraction is always accepted. If you use a decimal instead, it must FILL the grid: a short decimal like 0.33 leaves the grid under-filled and is not accepted, while the fraction 1/3 always is. Also write a fraction with a slash (a fraction bar), never a colon, and keep it right side up — do not flip 1/3 into 3/1.

Worked example

A problem's answer is 2/3, whose decimal repeats. How should it be entered on the grid?

Read. The answer is 2/3, and its decimal 0.6666... repeats forever. The grid accepts the exact fraction, which is the safest entry.

Plan.

1. Enter the exact fraction 2/3 with a slash.
2. If a decimal is used instead, fill the grid: .6666 or .6667, not .66.

Solve.

$$\frac{2}{3} = 0.6666\ldots \qquad \text{the decimal repeats and never ends}$$

$$\text{grid entry} = 2/3 \qquad \text{the exact fraction is always accepted}$$

Check. 2/3 is right side up and uses a slash; .66 would be too short, and 2:3 uses the wrong symbol.

Answer: enter 2/3 (or the filled decimal .6666).

Approach Read · Plan · Solve · Check

Read. Is the decimal form of the answer repeating?

Plan.

1. Enter the exact fraction, which is always accepted.
2. Or, if you use a decimal, fill the grid with digits rather than rounding short.

Solve. For a number over 3, enter it as that fraction, like 1/3.

Check. A slash, not a colon? The fraction right side up? Not rounded too short?

Common traps — every wrong answer is one of these

truncated too short.
Rounds a repeating decimal to just two places, like 0.33, which under-fills the grid and is not accepted. Enter the fraction, or fill the grid.

ratio notation.
Uses a colon, like 1:3, instead of a fraction bar. Grid answers use a slash, 1/3.

inverted.
Flips the fraction upside down, entering 3/1 instead of 1/3.

Practice

1. *Your answer is 2/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 2:3
(B) 3/2
(C) 0.66
(D) 2/3

2. *Your answer is 1/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 1/3
(B) 3/1
(C) 0.33
(D) 1:3

3. *Your answer is 1/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 1:3

(B) 1/3

(C) 3/1

(D) 0.33

4. *Your answer is 5/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 5:3

(B) 5/3

(C) 3/5

(D) 1.66

5. *Your answer is 2/3 (a repeating decimal). Which is an accepted SPR entry?*

Which is correct?

(A) 3/2

(B) 2:3

(C) 0.6

(D) 2/3

Converting percents & mixed numbers

node: sat.spr.S4

The skill. A percent answer must be entered as a decimal or a fraction, never with a percent sign. To convert, drop the percent sign and divide by 100 (not by 10): 25 percent = 25/100 = 0.25. So 20 percent is entered as 0.2, and 75 percent as 0.75 or 3/4. Leaving the percent sign in, entering the bare number, or dividing by 10 each give a wrong entry.

Worked example

A problem's answer is 40 percent (as a proportion). How should it be entered on the grid?

Read. The answer is 40 percent, and the grid does not accept a percent sign. It must be entered as a decimal or a fraction.

Plan.

1. Drop the percent sign.
2. Divide 40 by 100 to get the decimal.

Solve.

$$40\% = \frac{40}{100} \qquad \text{percent means out of 100}$$
$$= 0.4 \qquad \text{divide 40 by 100}$$

Check. 0.4 has no percent sign and comes from dividing by 100; entering 40, 40%, or 4 would all be wrong.

Answer: enter 0.4 (or 2/5).

Approach Read · Plan · Solve · Check

Read. Is the answer stated as a percent?

Plan.

1. Drop the percent sign.
2. Divide the number by 100 to get the decimal.

Solve. A percent p becomes p/100.

Check. Percent sign removed? Divided by 100, not 10?

Common traps — every wrong answer is one of these

no conversion.
Enters the bare percent number, like 25, without turning it into a decimal or fraction.

kept the % sign.
Leaves the percent sign in the entry, which the grid does not accept.

divided by 10.
Divides by 10 instead of 100, so 25 percent becomes 2.5 or 25/10 instead of 0.25.

Practice

1. *A problem's answer is 25% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 0.25
(B) 25
(C) 25%
(D) 25/10

2. *A problem's answer is 50% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 50/10
(B) 50%
(C) 0.50
(D) 50

3. *A problem's answer is 75% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 75
(B) 0.75
(C) 75%
(D) 75/10

4. *A problem's answer is 20% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 20%
(B) 0.20
(C) 20
(D) 20/10

5. *A problem's answer is 40% (as a proportion). Which is an accepted SPR entry?*

Which is correct?

(A) 40%
(B) 4
(C) 0.4
(D) 40

Answer Key — Student-Produced Response

SPR answer entry: format & symbols — Q1
Correct: **(D)**. *Traps:* **(A)** mixed-number entry error; **(B)** mixed-number entry error; **(C)** mixed-number entry error.

SPR answer entry: format & symbols — Q2
Correct: **(A)**. *Traps:* **(B)** symbol inclusion error; **(C)** symbol inclusion error; **(D)** symbol inclusion error.

SPR answer entry: format & symbols — Q3
Correct: **(C)**. *Traps:* **(A)** decimal precision error; **(B)** decimal precision error; **(D)** decimal precision error.

SPR answer entry: format & symbols — Q4
Correct: **(C)**. *Traps:* **(A)** percent-not-converted error; **(B)** percent-not-converted error; **(D)** percent-not-converted error.

SPR answer entry: format & symbols — Q5
Correct: **(B)**. *Traps:* **(A)** mixed-number entry error; **(C)** decimal precision error; **(D)** mixed-number entry error.

Entering negatives & decimals — Q1
Correct: **(C)**. *Traps:* **(A)** space not decimal; **(B)** dropped the decimal part; **(D)** dropped the sign.

Entering negatives & decimals — Q2
Correct: **(B)**. *Traps:* **(A)** space not decimal; **(C)** dropped the decimal part; **(D)** dropped the sign.

Entering negatives & decimals — Q3
Correct: **(C)**. *Traps:* **(A)** dropped the sign; **(B)** space not decimal; **(D)** dropped the decimal part.

Entering negatives & decimals — Q4
Correct: **(A)**. *Traps:* **(B)** dropped the decimal part; **(C)** dropped the sign; **(D)** space not decimal.

Entering negatives & decimals — Q5
Correct: **(C)**. *Traps:* **(A)** space not decimal; **(B)** dropped the sign; **(D)** dropped the decimal part.

Fractions vs decimals: rounding & truncation — Q1
Correct: **(D)**. *Traps:* **(A)** ratio notation; **(B)** inverted; **(C)** truncated too short.

Fractions vs decimals: rounding & truncation — Q2
Correct: **(A)**. *Traps:* **(B)** inverted; **(C)** truncated too short; **(D)** ratio notation.

Fractions vs decimals: rounding & truncation — Q3
Correct: **(B)**. *Traps:* **(A)** ratio notation; **(C)** inverted; **(D)** truncated too short.

Fractions vs decimals: rounding & truncation — Q4
Correct: **(B)**. *Traps:* **(A)** ratio notation; **(C)** inverted; **(D)** truncated too short.

Fractions vs decimals: rounding & truncation — Q5
Correct: **(D)**. *Traps:* **(A)** inverted; **(B)** ratio notation; **(C)** truncated too short.

Converting percents & mixed numbers — Q1
Correct: **(A)**. *Traps:* **(B)** no conversion; **(C)** kept the % sign; **(D)** divided by 10.

Converting percents & mixed numbers — Q2
Correct: **(C)**. *Traps:* **(A)** divided by 10; **(B)** kept the % sign; **(D)** no conversion.

Converting percents & mixed numbers — Q3
Correct: **(B)**. *Traps:* **(A)** no conversion; **(C)** kept the % sign; **(D)** divided by 10.

Converting percents & mixed numbers — Q4
Correct: **(B)**. *Traps:* **(A)** kept the % sign; **(C)** no conversion; **(D)** divided by 10.

Converting percents & mixed numbers — Q5
Correct: **(C)**. *Traps:* **(A)** kept the % sign; **(B)** divided by 10; **(D)** no conversion.

Formula Reference

The Digital SAT provides the reference below on *every* math question. Your job is to **select** the right formula, not to recall it — so practice recognizing which one a problem calls for. Two things are **not** provided and must be memorized: **surface-area** formulas and **trigonometry** (SOH-CAH-TOA).

Area & circumference	**Volume**
Circle: $A = \pi r^2,\ C = 2\pi r$	Box: $V = \ell w h$
Rectangle: $A = \ell w$	Cylinder: $V = \pi r^2 h$
Triangle: $A = \frac{1}{2}bh$	Sphere: $V = \frac{4}{3}\pi r^3$
Pythagorean: $a^2 + b^2 = c^2$	Cone: $V = \frac{1}{3}\pi r^2 h$
	Pyramid: $V = \frac{1}{3}\ell w h$
Special right triangles	**Angle measure**
30–60–90: sides $x,\ x\sqrt{3},\ 2x$	Circle: $360^\circ = 2\pi$ radians
45–45–90: sides $s,\ s,\ s\sqrt{2}$	Triangle: angles sum to 180°

Memorize (not on the sheet): surface area (e.g. box $= 2(\ell w + \ell h + wh)$, cylinder $= 2\pi r^2 + 2\pi rh$, sphere $= 4\pi r^2$); and right-triangle trig: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$.